Public Key Function-Private Hidden Vector Encryption (and More)

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Zhengzhong Jin	(Johns Hopkins)
Tancrède Lepoint	(Google)
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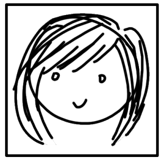
Tal Malkin	(Columbia)
Alex J. Malozemoff	(Galois)
Mariana Raykova	(Google)

Hey Alice,
It’s me, Bob.



Alice’s homepage
alice@gmail.com

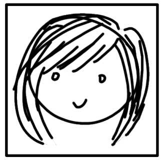
my public key is:
8h9f8he9
ak928ads

Enc(8h9f8he9 ak928ads , Hey Alice, It's me, Bob.)

pk

↓

ct



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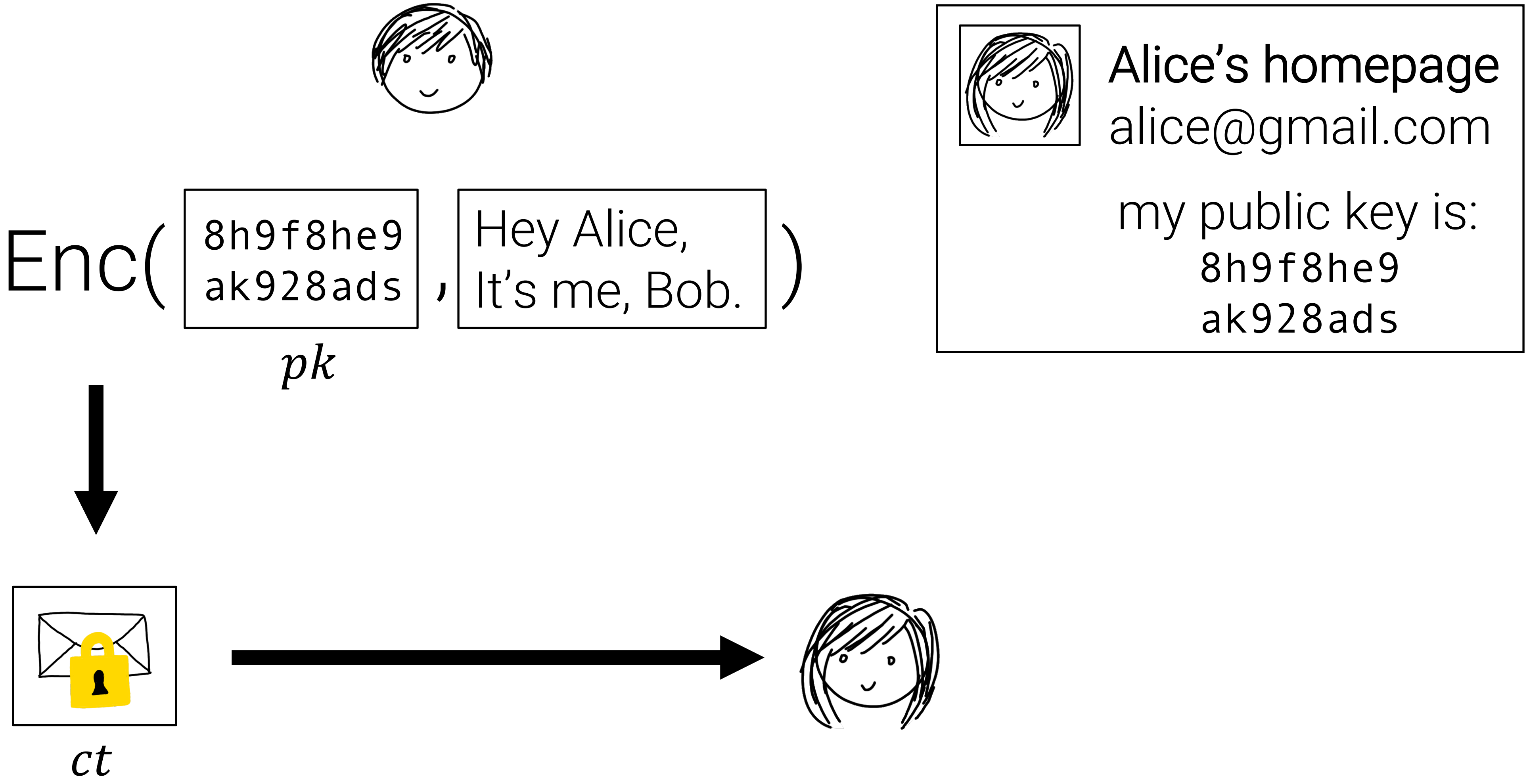
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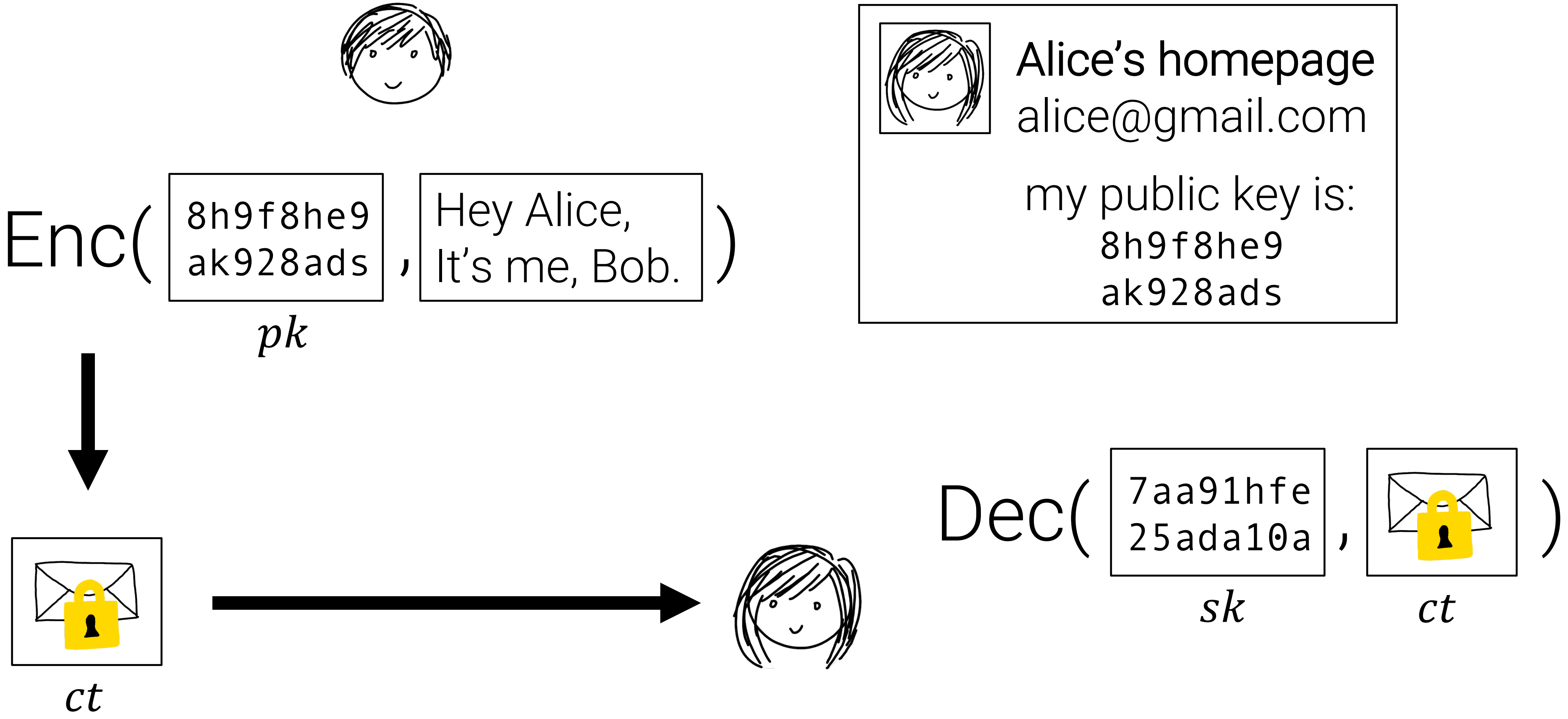

