

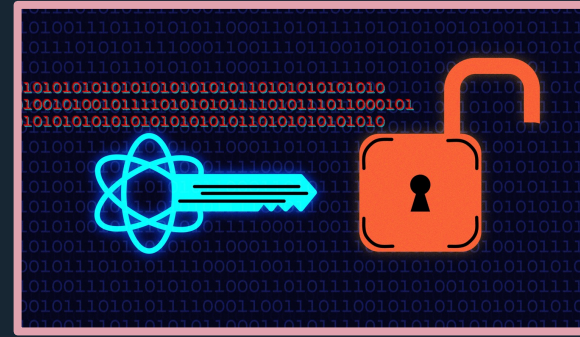
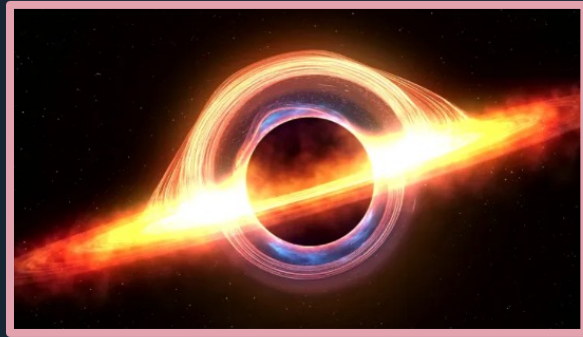
Quantum Commitments and Black Hole Radiation Decoding

Fermi Ma

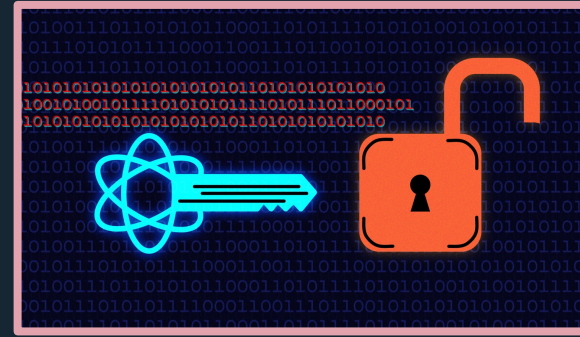
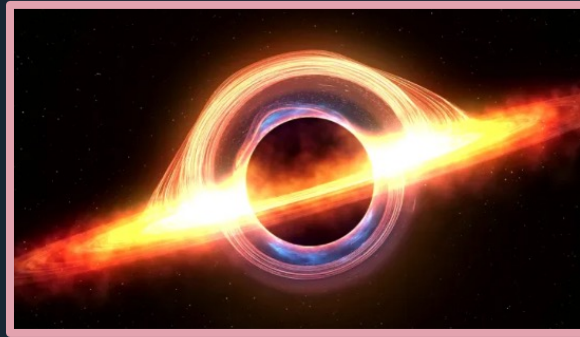
(Simons and Berkeley)

Based on discussions with Sam Gunn (Berkeley)
and Alex Lombardi (Berkeley → Princeton)

Question: What does black hole radiation decoding have to do with quantum cryptography?



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Answer:

Black-Hole Radiation Decoding is Quantum Cryptography

Zvika Brakerski*

Abstract

We propose to study equivalence relations between phenomena in high-energy physics and the existence of standard cryptographic primitives, and show the first example where such an

(building on [Harlow-Hayden13, Aaronson16])

You might be wondering...

“Black-Hole Radiation Decoding is Quantum Cryptography.”

You might be wondering...

1) What does this mean?



“Black-Hole Radiation Decoding is Quantum Cryptography.”

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Goal: understand the title of Zvika's paper

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(3) + (4) is an alternative view of [Brakerski23].

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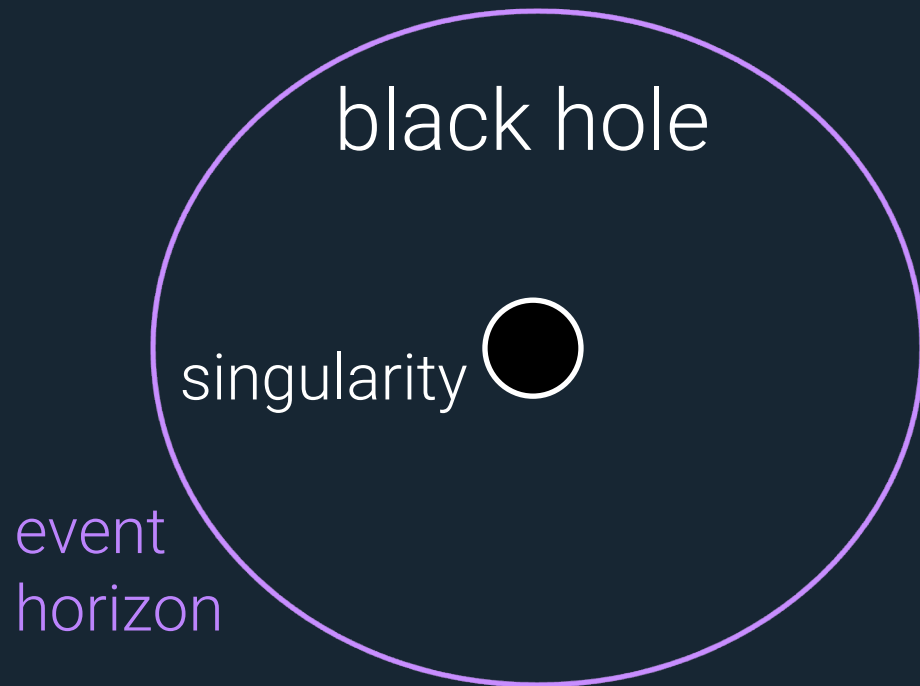
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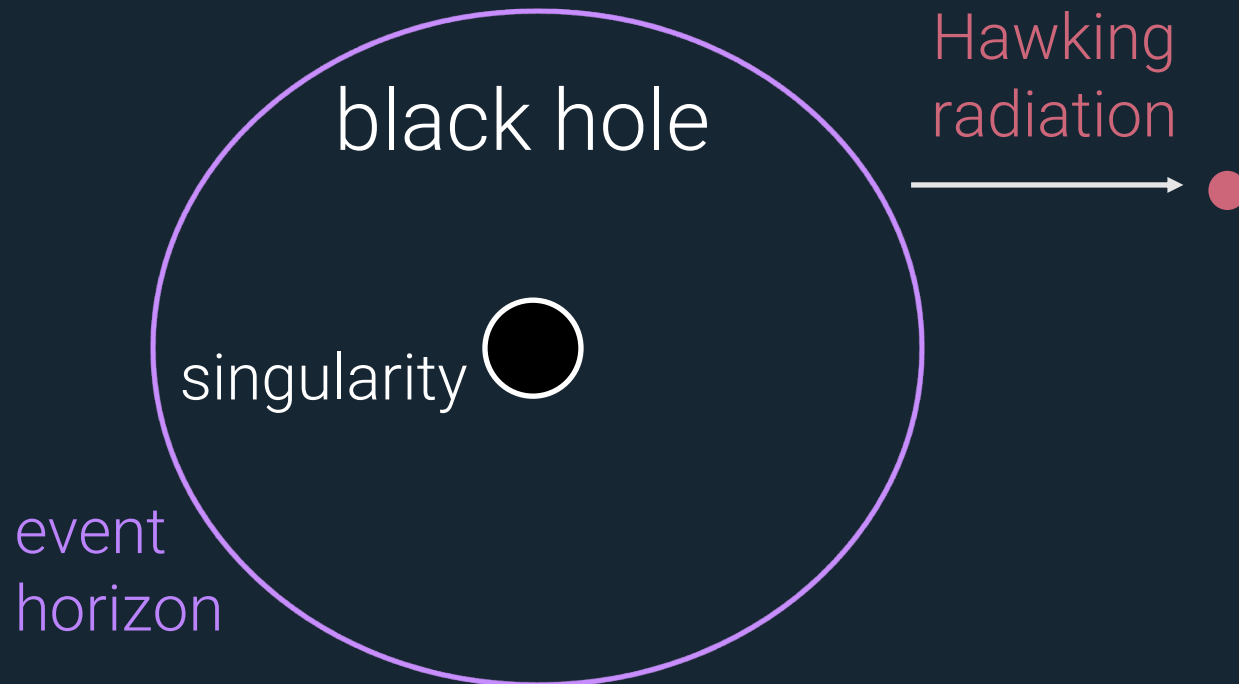
Everything I'm about to say about black hole physics is from Scott Aaronson's Barbados lecture notes (any mistakes are my own).

Black Hole Radiation



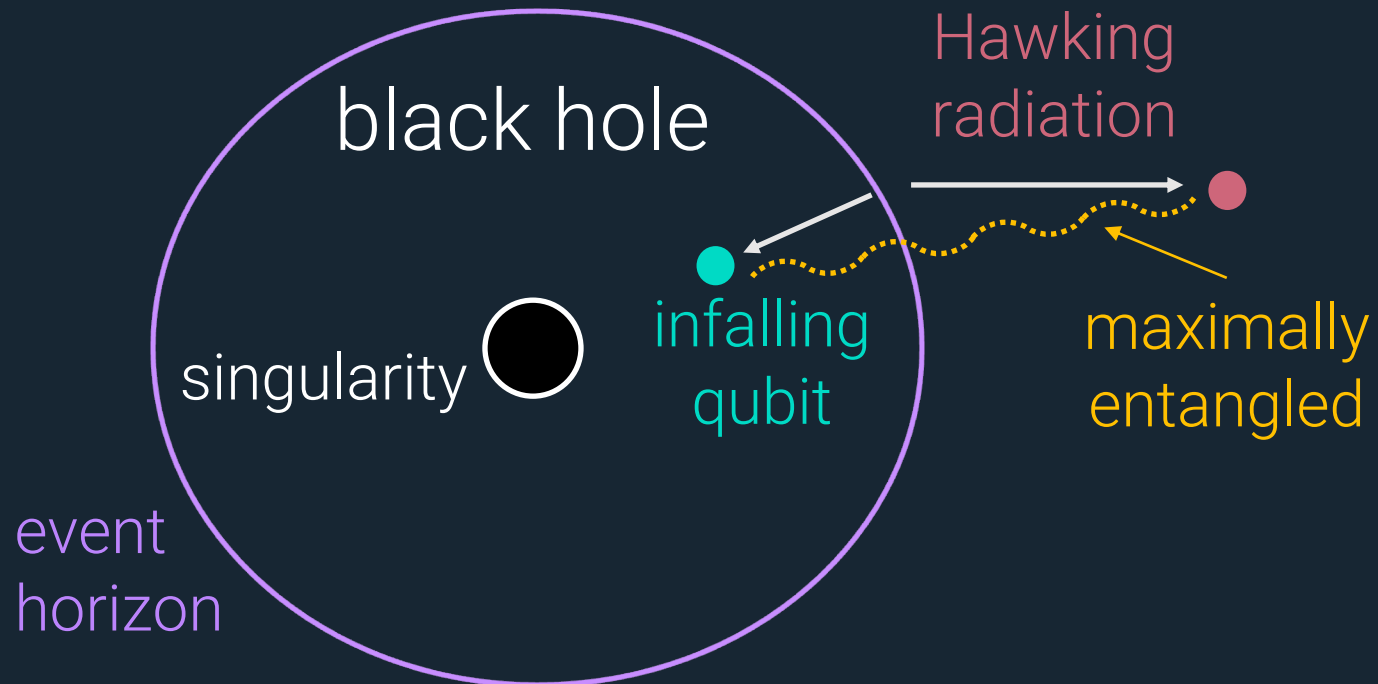
Black Hole Radiation

- Black holes emit qubits of Hawking radiation.



