Post-Quantum Zero Knowledge, Revisited
(or: How to do Quantum Rewinding Undetectably)

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Big Question:

When are *classical* cryptosystems secure against *quantum* attacks?

(i.e., post-quantum cryptography)

Why aren’t post-quantum assumptions enough?

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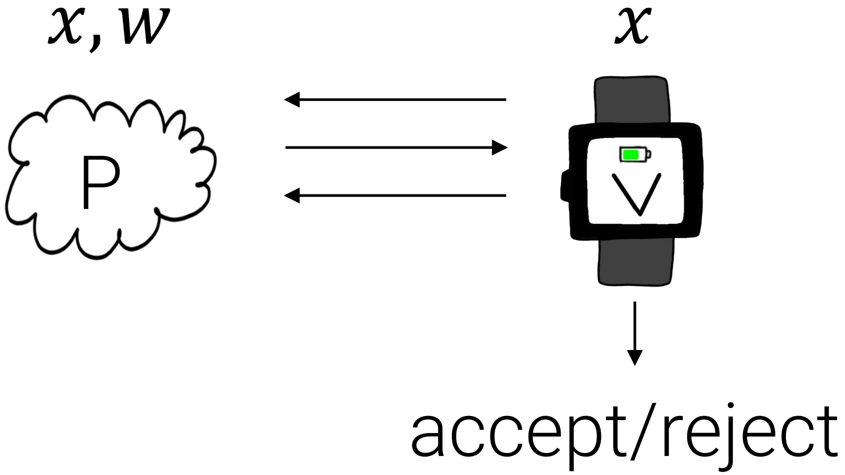
post-quantum security = **post-quantum** definition + **post-quantum** reduction + **post-quantum** assumption

In reality, we also need **definitions** and **reductions** that capture post-quantum security. This can be surprisingly difficult!

This work: zero-knowledge protocols [GMR85]



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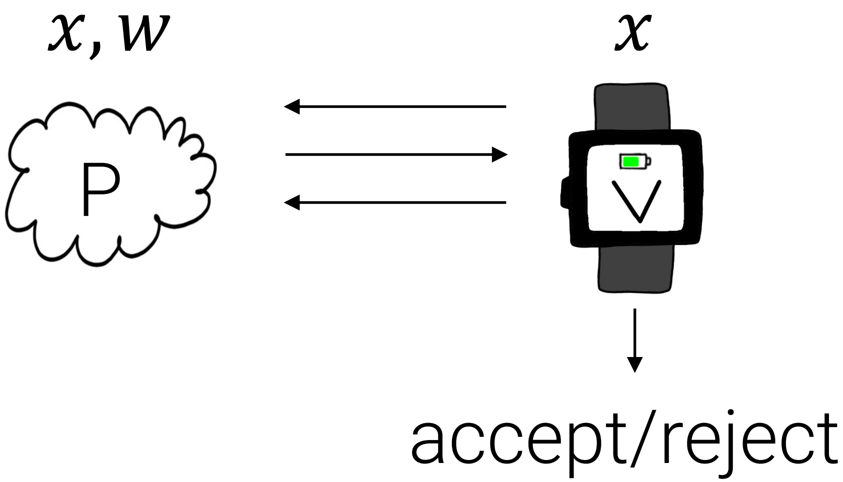