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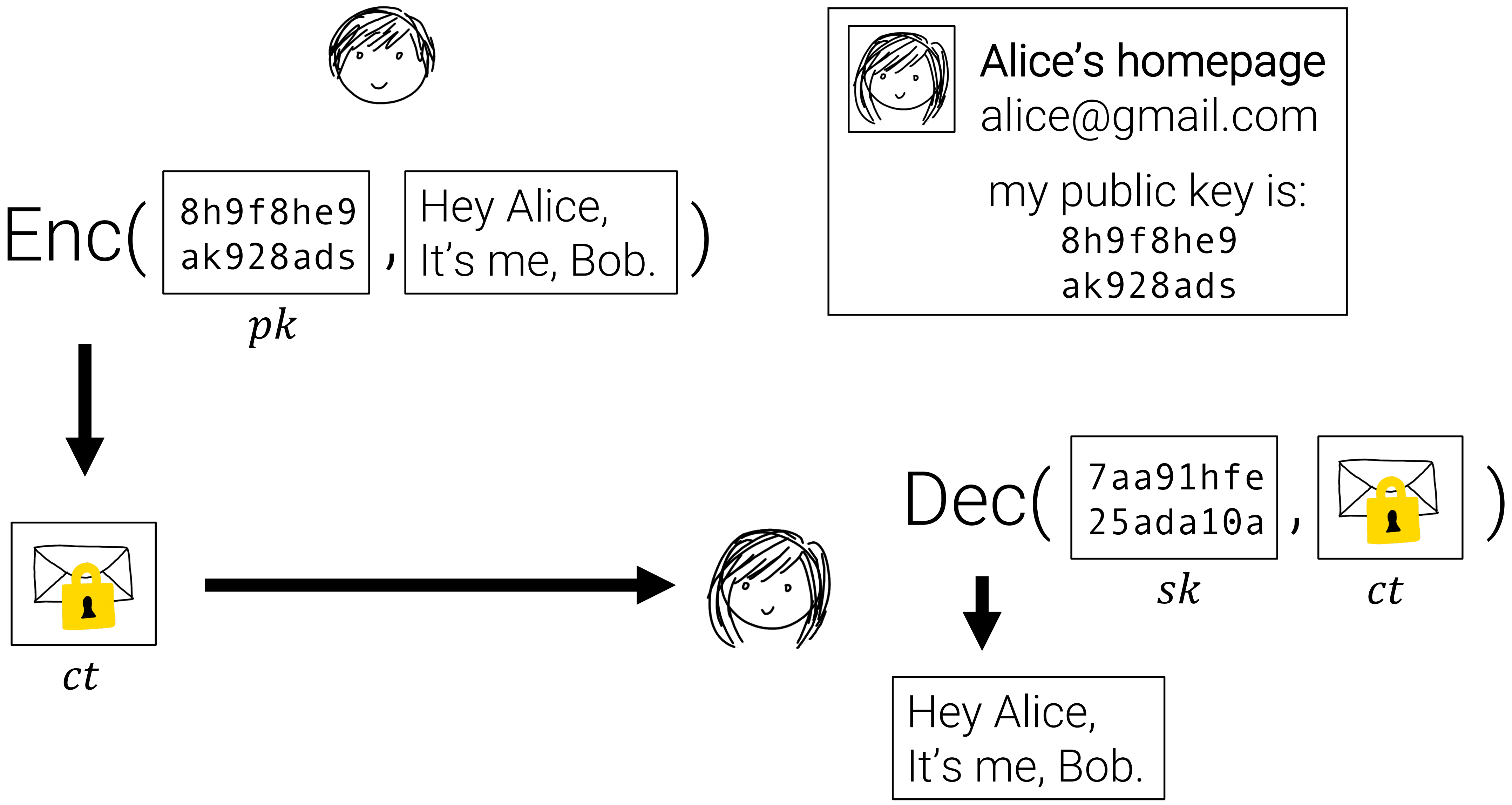
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