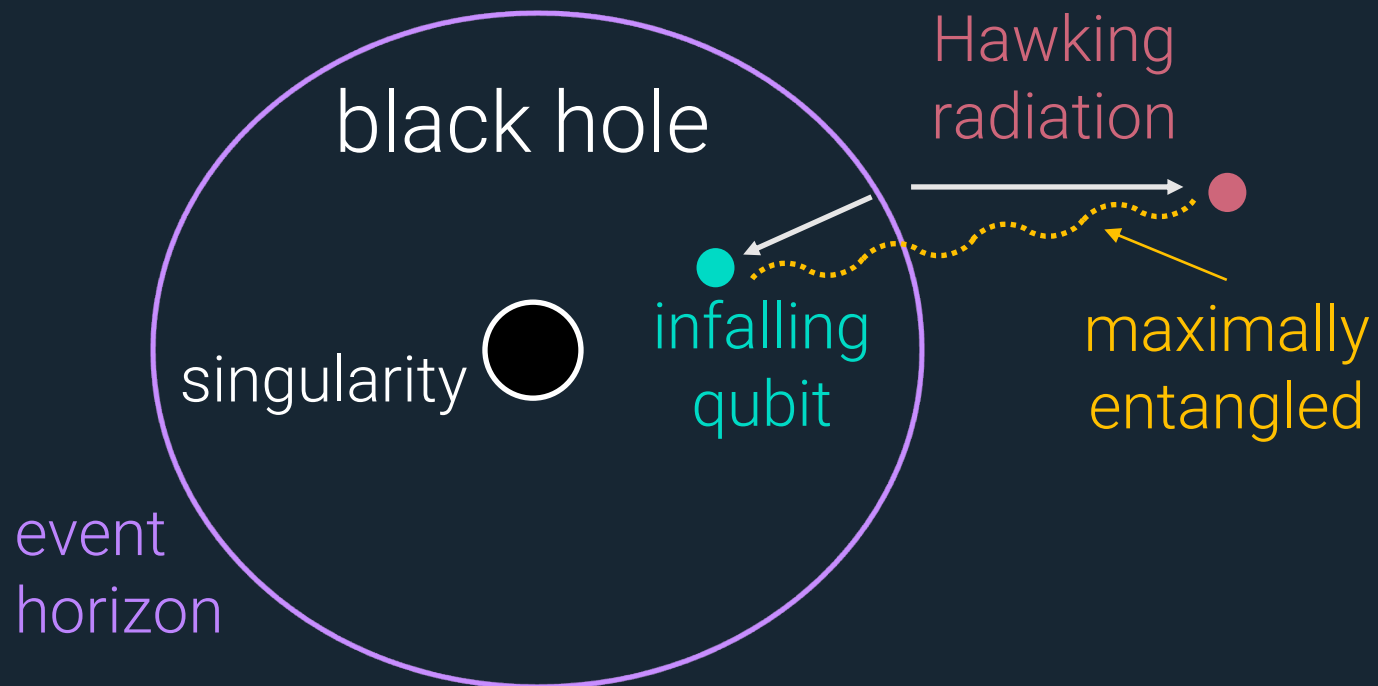
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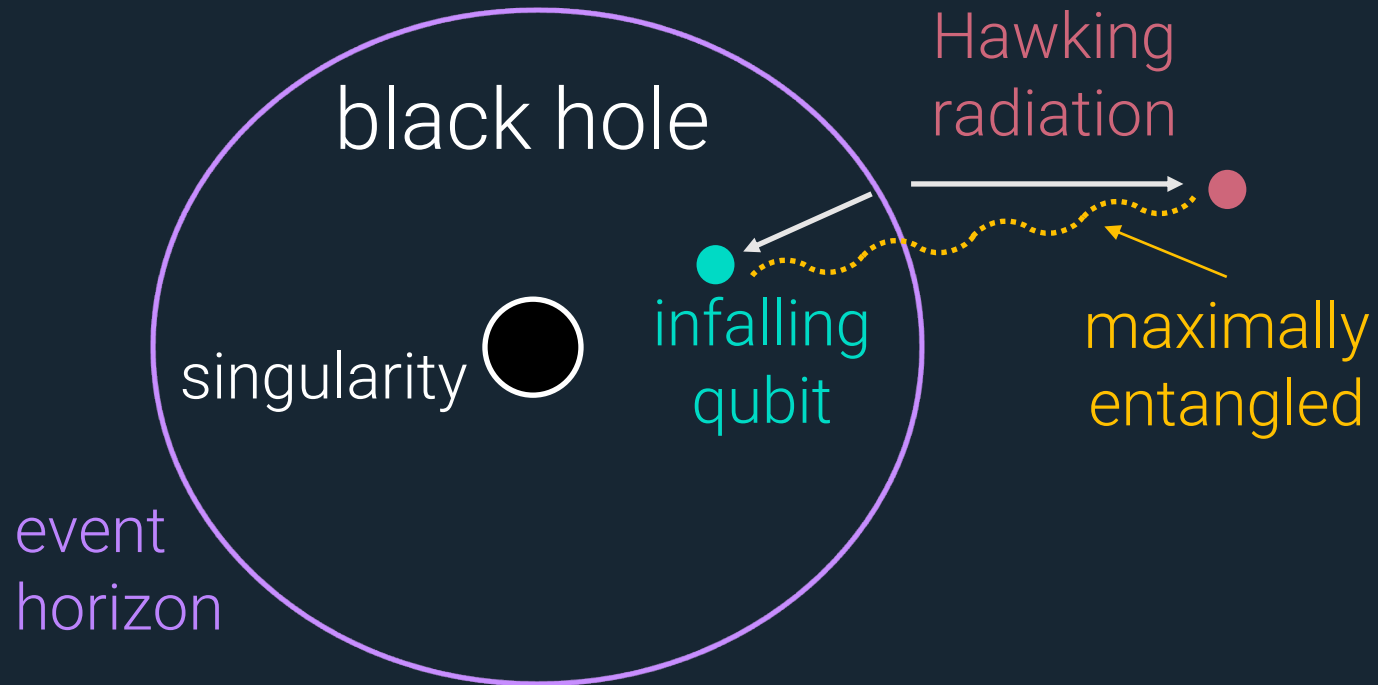


Black Hole Radiation

- Black holes emit qubits of Hawking radiation.
- Each outgoing qubit is maximally entangled with an infalling qubit.
- After long enough, black hole evaporates completely.

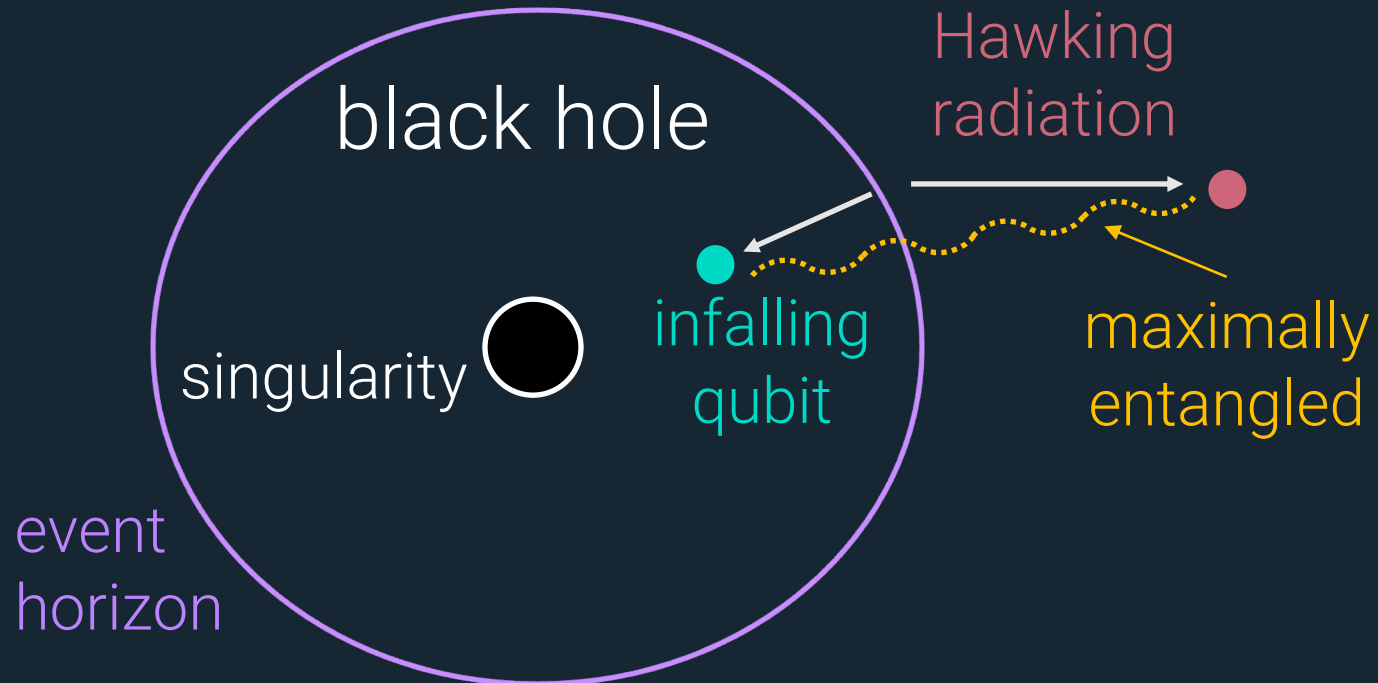


Emitted radiation comes out “scrambled.”



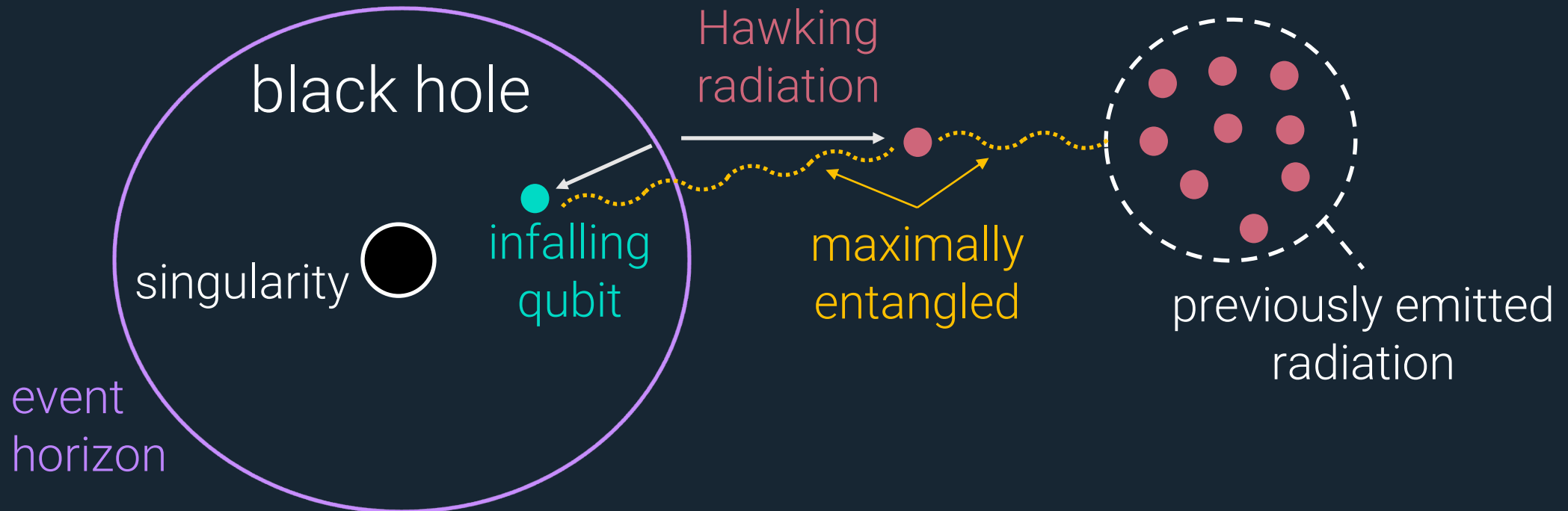
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- Post-evaporation state is a (roughly) a random pure state.



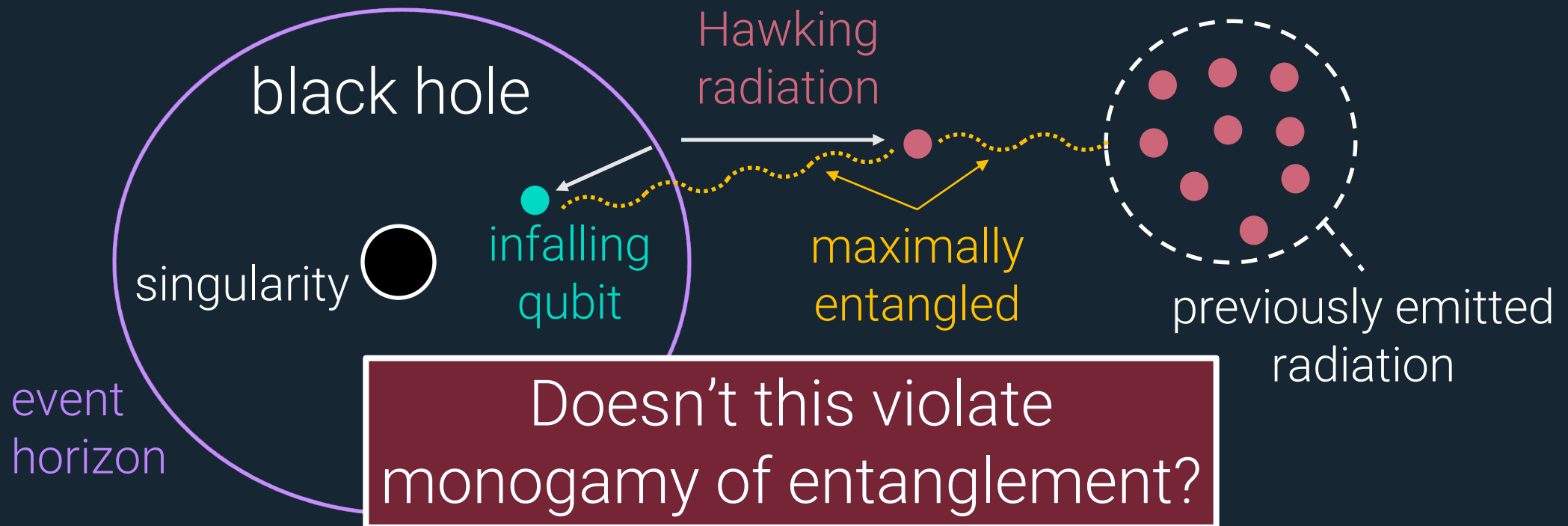
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- Consequence: after $\sim 1/2$ of the black hole has evaporated, outgoing qubits are *maximally entangled* with previously emitted radiation.