x, w

x

accept/reject

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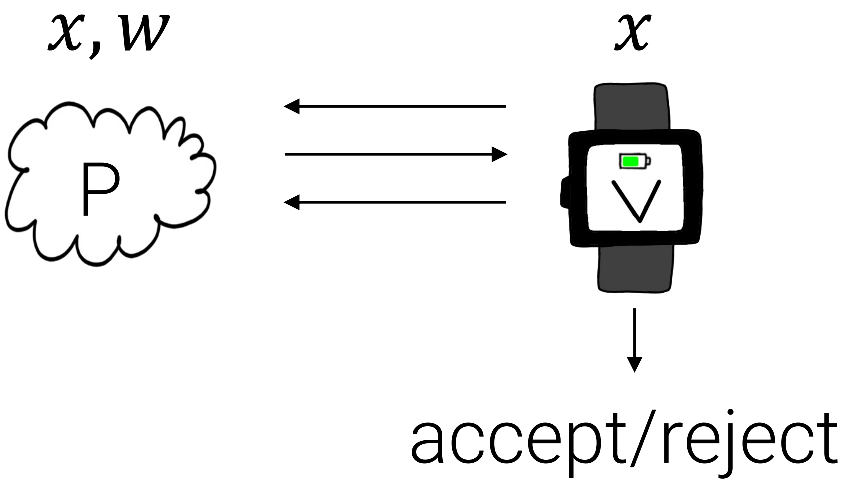

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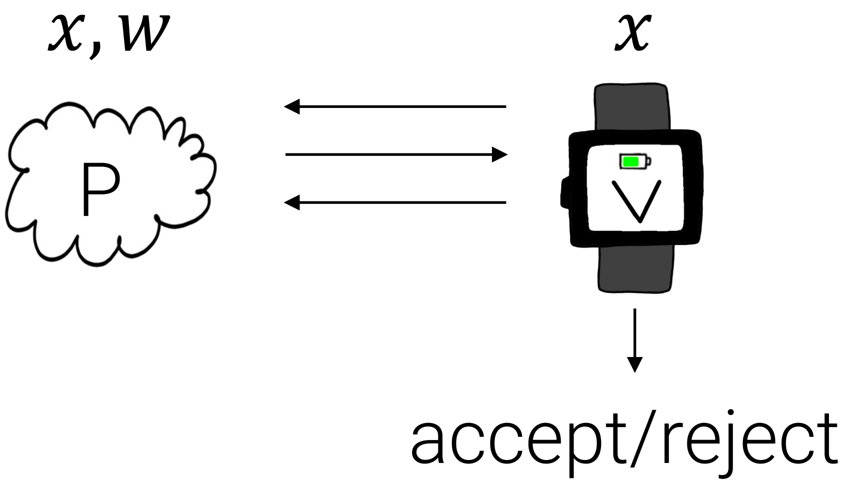

- **Zero knowledge:** proof (of a true statement) can be *efficiently simulated* without the witness.
- **Completeness:** If x is true and P is honest, V accepts.
- **Soundness:** If x is false, P^* cannot make V accept.

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Will this theory hold up against quantum attacks?

Positive results exist [W07,CCY20,BS20], but a cohesive theory is elusive. Even many *textbook ZK protocols* are still not understood!

- [Goldreich-Micali-Wigderson86]: graph non-isomorphism
- [Feige-Shamir90]: ZK via “trapdoor extraction”
- [Goldreich-Kahan96]*: five-message proofs for NP

*[CCY21] proved that [GK96] is post-quantum ε-ZK (a weakening of ZK).

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See paper: [GK96] protocol for NP is post-quantum ZK.

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1. Recap: GNI protocol + “extract-and-simulate”

2. Challenges in the post-quantum setting

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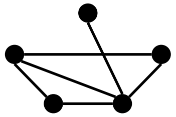
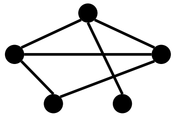
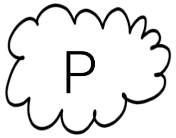
Instance: $G_0 =$  $G_1 =$ 



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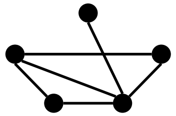
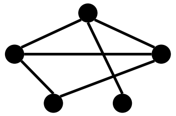
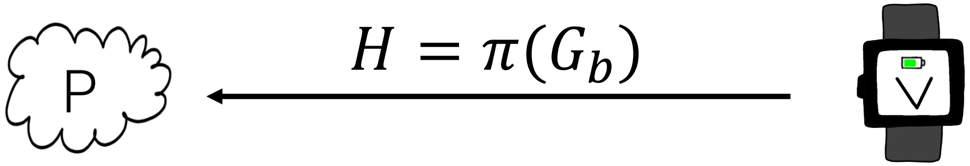
Instance: $G_0 =$  $G_1 =$ 