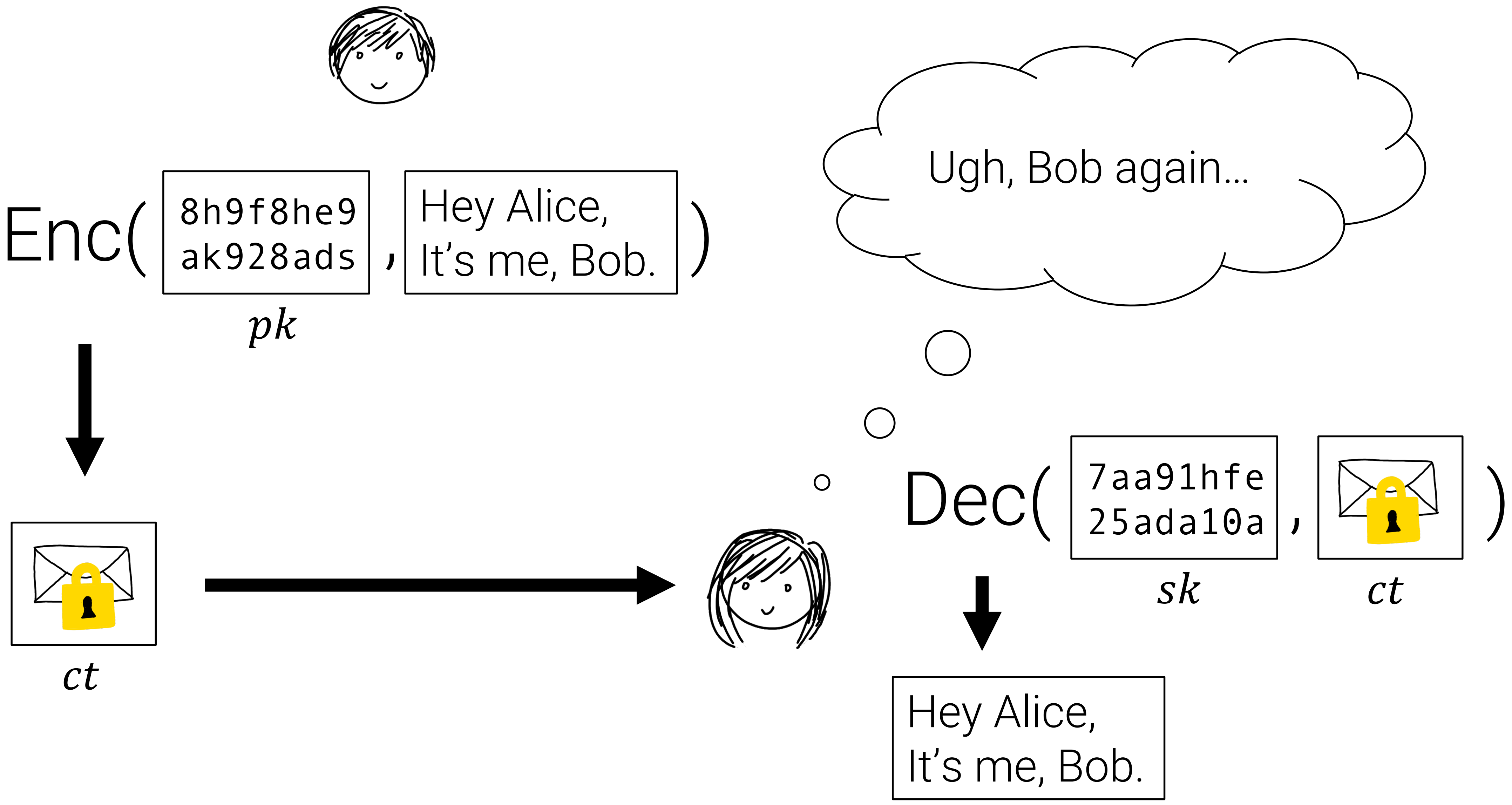
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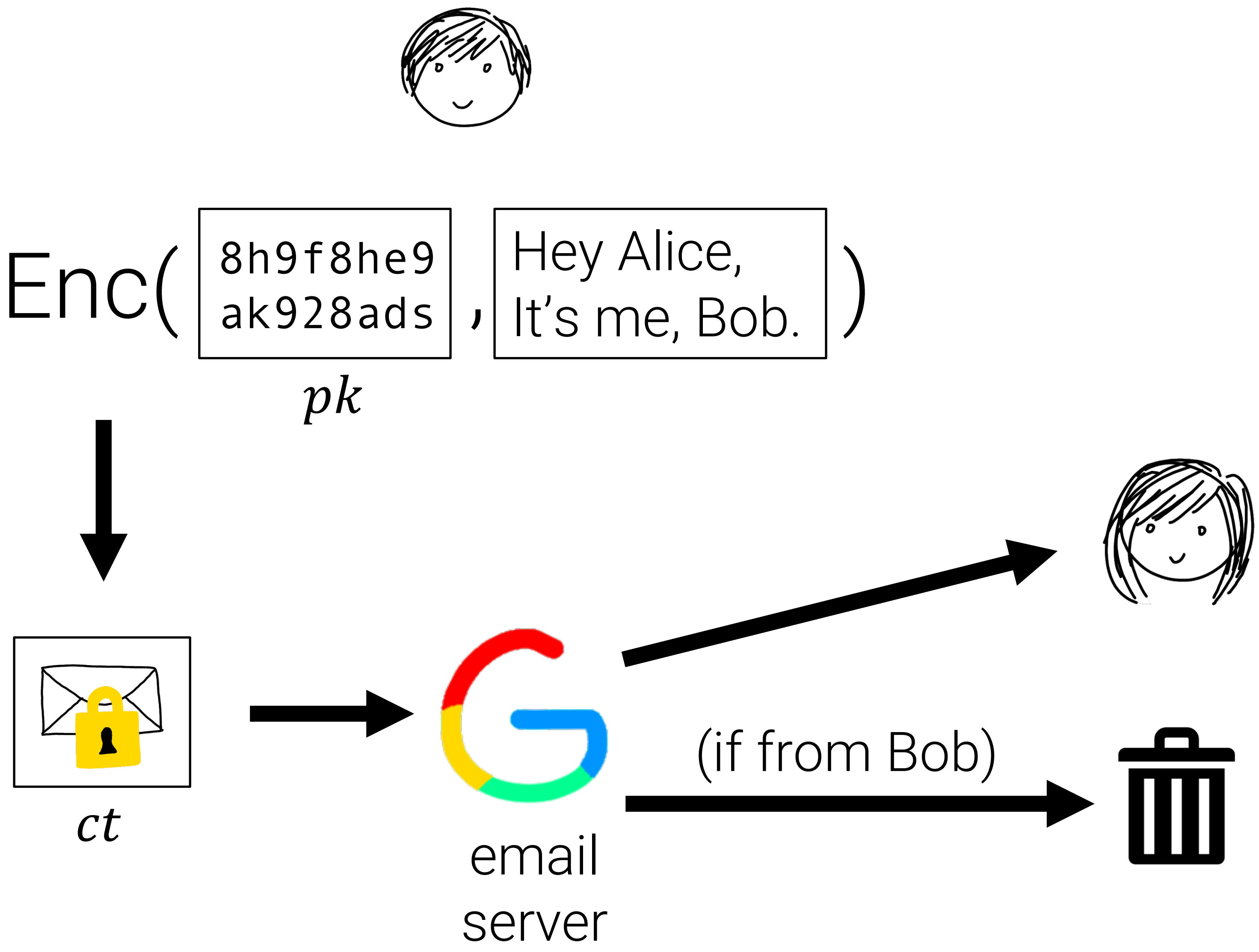
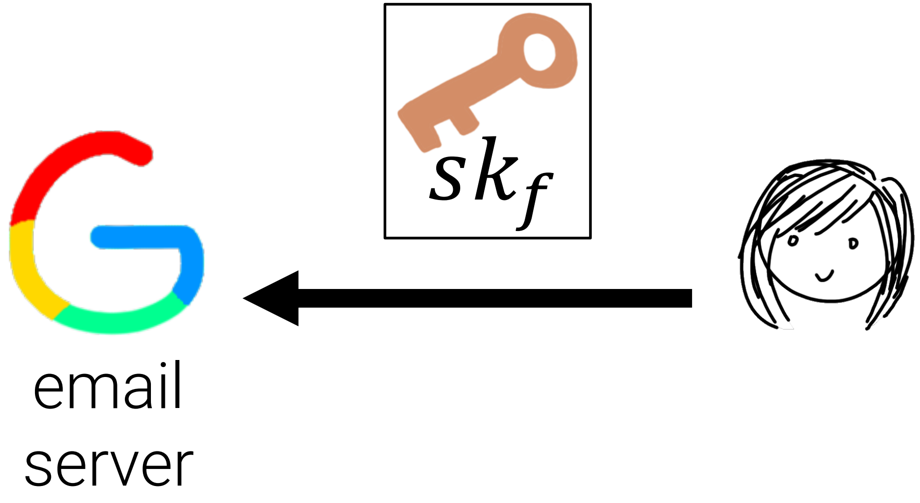