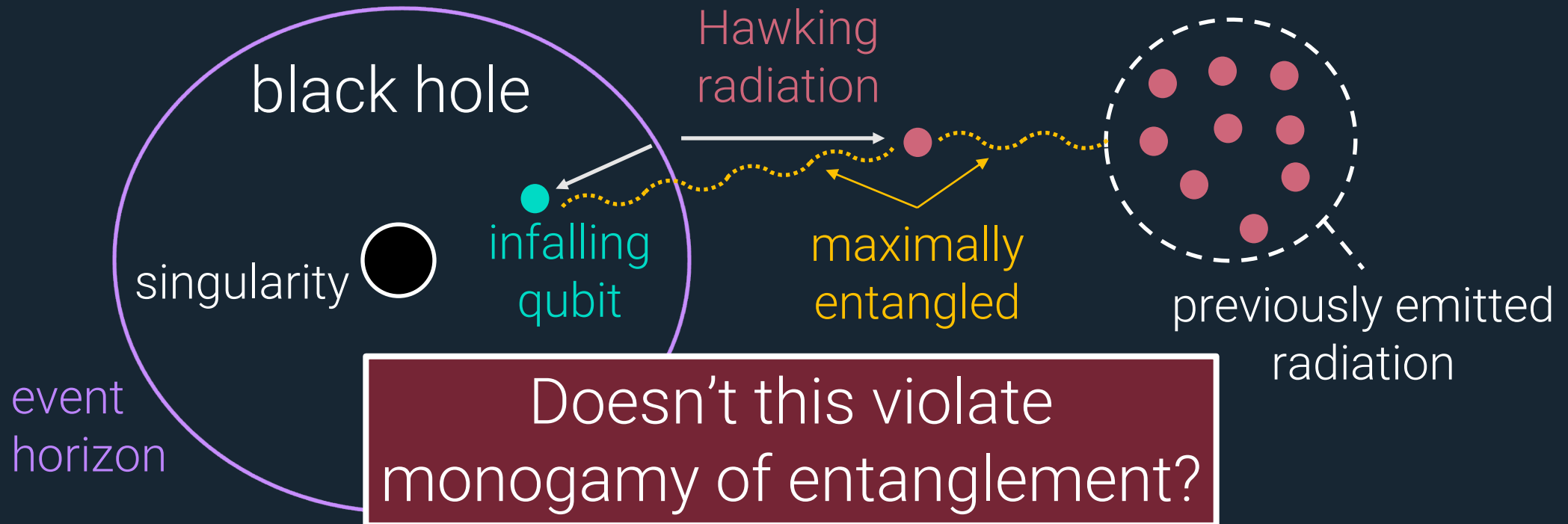
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Black hole complementarity [Susskind-'t Hooft, 90s]

If radiation is maximally entangled with two systems, they're the **same system**.

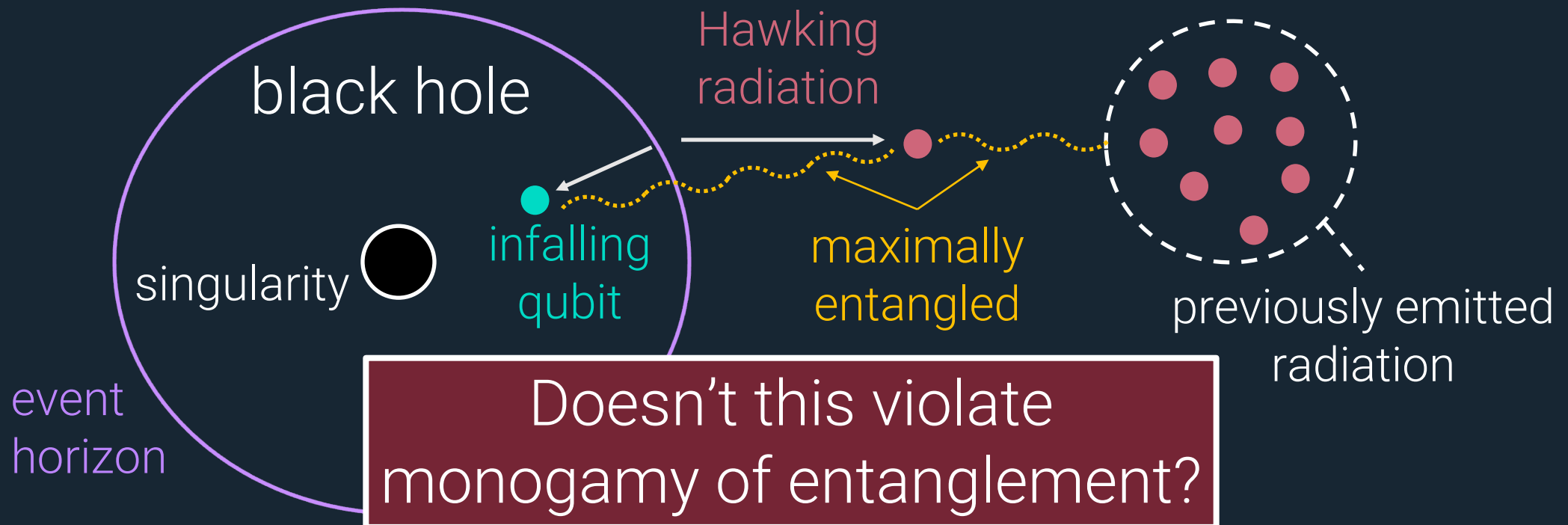


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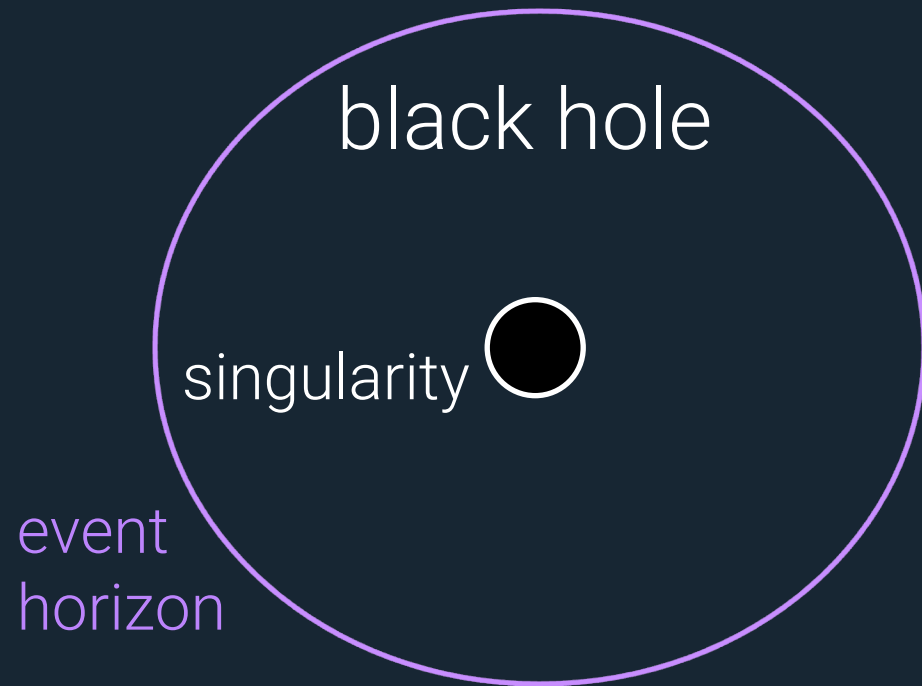
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Firewall paradox [Almheiri-Marolf-Polchinski-Sully, 11]

Thought experiment in which an observer **detects** the monogamy violation.

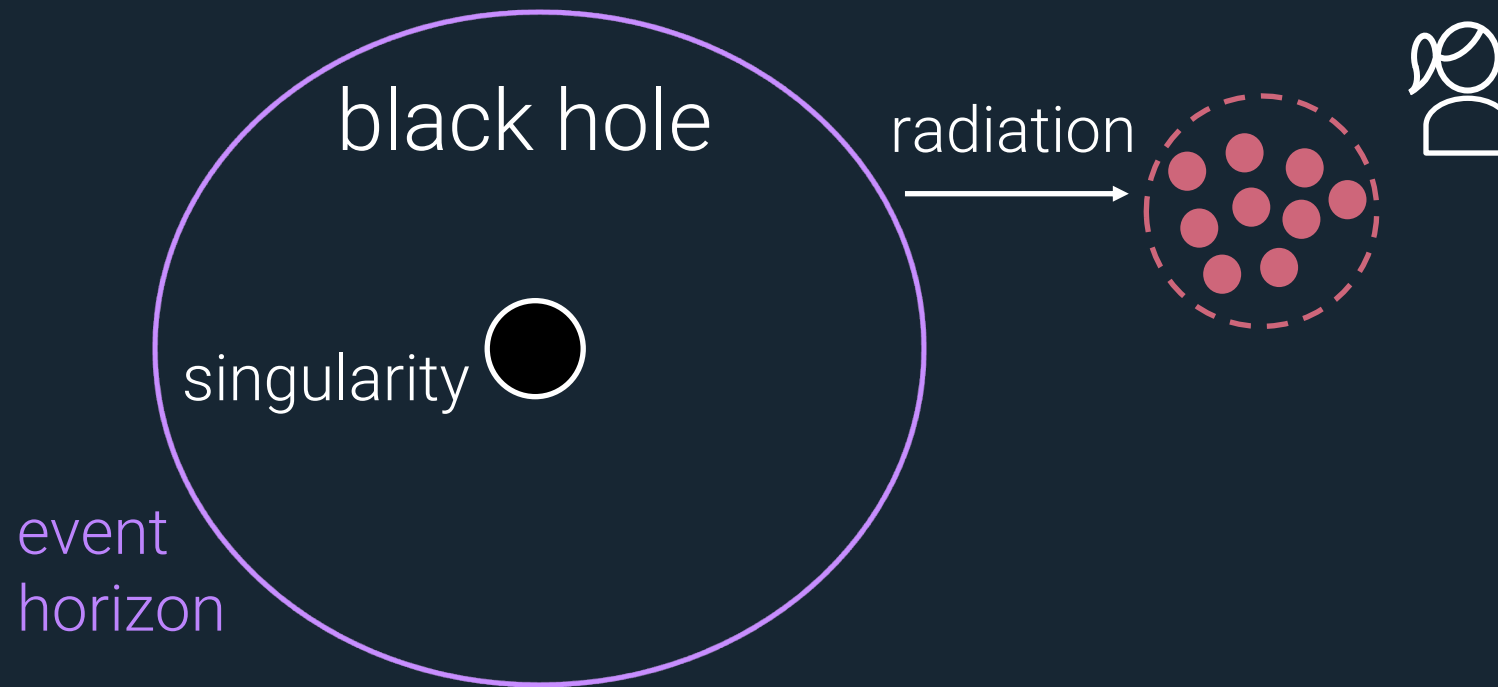


[AMPS11] experiment:



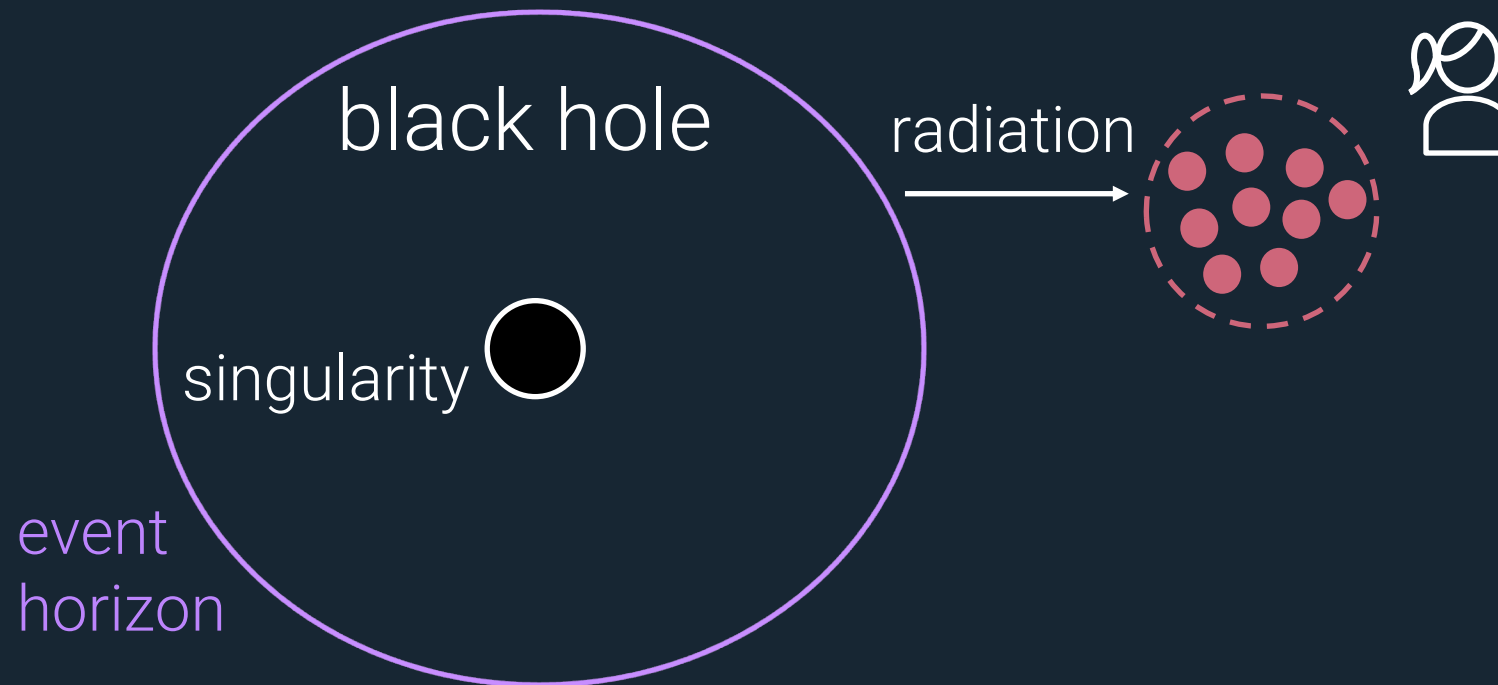
[AMPS11] experiment:

1) Alice collects radiation until $2/3$ of black hole has evaporated.