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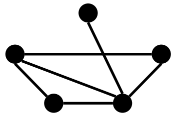
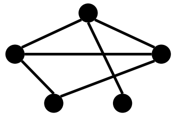
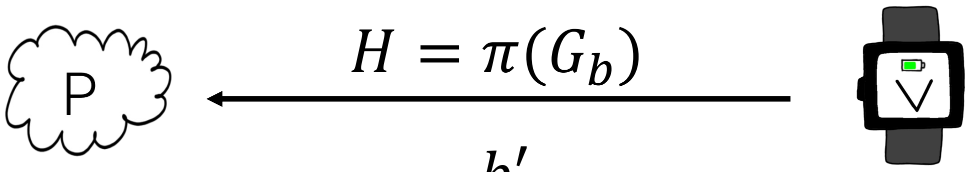
Instance: $G_0 =$  $G_1 =$ 



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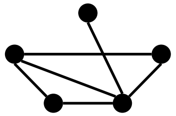
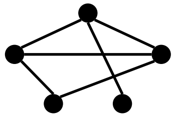
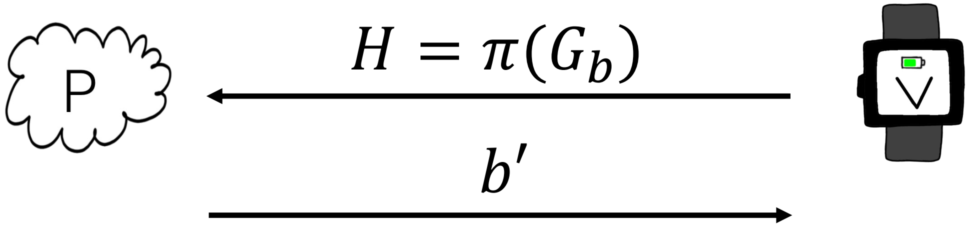
Instance: $G_0 =$  $G_1 =$ 



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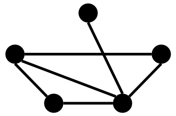
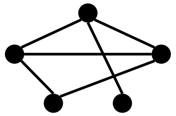
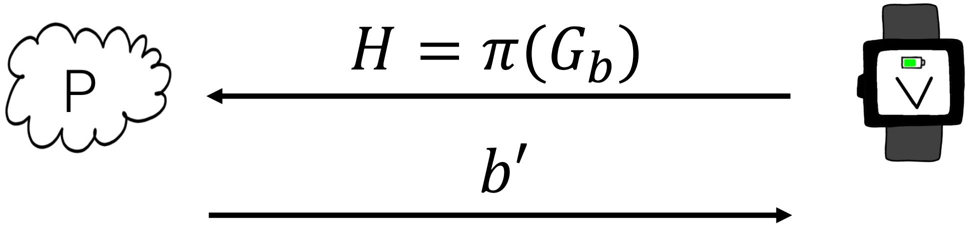
Instance: $G_0 =$  $G_1 =$ 



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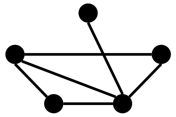
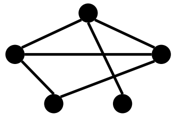
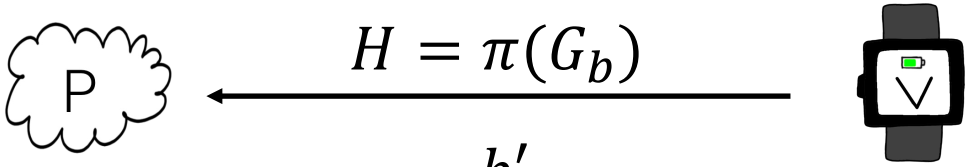
Instance: $G_0 =$  $G_1 =$ 