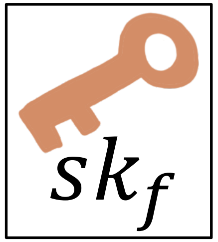
Goal: Allow G to filter emails, without sacrificing privacy

Predicate Encryption

[BCOP04,SW05,BW07,KSW08]



$$f(x) = \begin{cases} 1 & \text{if } x \text{ contains "It's me, Bob"} \\ 0 & \text{else} \end{cases}$$

Given 

, can learn $f(x)$ given encryption of x

Security: Nothing else about x is leaked

$$f(x) = \begin{cases} 1 & \text{if } x \text{ contains "It's me, Bob"} \\ 0 & \text{else} \end{cases}$$

Enc( mpk , Hey Alice, It's me, Bob.)

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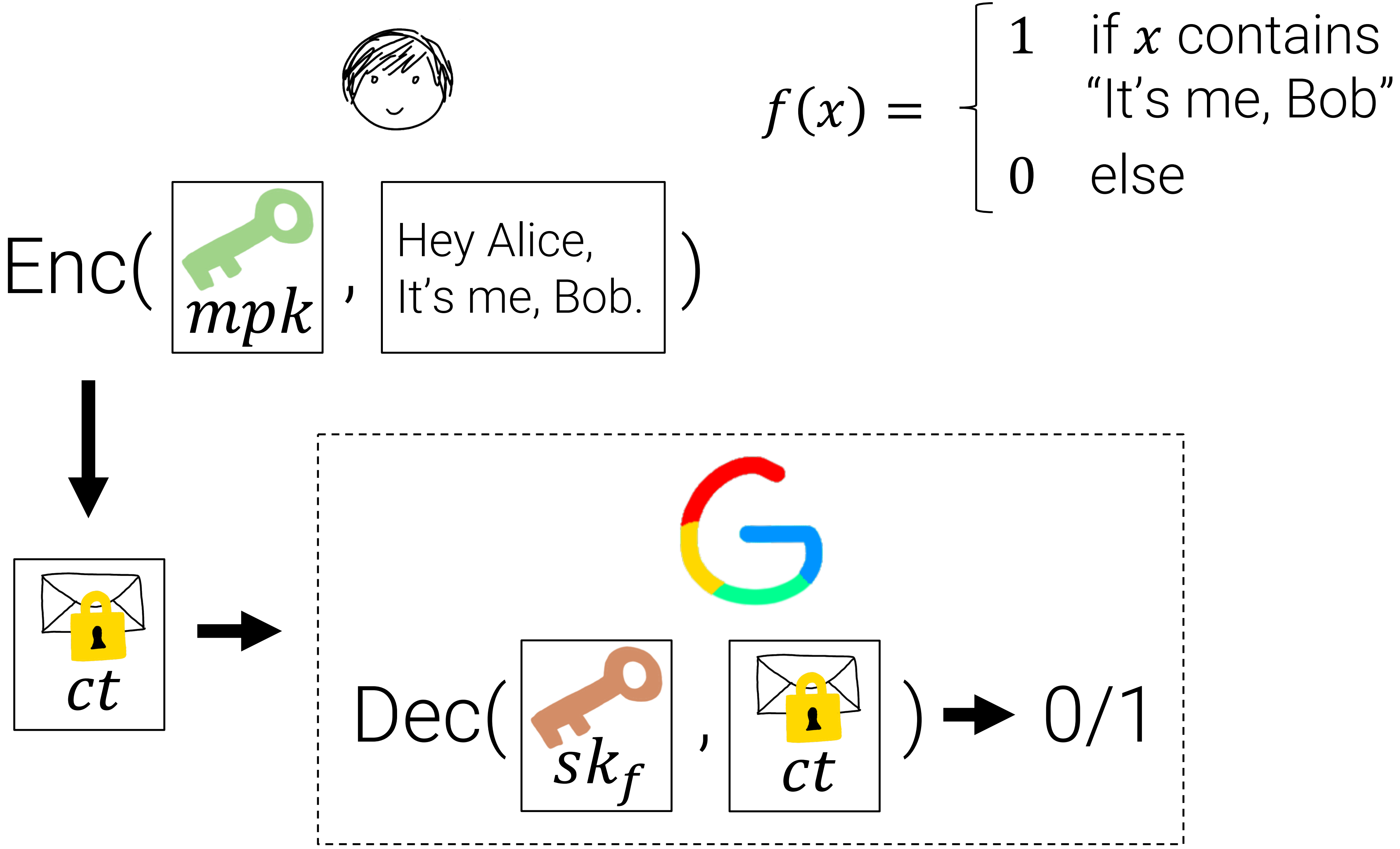

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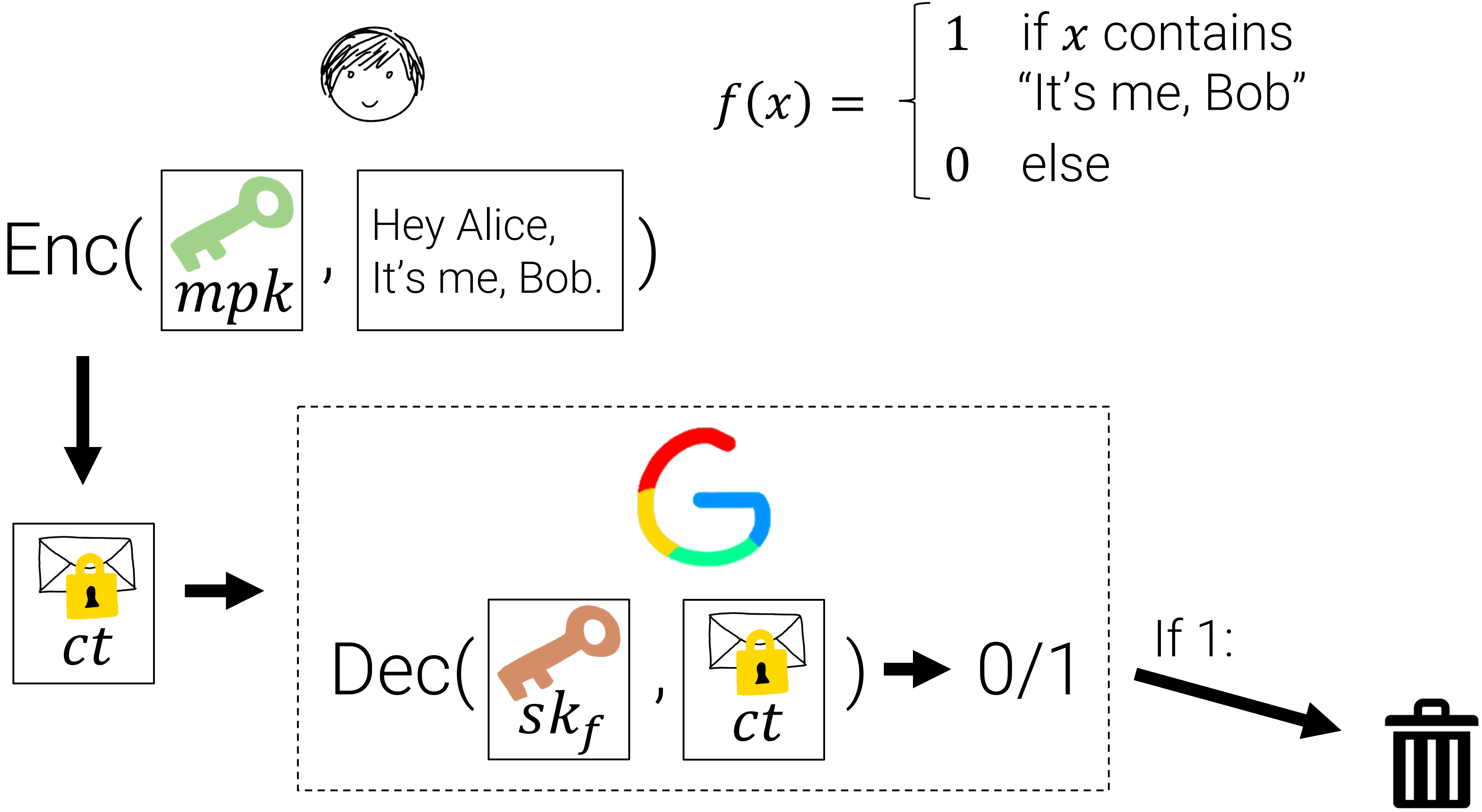