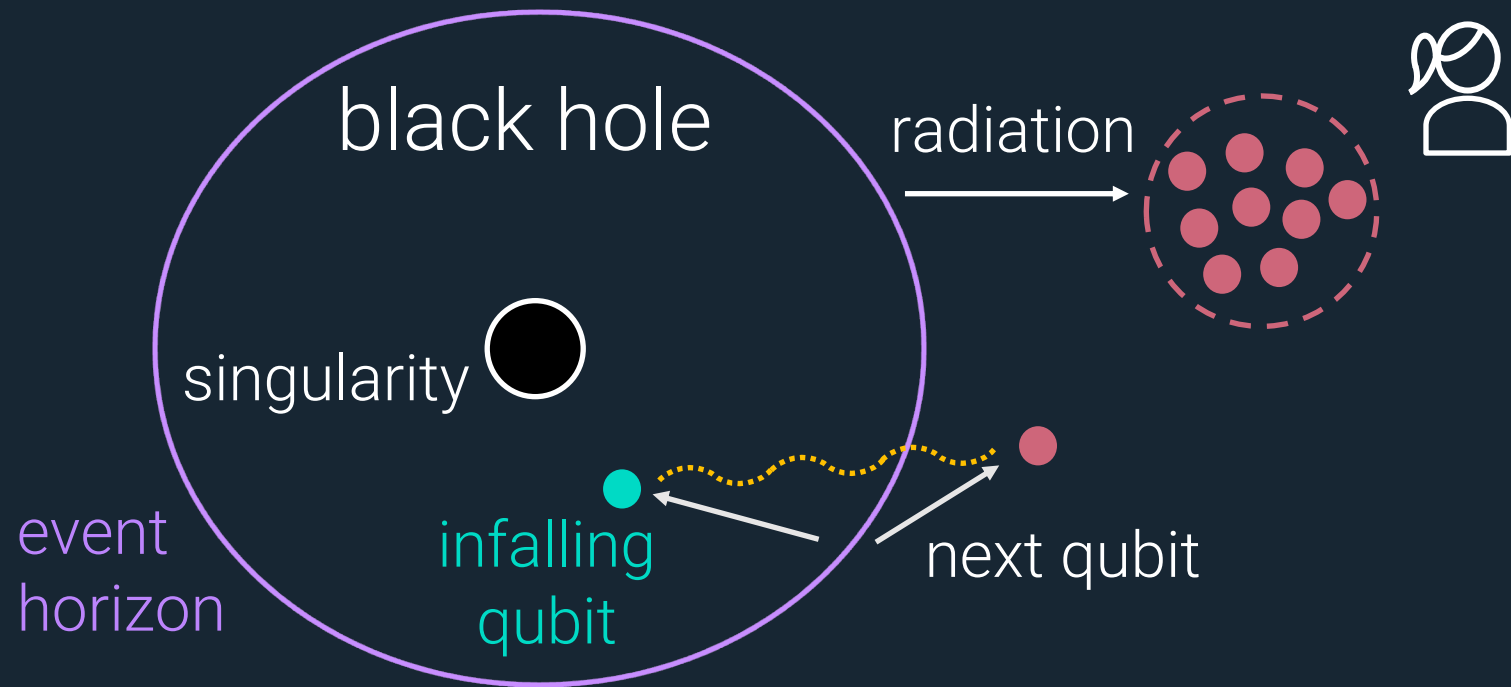
[AMPS11] experiment:

- 1) Alice collects radiation until $2/3$ of black hole has evaporated.
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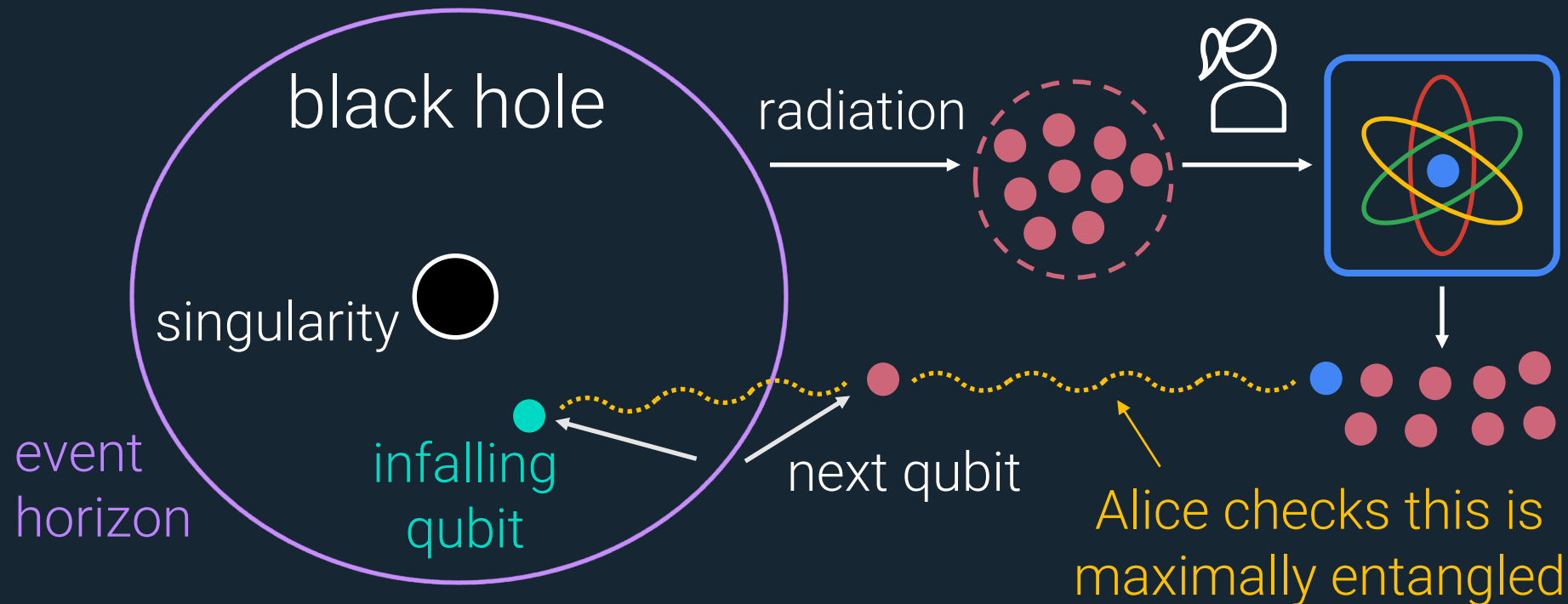
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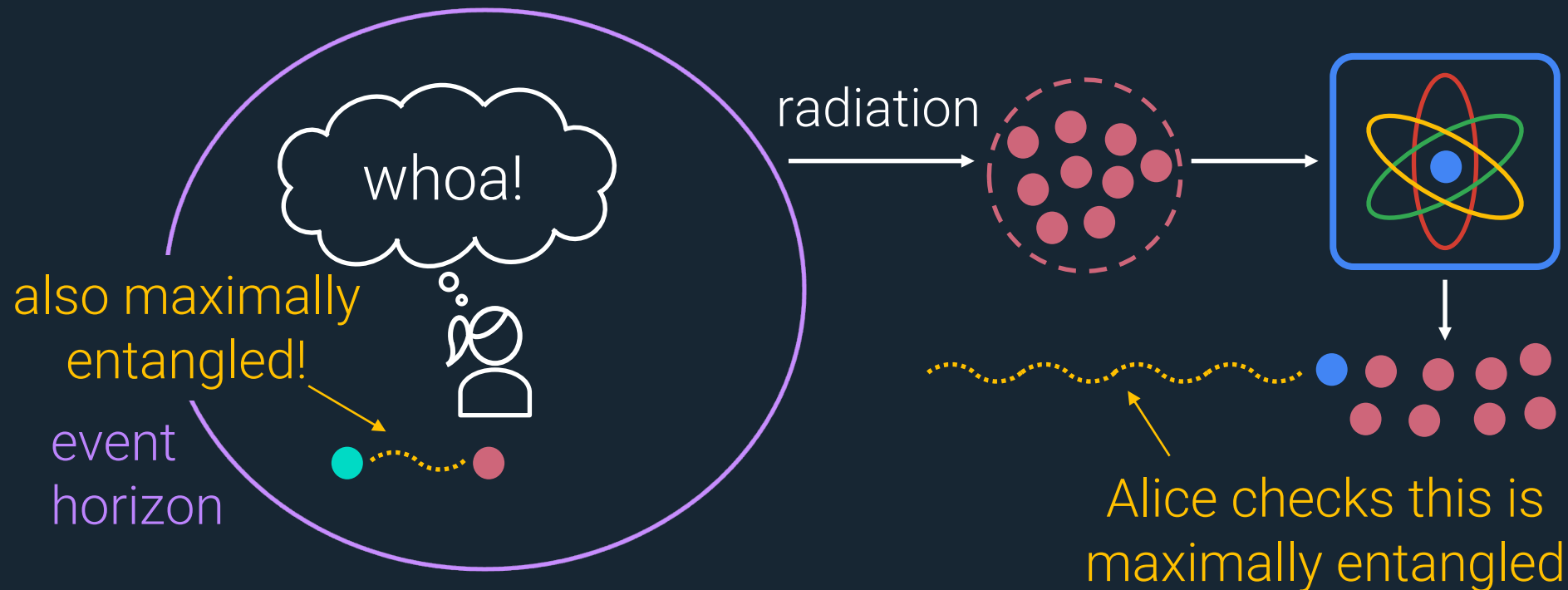
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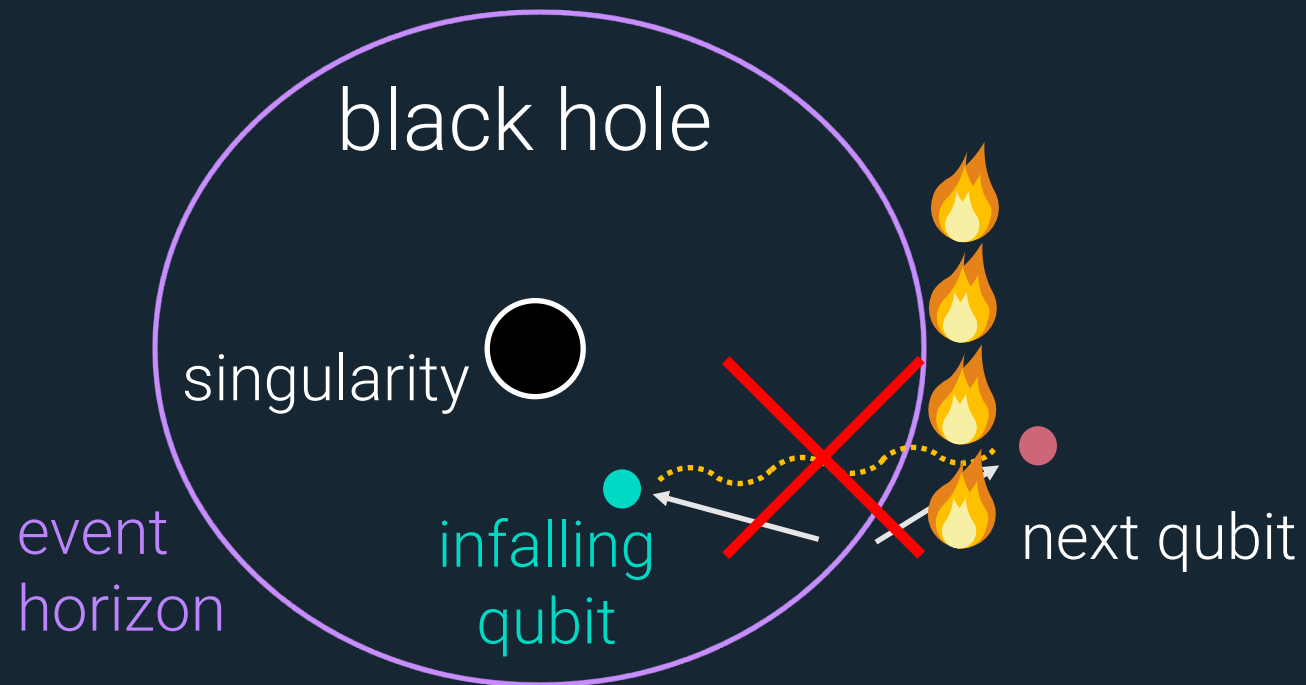
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- 3) Alice jumps into the black hole.



AMPS11 proposed resolution:

“Firewall” outside event horizon (breaking entanglement)



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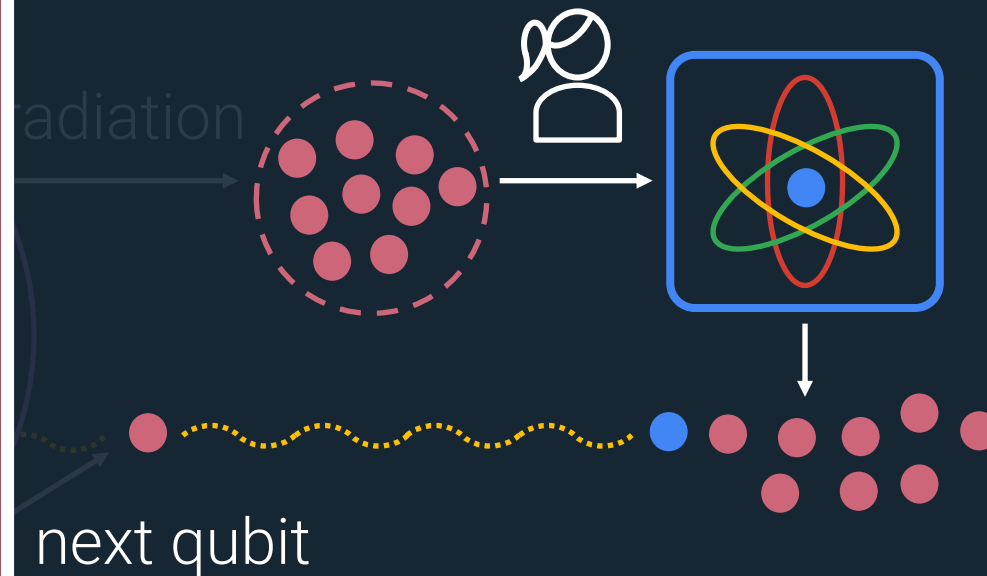
Very cool and surprising!!

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[Harlow-Hayden 2013]

Under certain cryptographic assumptions, this step can require **exponential** time.



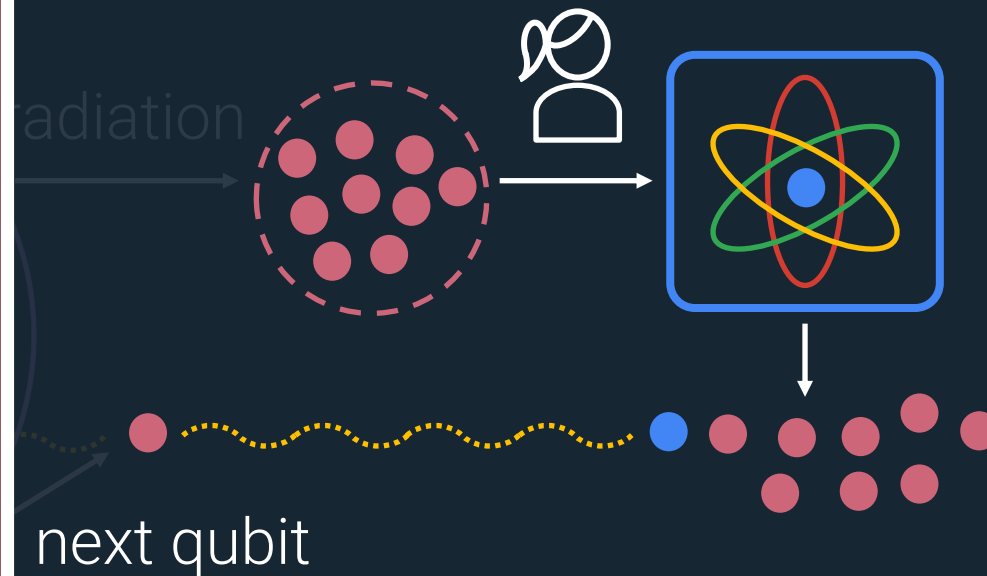
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By the time she's done decoding, the black hole will have evaporated!



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(2) Radiation decoding problem [Harlow-Hayden13]

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The Radiation Decoding Problem [HH13]

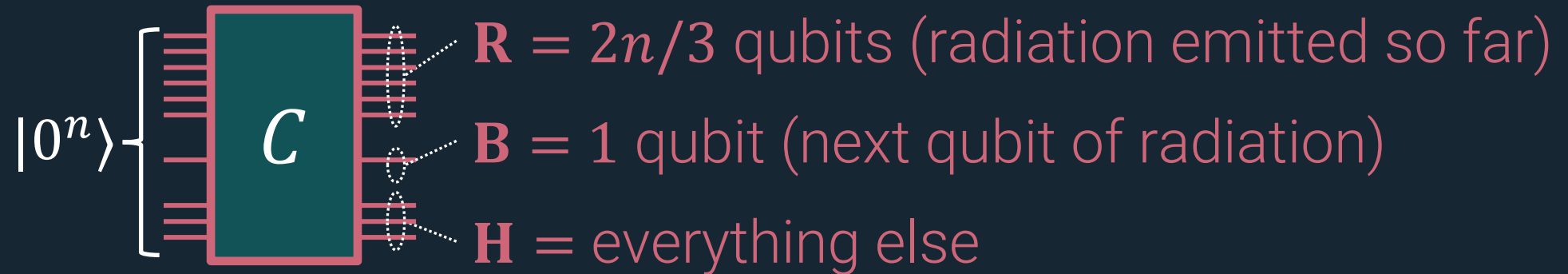
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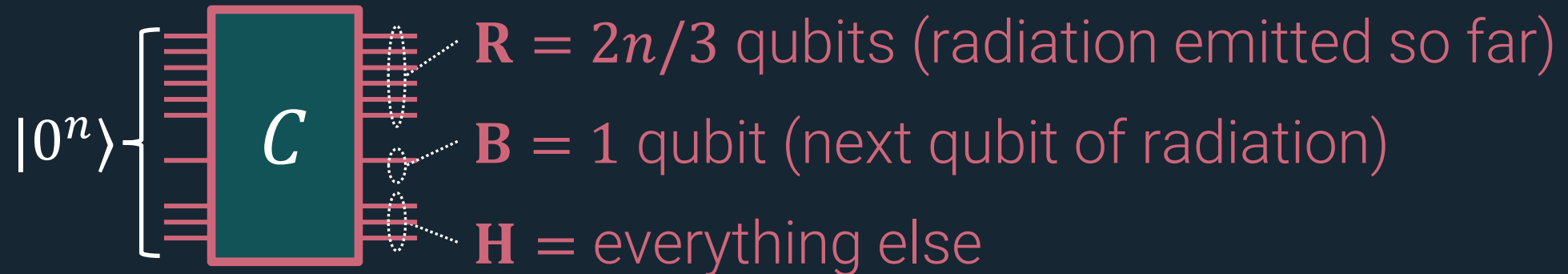
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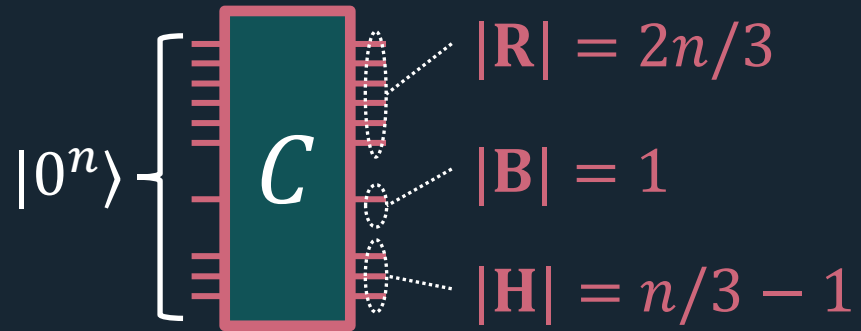
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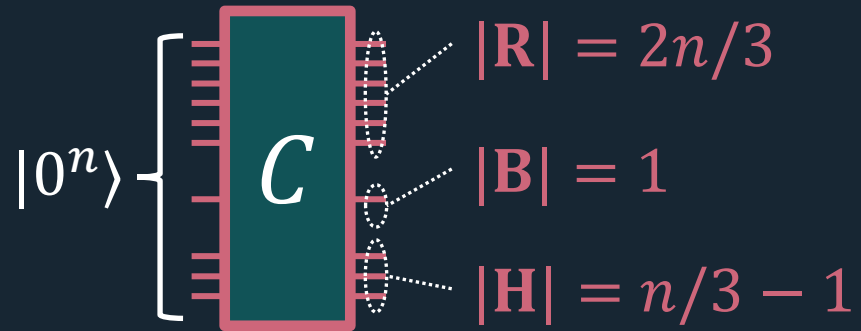
Task: Given \mathbf{R} register of $|\psi\rangle_{\mathbf{RBH}} = C|0^n\rangle$, output a single qubit \mathbf{A} such that (\mathbf{A}, \mathbf{B}) is the EPR state $|00\rangle + |11\rangle$.

(promised that \mathbf{R} and \mathbf{B} are maximally entangled)



Radiation Decoding Problem:

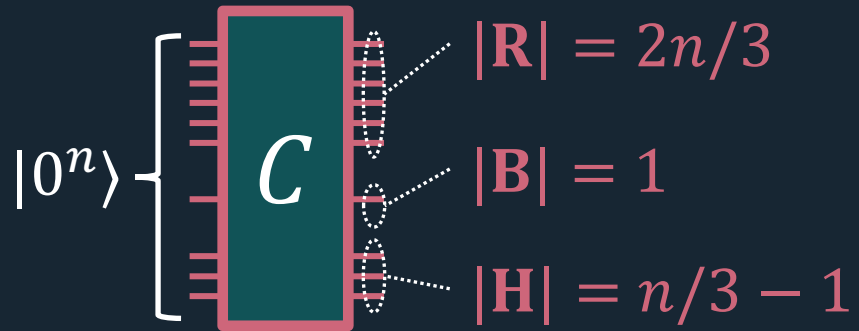
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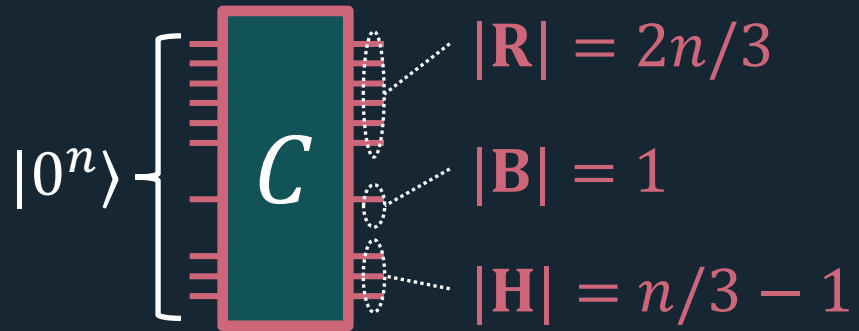


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“Hard” means no QPT adversary can win with probability $\geq \frac{1}{4} + \text{negl}(n)$
(formalized by [Brakerski23])



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Later works *weakened* the assumptions needed:

- [Aaronson16]: quantum-secure one-way functions
- [Brakerski23]: quantum bit commitment

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Why cryptographers care: quantum commitments imply many important primitives, e.g., quantum oblivious transfer, multi-party computation, and zero knowledge.

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Why cryptographers care: quantum commitments imply many important primitives, e.g., quantum oblivious transfer, multi-party computation, and zero knowledge.

“This can be viewed (with proper disclaimers, as we discuss) as providing a physical justification for the existence of secure cryptography” – [Brakerski23]

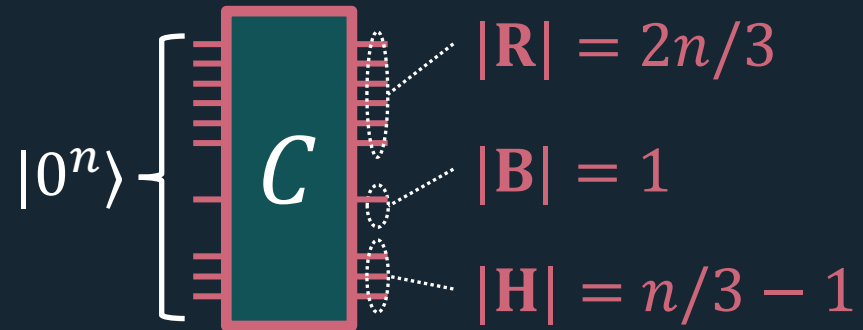
Rest of today: new perspective on Brakerski's result/proof.

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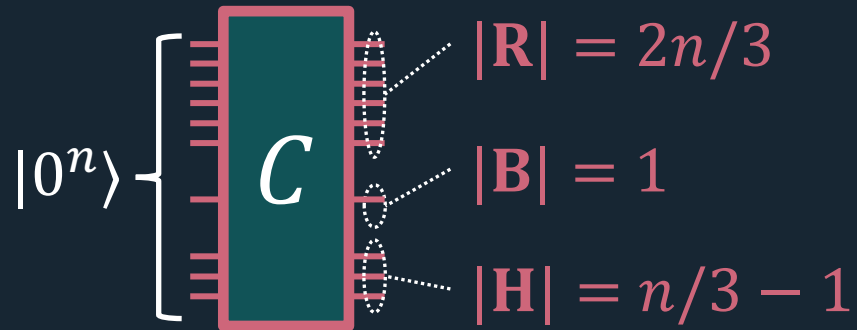
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Instead of studying the [HH13] radiation **decoding** problem,
we'll define a new radiation **distinguishing** problem.



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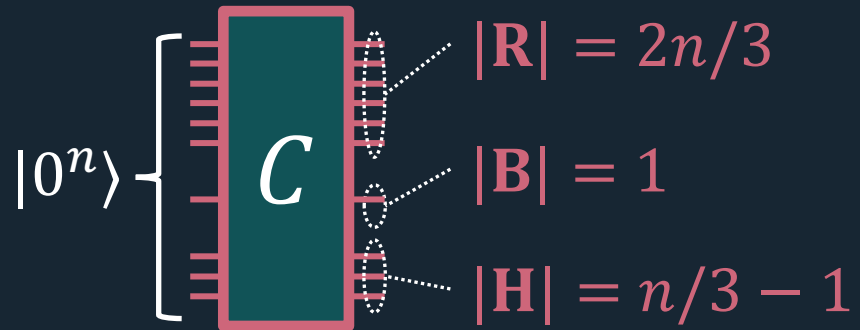


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The point:

R and B are maximally entangled, but this entanglement isn't efficiently detectable.