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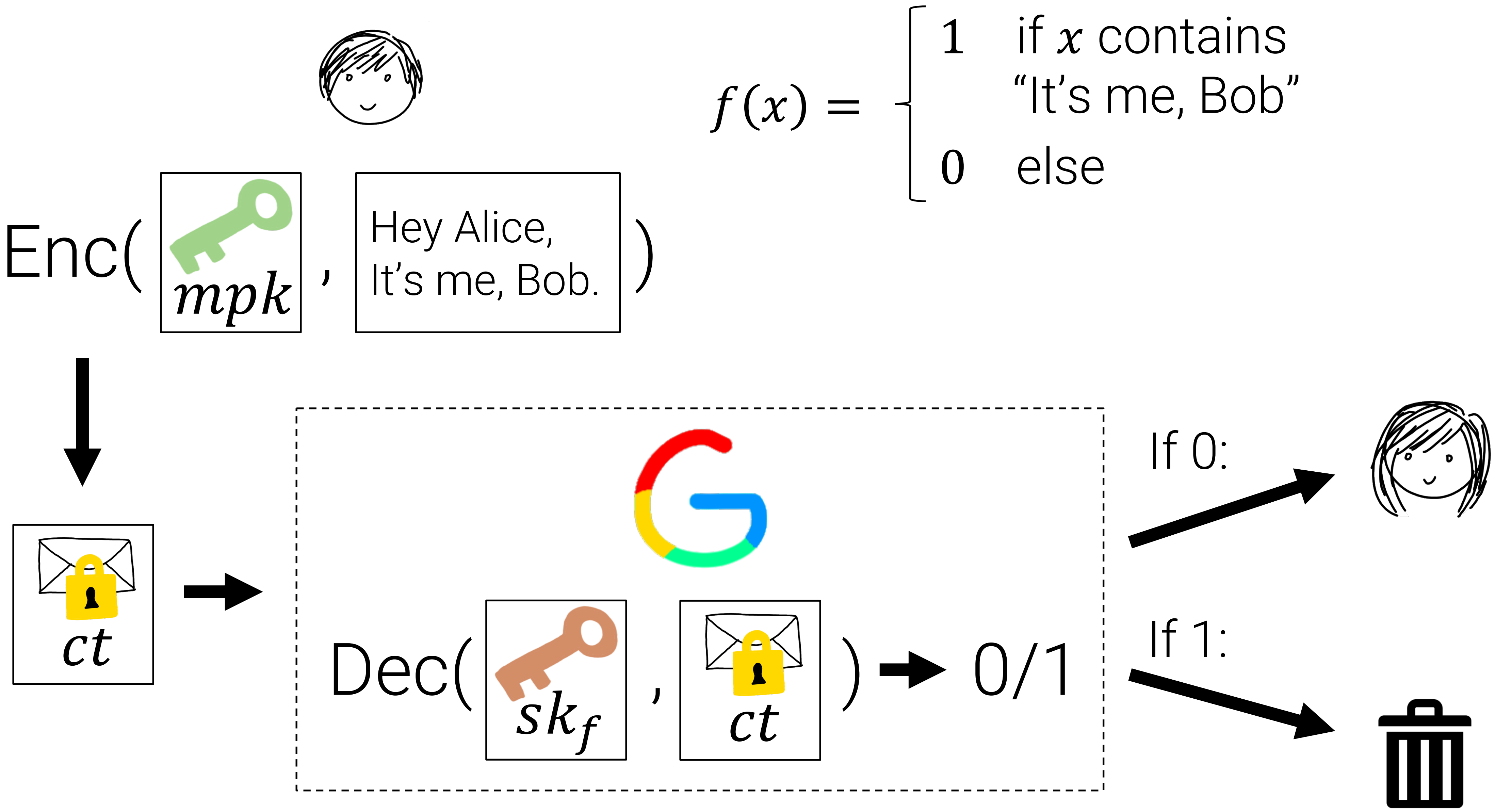

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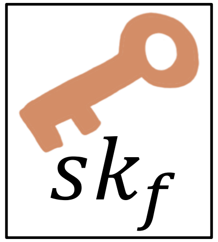
If 0:
If 1:

In many schemes, 

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Function Privacy

[SWP00,OS07,BSW09,SSW09]

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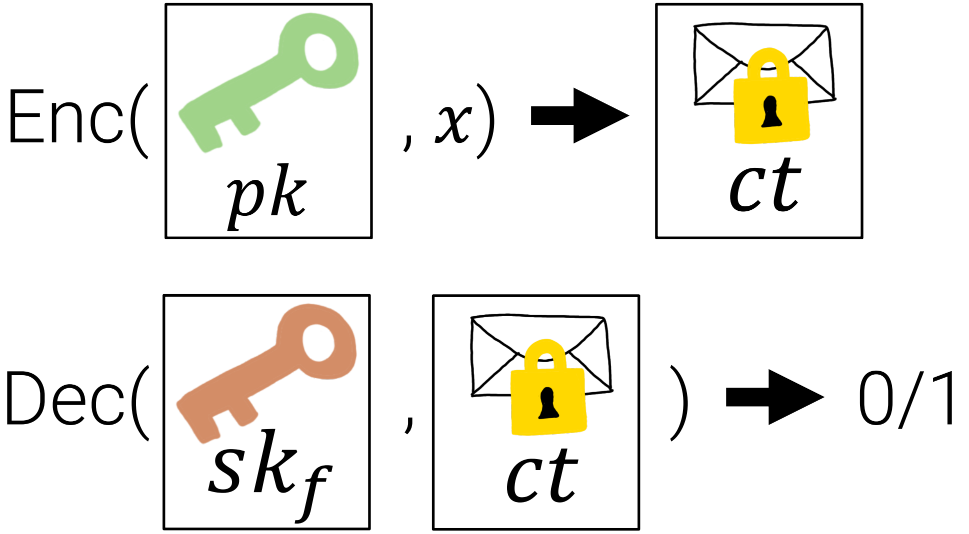

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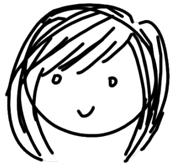
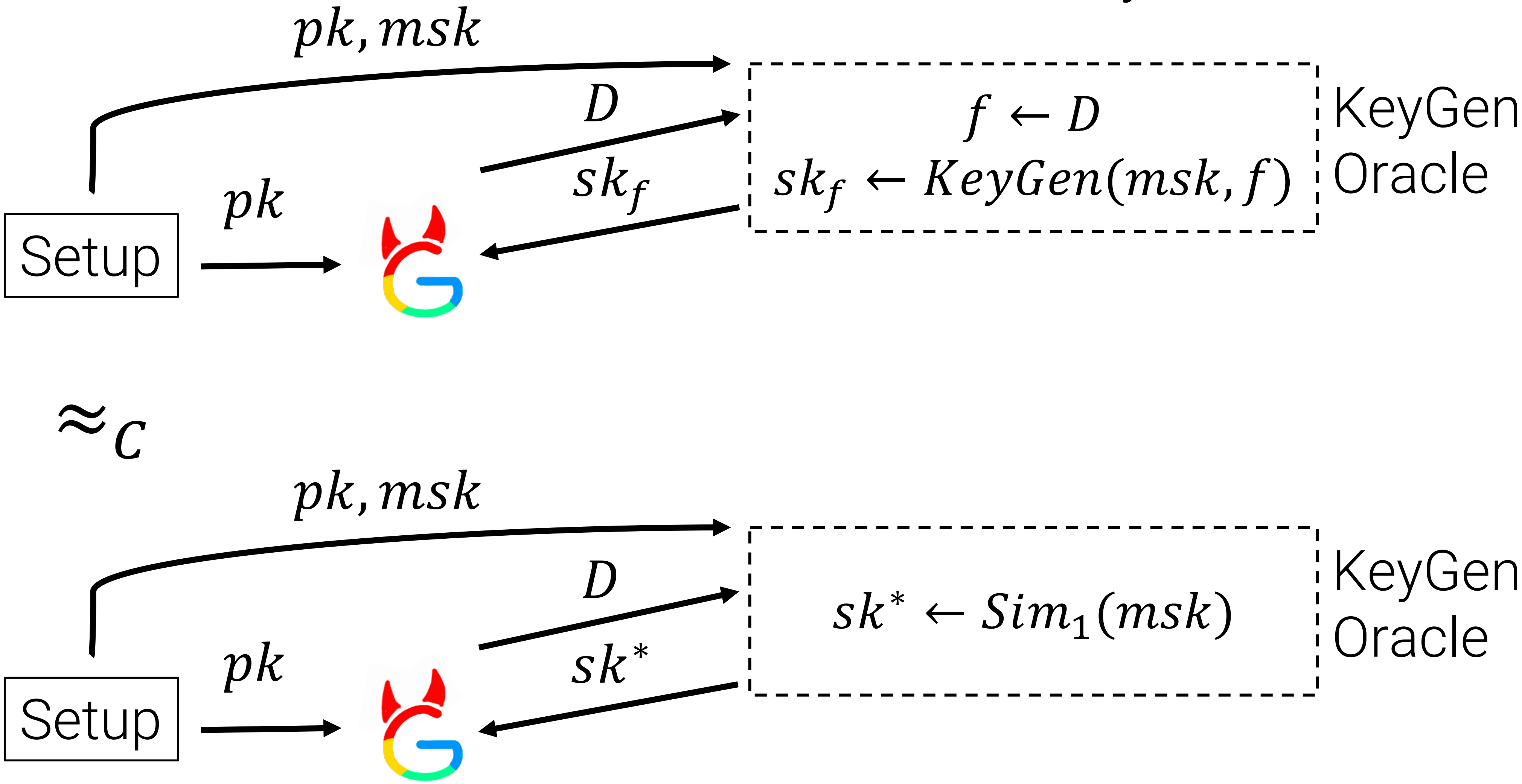
Why is this reasonable?