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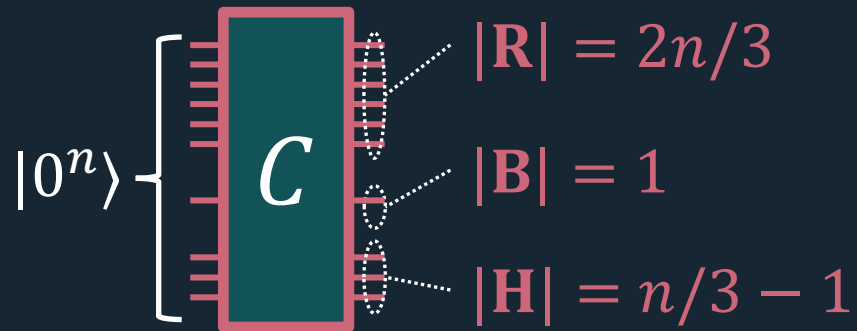
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Radiation **Distinguishing** Problem:

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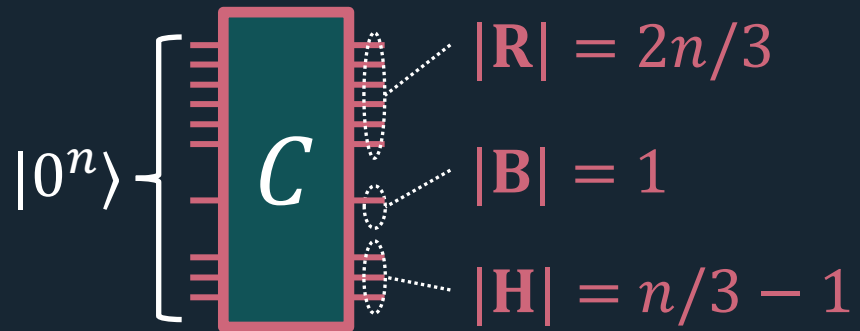
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If you can solve the decoding problem with advantage $1/4 + \varepsilon$, you can distinguish with advantage ε .



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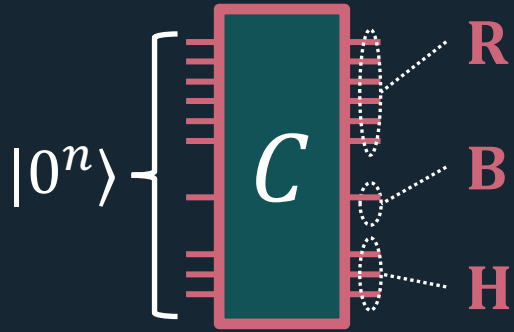
Claim 2: **Distinguishing** should still be hard.

If Alice can't trigger a firewall, then she shouldn't be able to detect entanglement between B and R in the AMPS experiment.

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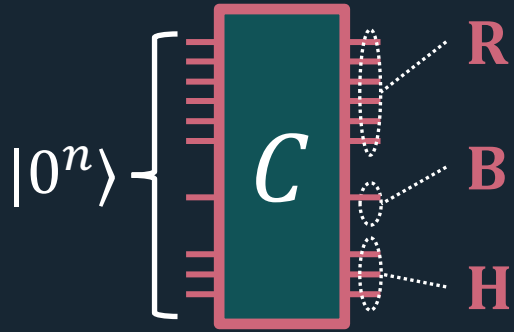
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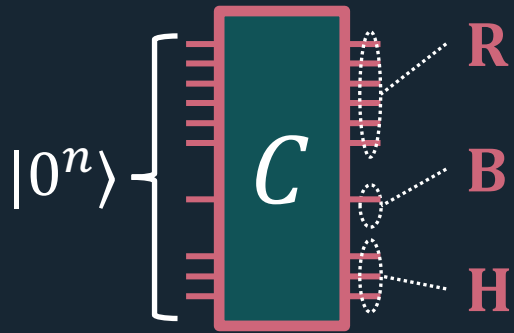
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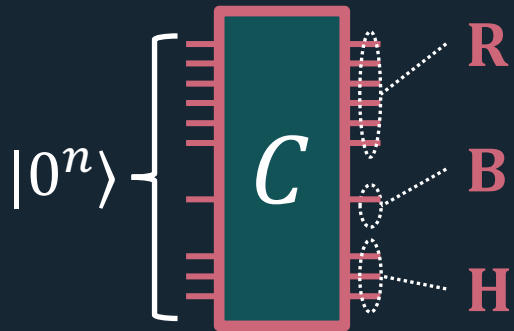


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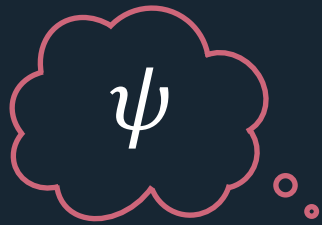
Radiation Distinguishing is hard if and only if *quantum commitments to the EPR state* exist.

Up next: define commitments to quantum states

Quantum State Commitments

[Gunn-Ju-M-Zhandry23]

Protocol that lets a sender commit to a (possibly entangled) quantum state ψ , with the ability to reveal ψ later.



Sender



Receiver

Quantum State Commitments

[Gunn-Ju-M-Zhandry23]

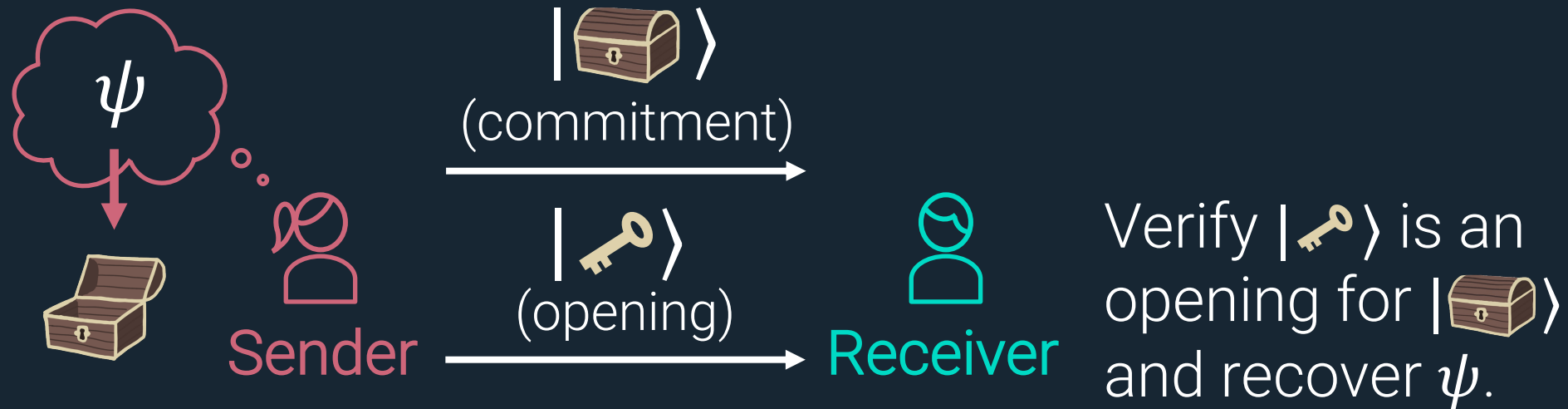
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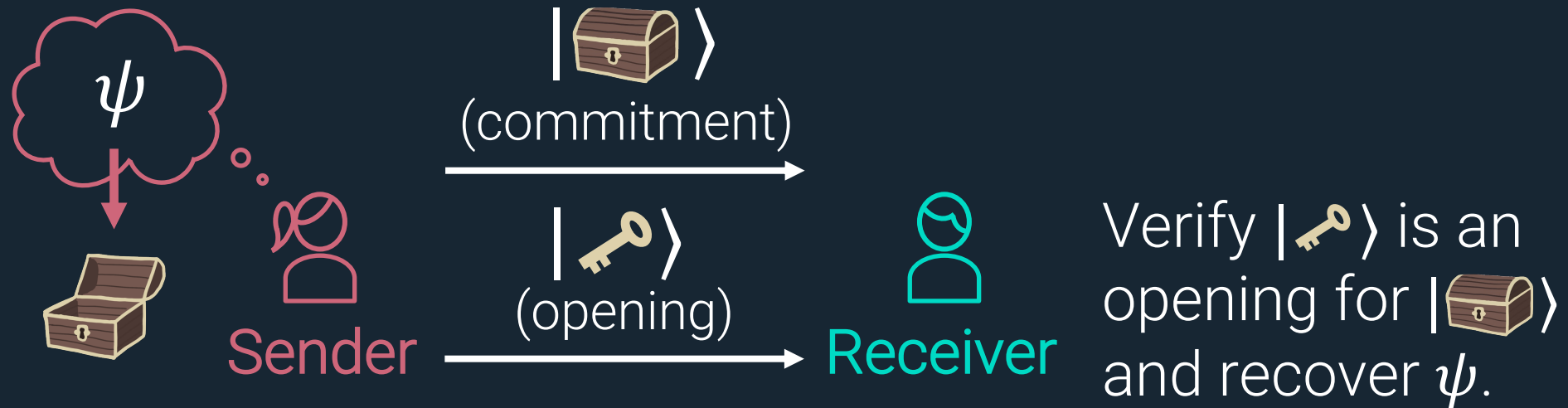
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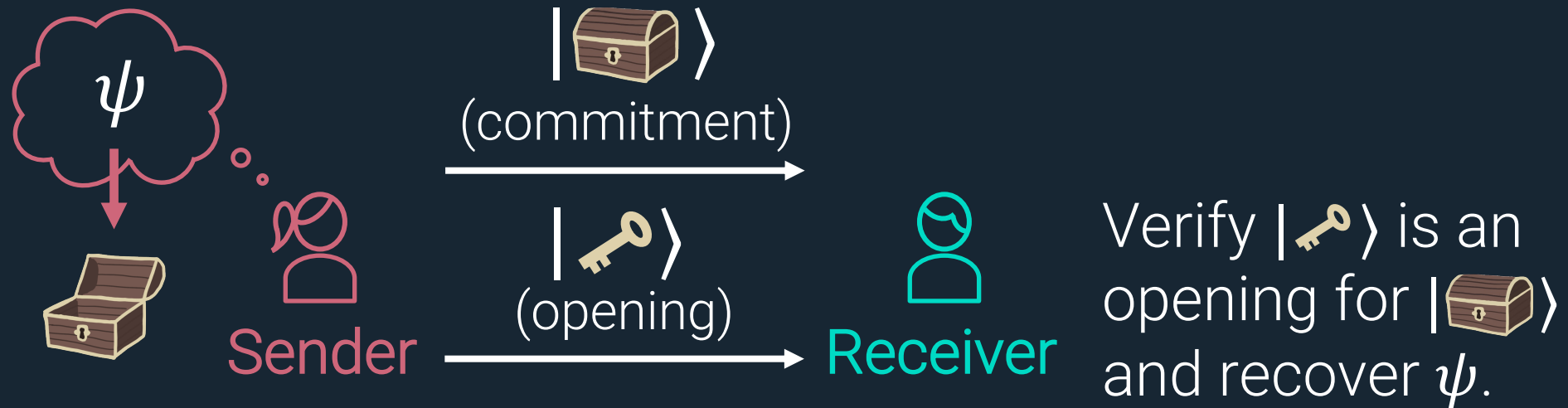
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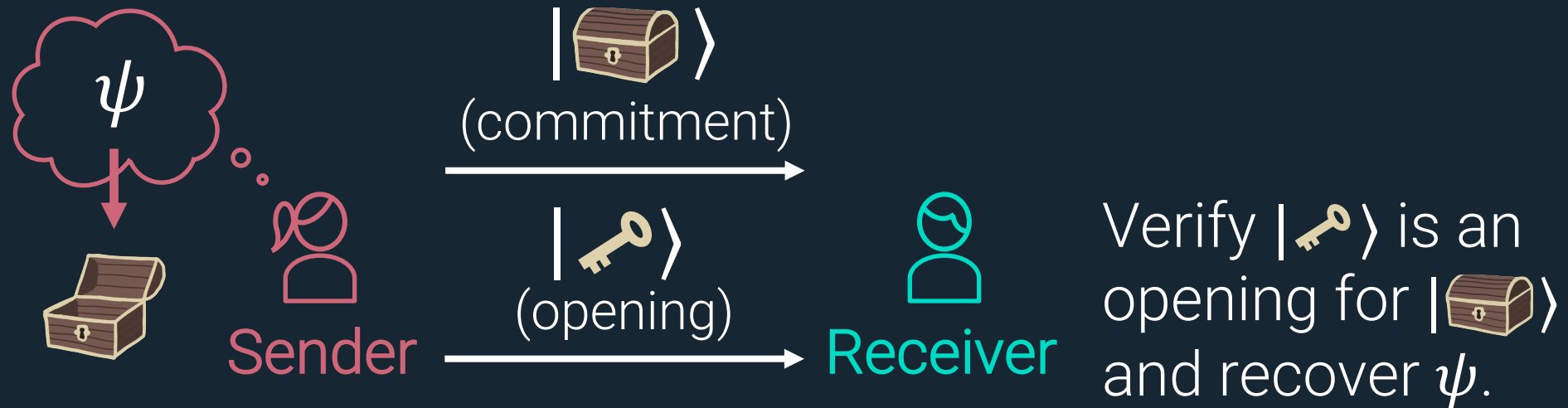
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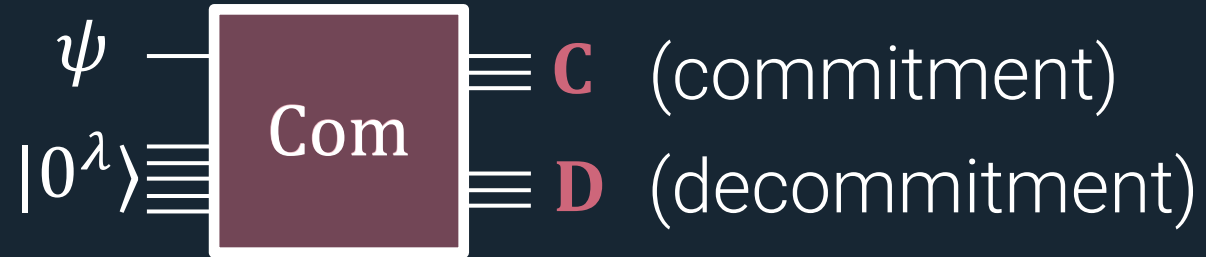


- Requires computational assumptions [M96, LC96].
- Exist if and only if quantum bit commitments exist.

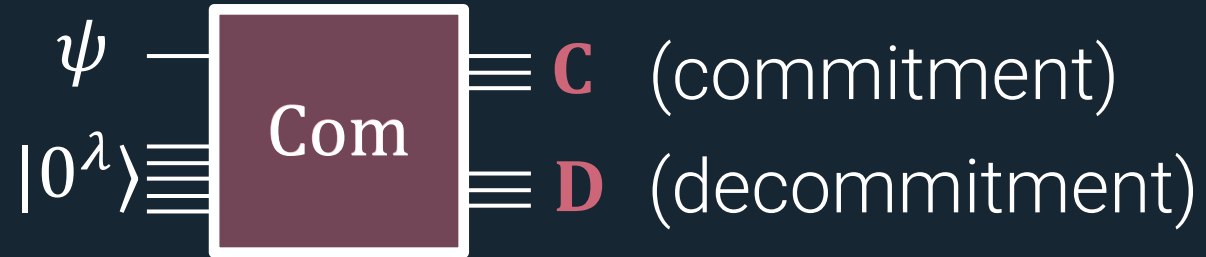
Commitment Syntax



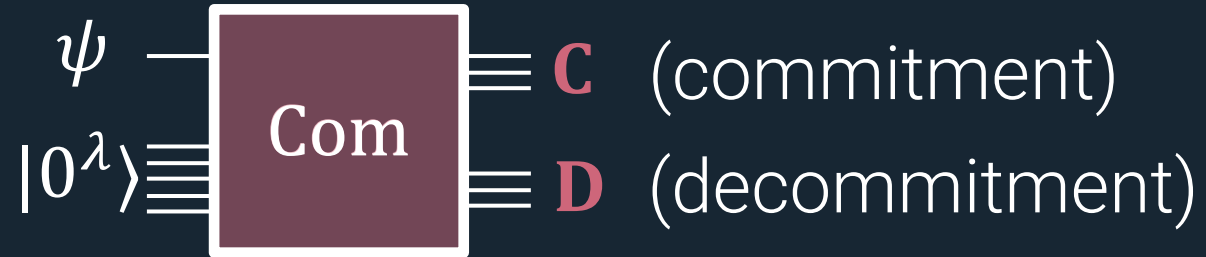
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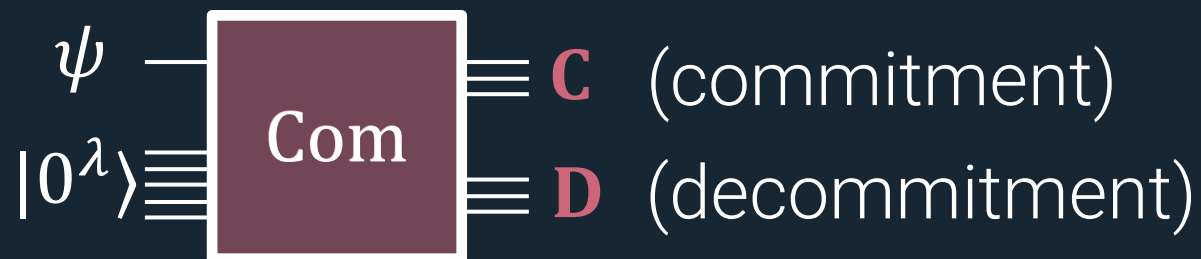
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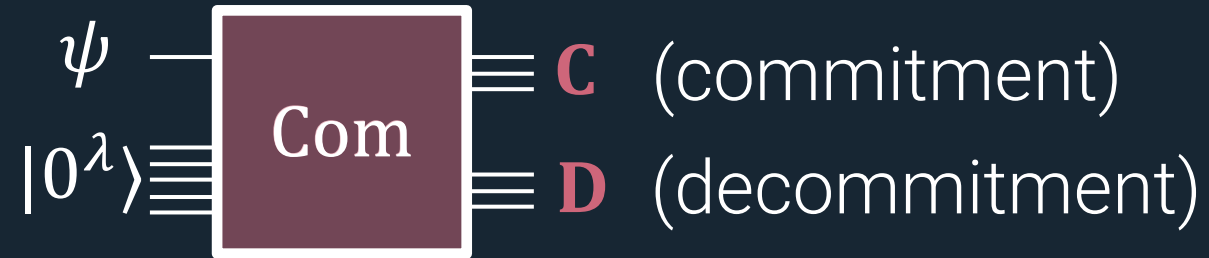


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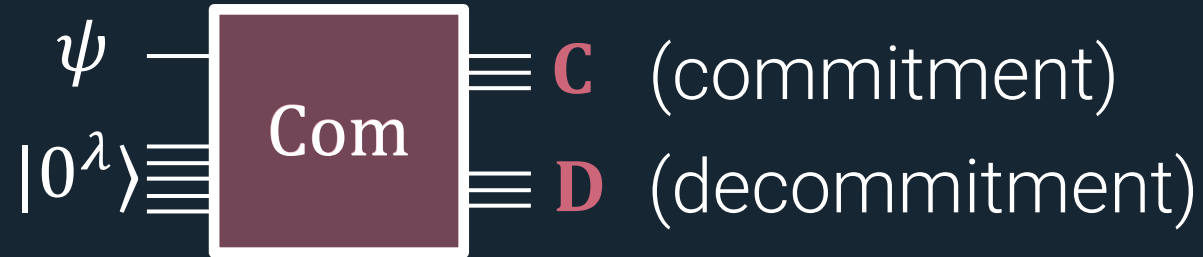


To verify **(C, D)**, receiver applies Com^\dagger and checks if last λ bits are 0.

Security: Binding and Hiding

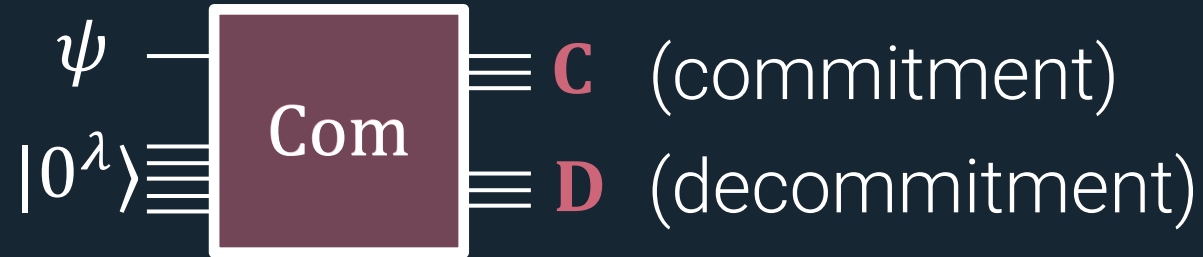


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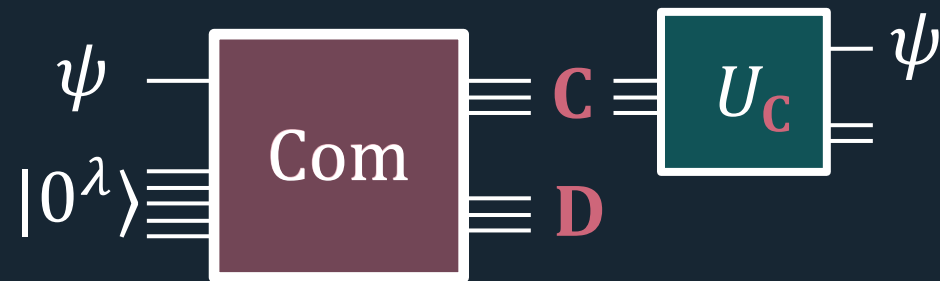
Statistical binding: \mathbf{C} info-theoretically determines/contains ψ .

Security: Binding and Hiding

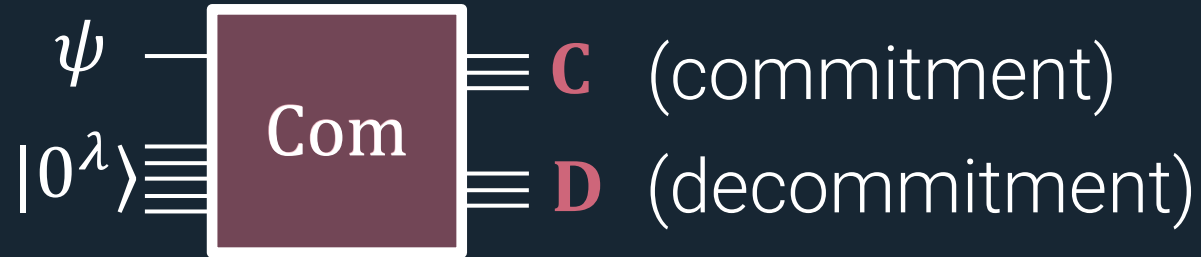


Statistical binding: \mathbf{C} info-theoretically determines/contains ψ .

Exists an inefficient unitary $U_{\mathbf{C}}$ that recovers ψ from \mathbf{C} alone.



Security: Binding and Hiding

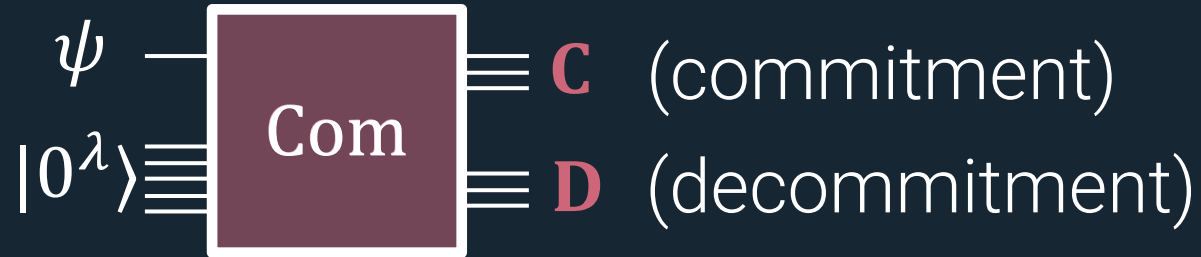


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- (1) commitment to ψ of the adversary's choice
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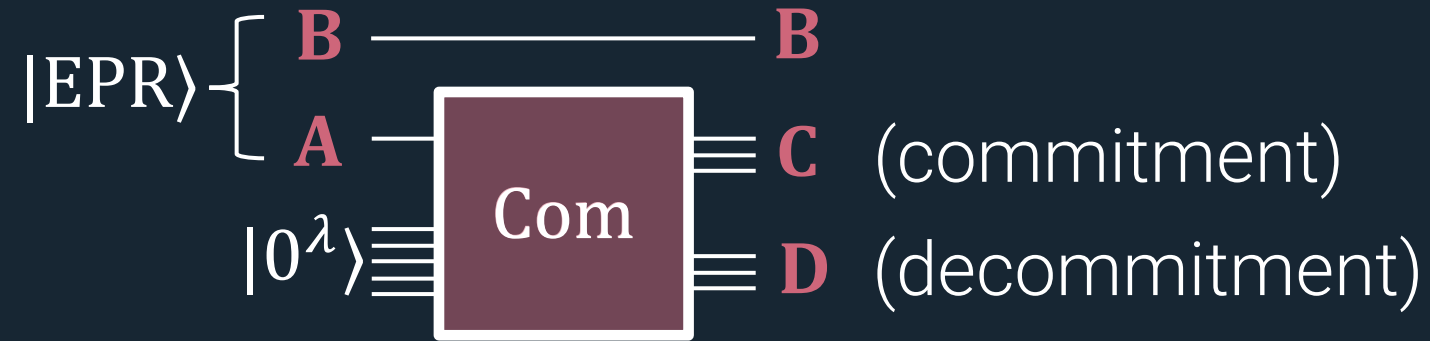
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Crucial point: since adversary picks ψ , indistinguishability holds even if the adversary has a state entangled with ψ .