Intuition:  unlikely to blindly guess what  is filtering for (i.e. will never find an x s.t. $f(x) = 1$).

Function Privacy [BRS13a]



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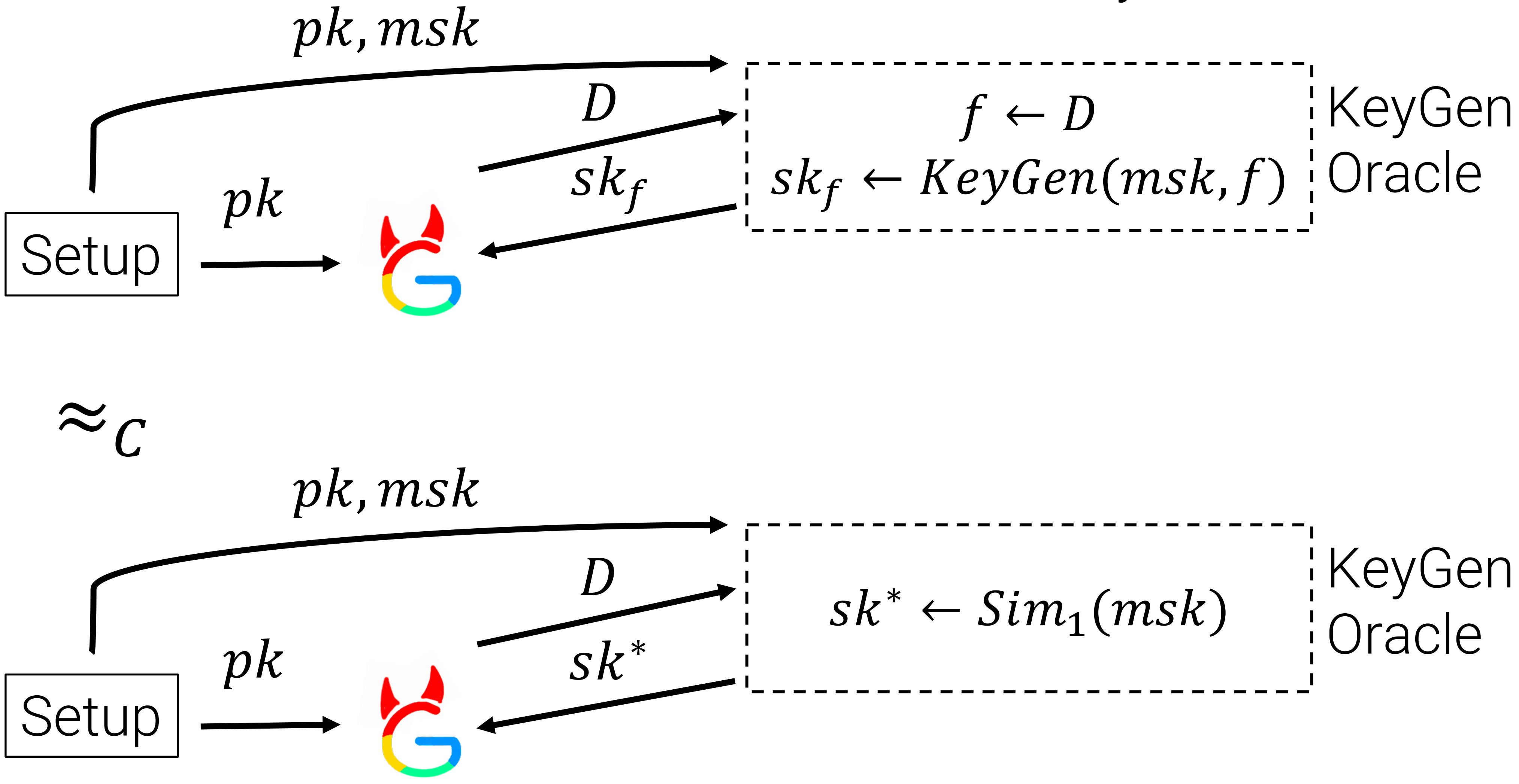
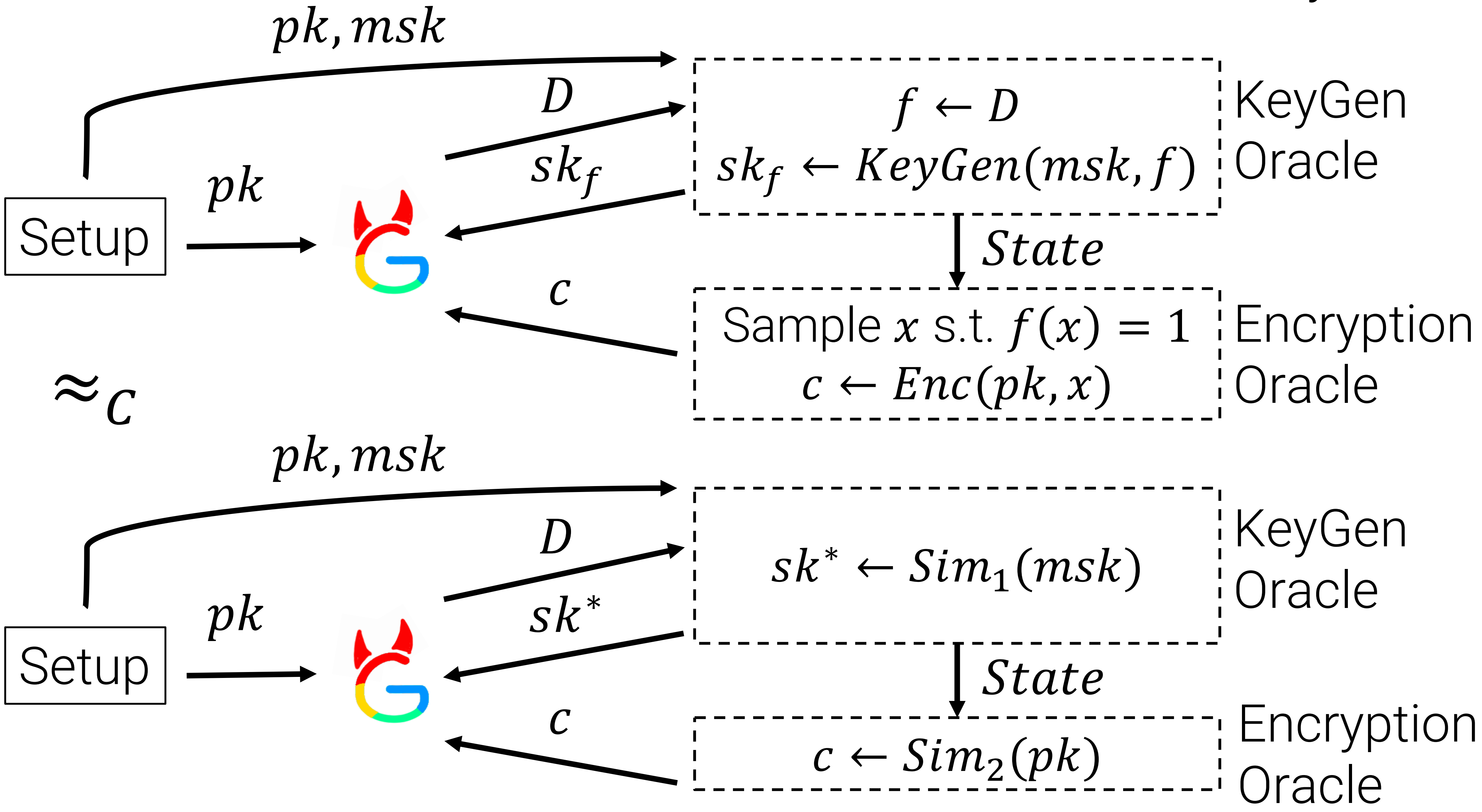
[BRS13a] address this with **enhanced function privacy**, where an "encryption oracle" is provided

Function Privacy [BRS13a]



“Enhanced” Function Privacy [BRS13a]



	Predicates	Assumption	Enhanced Function Privacy?
[BRS13a]	• Equality (IBE)	None (statistical)	Yes
[BRS13b]	• Subspace Membership	None (statistical)	No
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**leaks positions of "wildcards"

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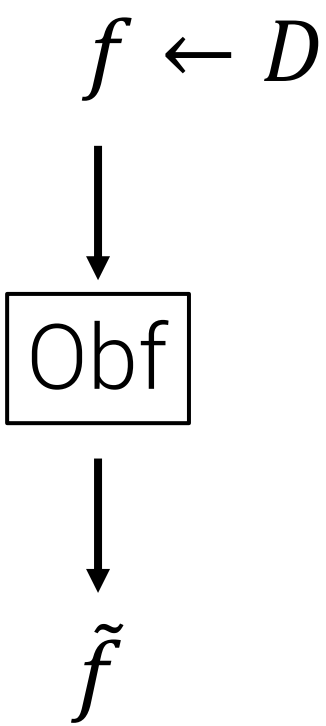
This work	• Equality (IBE) • Conjunctions (Hidden Vector Encryption) • “Small Superset” [BW19]	Generic Group Model	Yes

**leaks positions of “wildcards”

Our Techniques: Obfuscation in the Distributional Setting



1) Functionality: Given $\tilde{f}$, possible to learn $f(x)$ for any x
2) Distributional Security: If D is high-entropy, then $\tilde{f}$ leaks **nothing** about f

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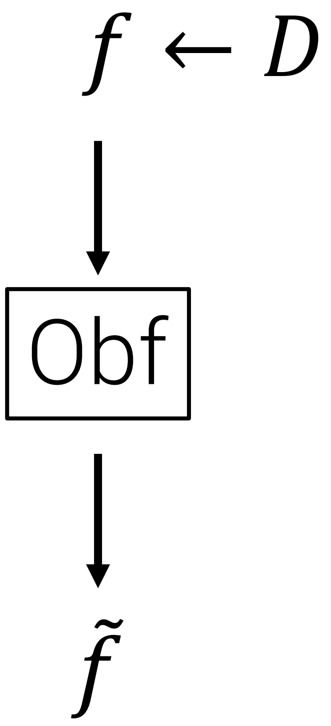

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Point Functions: $f_x(y) = 1$ iff $y = x$.
Sample x with high entropy.
(x, y length n bitstrings)

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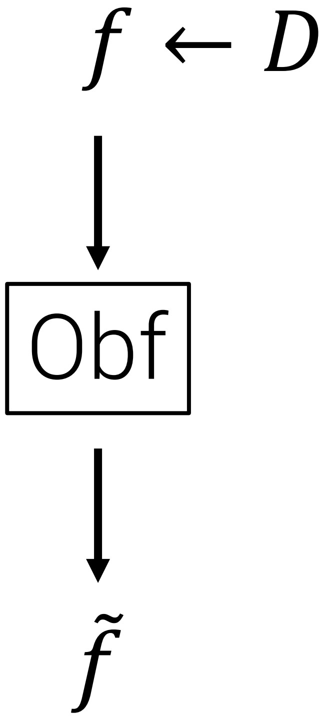

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Conjunctions: $f_{pat}(\cdot)$. $pat =$ 0*1**10.
$f_{pat}(y) = 1$ if y agrees with pat on all 0/1 positions (y is length n bitstring)

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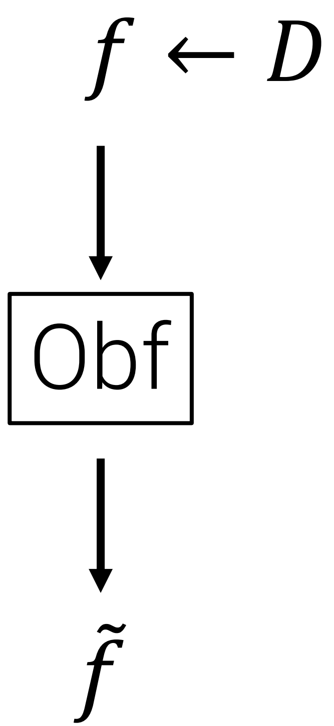