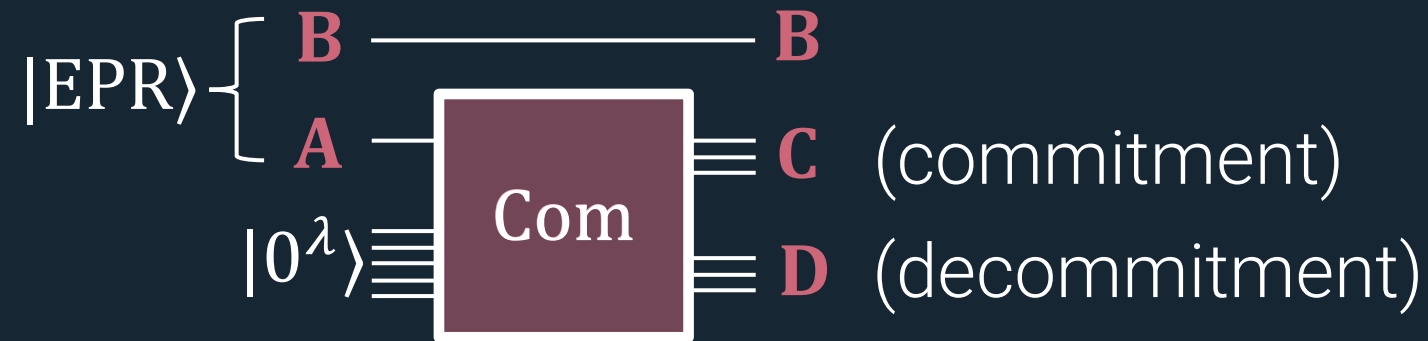
Commitments to the EPR State

Setup: Prepare $|EPR\rangle_{AB}$ and commit to **A**.



Commitments to the EPR State

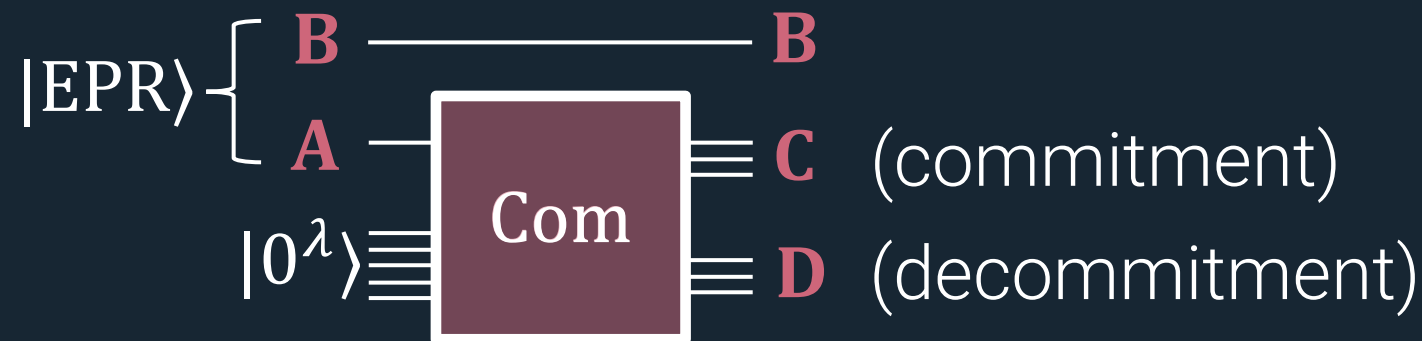
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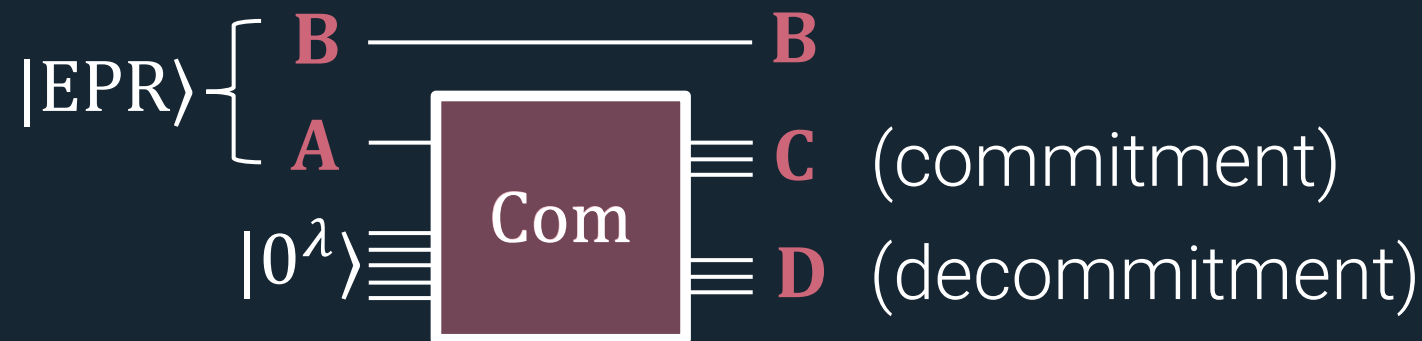


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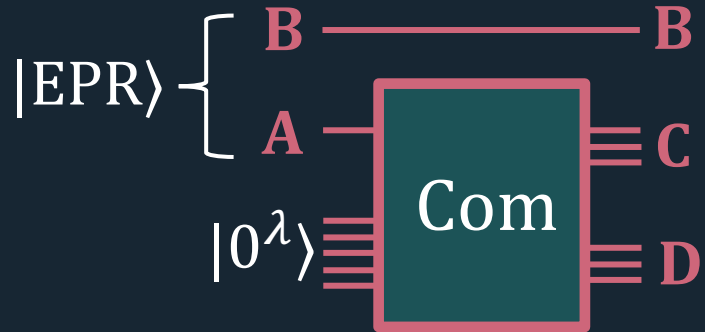
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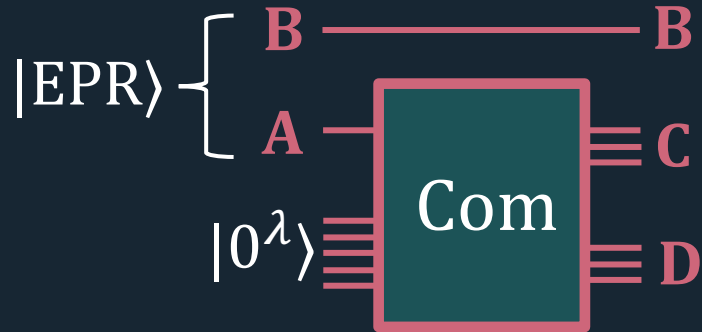
Computational Hiding: (\mathbf{B}, \mathbf{C}) indistinguishable from $(\mathbf{B}, \mathbf{C}')$ where \mathbf{C}' is a commitment to a maximally mixed state

Fact: $(\mathbf{B}, \mathbf{C}')$ is distributed as $(\mathbf{B}', \mathbf{C})$ for \mathbf{B}' maximally mixed.



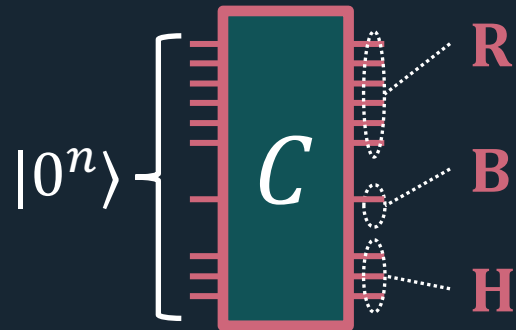
Breaking Hiding of EPR Commitment:

Promised that B and C are maximally entangled, distinguish (B, C) from (B', C) where B' is an unentangled, maximally mixed qubit.



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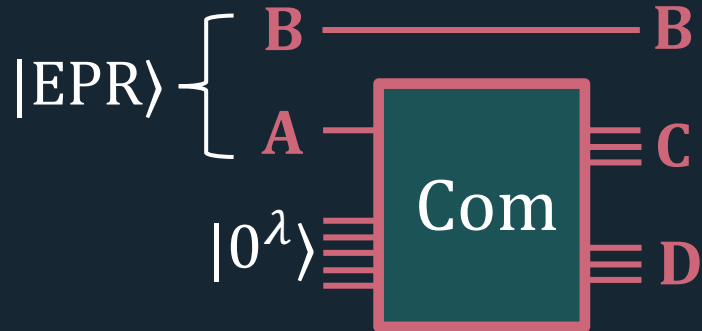
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Radiation Distinguishing Problem:

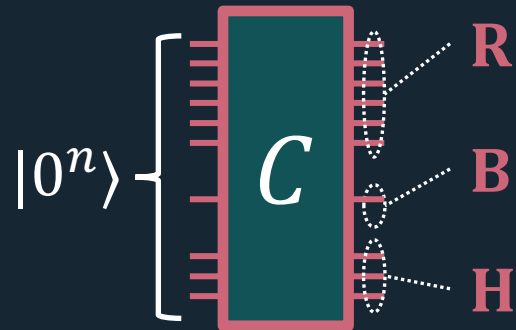
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Thus, quantum commitments \rightarrow hard radiation distinguishing.



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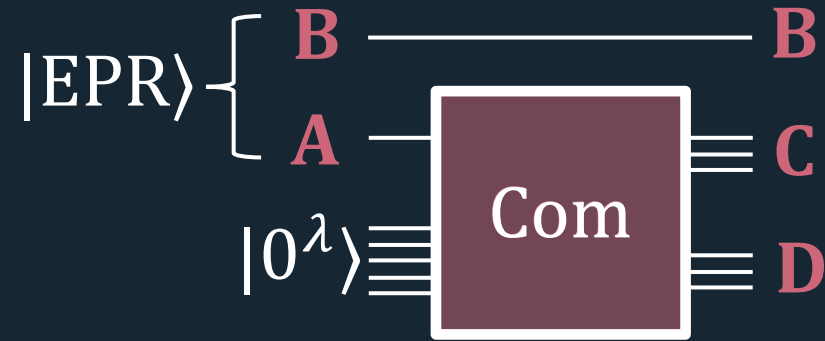


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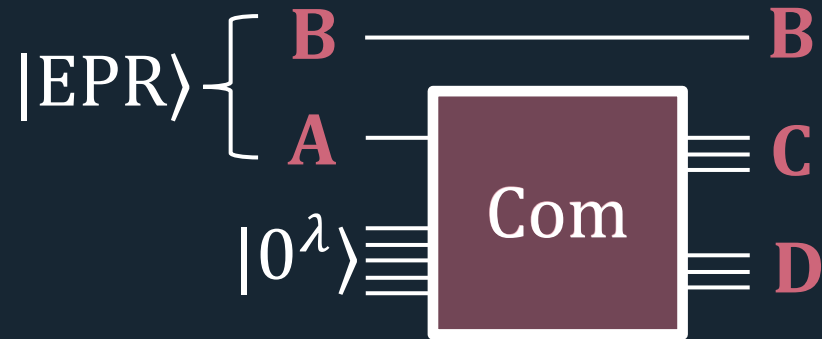
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One last thing: to show hard radiation distinguishing \rightarrow crypto,
need to show EPR commitments \rightarrow commitments to any state.

EPR Commitments → Commitment to Any State

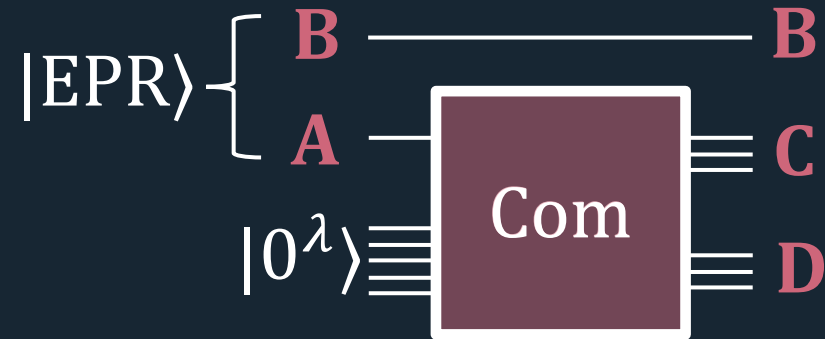


EPR Commitments → Commitment to Any State



Just teleport ψ into C : to commit to ψ , measure (ψ, B) in the Bell basis to get classical bits (x, z) , and send (C, x, z) .

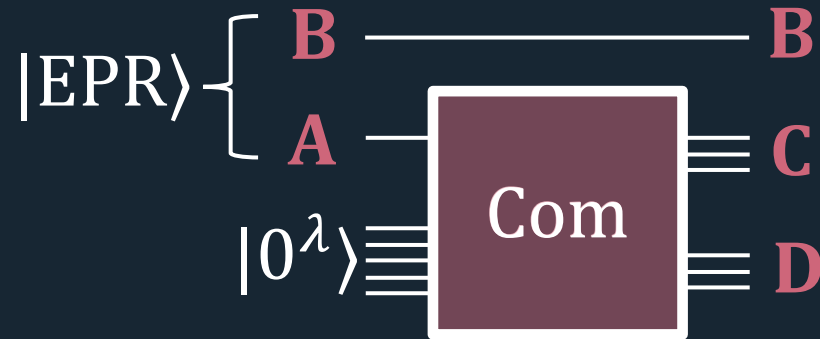
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- **Statistical Binding:** **C** determines **A**. (\mathbf{A}, x, z) determines ψ .
- **Computational Hiding:** (\mathbf{C}, x, z) indistinguishable from (\mathbf{C}', x, z) where **C'** is a commitment to junk, but this is independent of ψ .

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Tight relationship between a problem from black hole physics and quantum cryptography.

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Future research direction: give more evidence for hardness.

Given description of a random circuit \mathcal{C} , how hard is it to distinguish $\mathcal{C}|0^n\rangle$ from $\mathcal{C}|1^n\rangle$ given $2n/3$ of the qubits?