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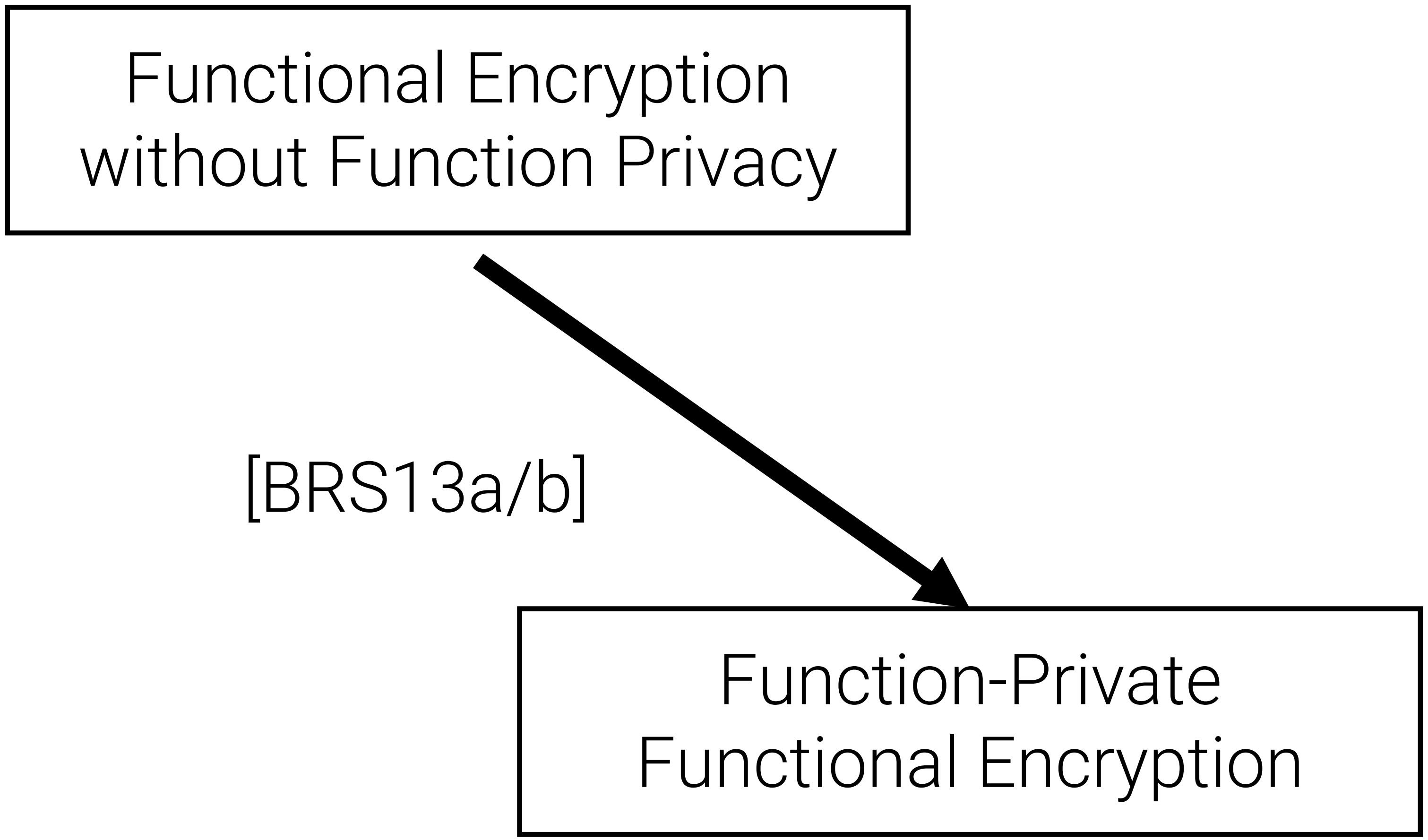
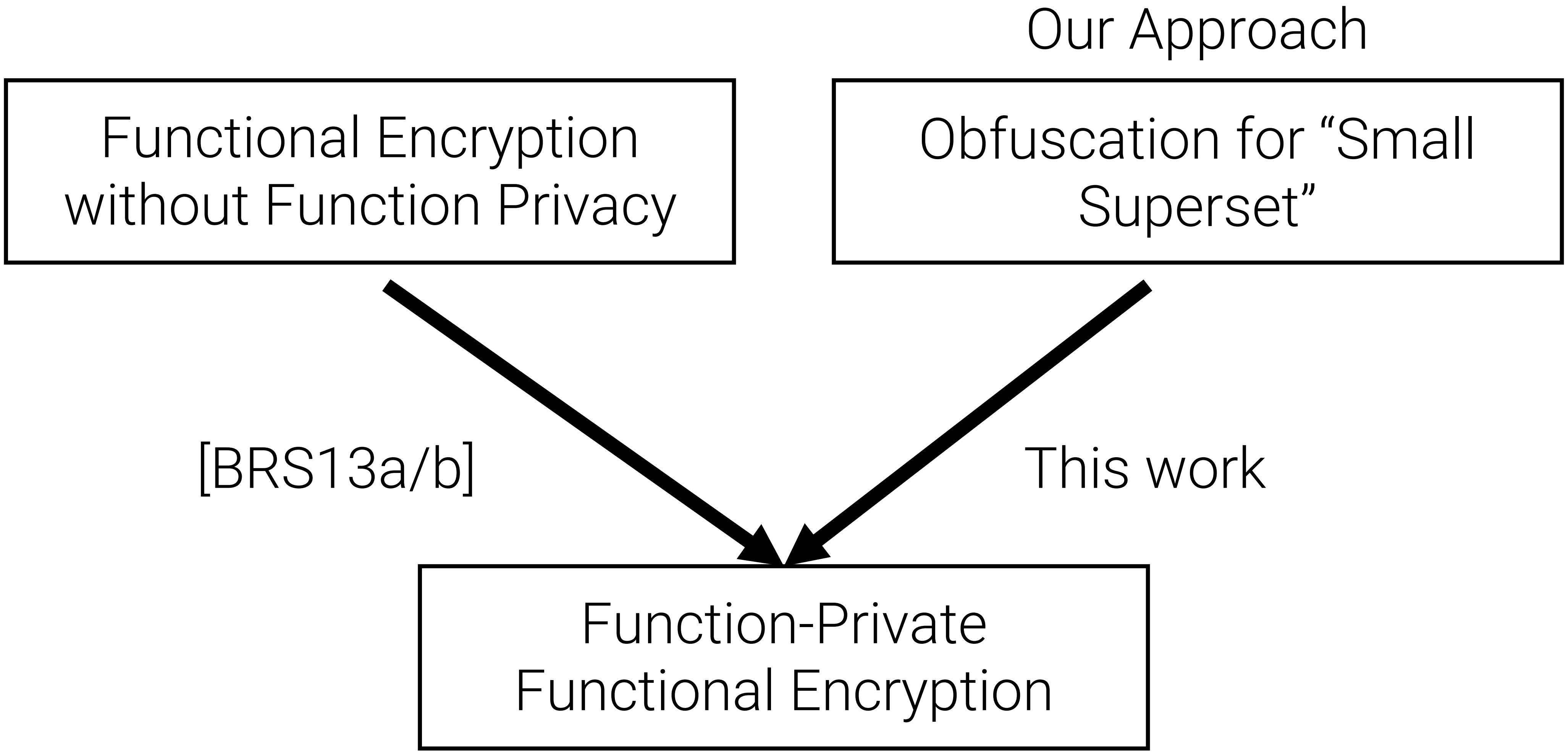
Observation: Think of $\tilde{f}$ as sk_f (a function decryption key) which is evaluated on "trivially insecure" ciphertexts

Can we “upgrade” specific obfuscation constructions so that they can be evaluated on **encrypted** inputs?


Functional Encryption without Function Privacy
[BRS13a/b]
Function-Private Functional Encryption


Our Approach
Functional Encryption without Function Privacy
Obfuscation for "Small Superset"
[BRS13a/b]
This work
Function-Private Functional Encryption

“Small Superset” Obfuscation [BKMPRS18, BLMZ19, BW19]

For subset $X \subseteq [n]$, define $f_{t,X}(S) = \begin{cases} 1 \text{ if } X \subseteq S \text{ AND } |S| \leq t. \\ 0 \text{ otherwise} \end{cases}$

(Generalizes point functions, conjunctions, and more)

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Obfuscation for $t = 4, X = \{1,5,11\}$

1) Sample random $\vec{v}$ in rowspace of $\underbrace{\begin{bmatrix} 1 & 1^2 & 1^3 & 1^4 & 1^5 \\ 5 & 5^2 & 5^3 & 5^4 & 5^5 \\ 11 & 11^2 & 11^3 & 11^4 & 11^5 \end{bmatrix}}_{t+1}$
2) Output $g^{\vec{v}}$.

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- Use bilinear map $e: G_1 \times G_2 \rightarrow G_T$

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In the generic group model, random R forces "honest" pairing

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Simulate with random group elements

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Simulate with random vectors orthogonal to matching keys

Thank You!

Questions?

Slide Artwork by Eysa Lee