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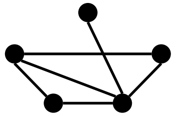
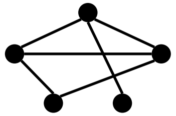
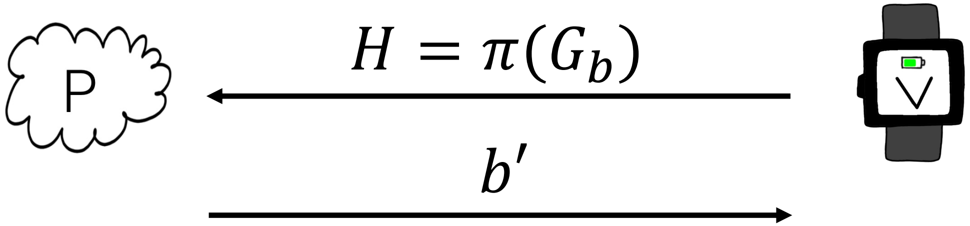
Instance: $G_0 =$  $G_1 =$ 



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In the full [GMW86] protocol, V *proves that it knows b* before P sends b'.

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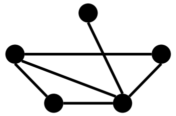
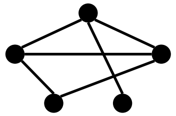
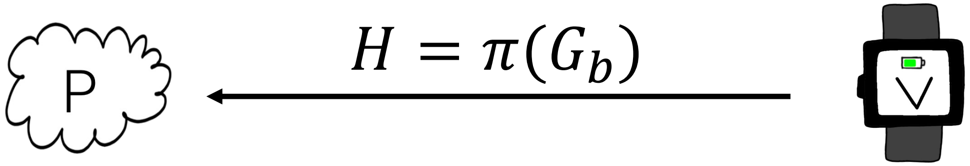
Instance: $G_0 =$  $G_1 =$ 



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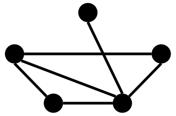
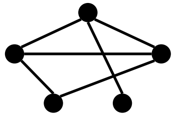
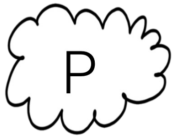
Instance: $G_0 =$  $G_1 =$ 



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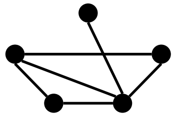
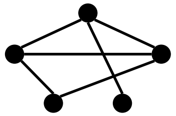
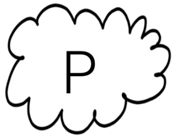
Instance: $G_0 =$  $G_1 =$ 



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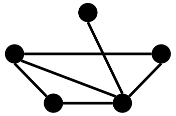
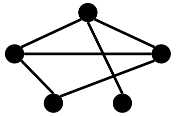
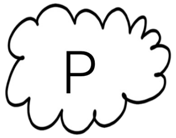
Instance: $G_0 =$  $G_1 =$ 



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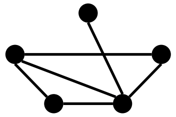
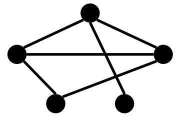
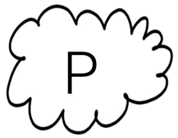
Instance: $G_0 =$  $G_1 =$ 



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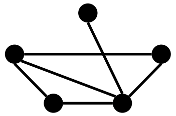
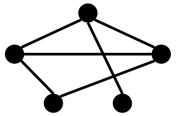
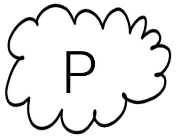
Instance: $G_0 =$  $G_1 =$ 



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Why does this prove knowledge of b? If V^* can answer both challenges correctly, it must know which of G_0, G_1 is isomorphic to H

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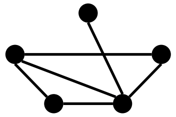
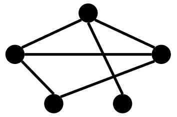
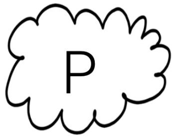
Instance: $G_0 =$  $G_1 =$ 



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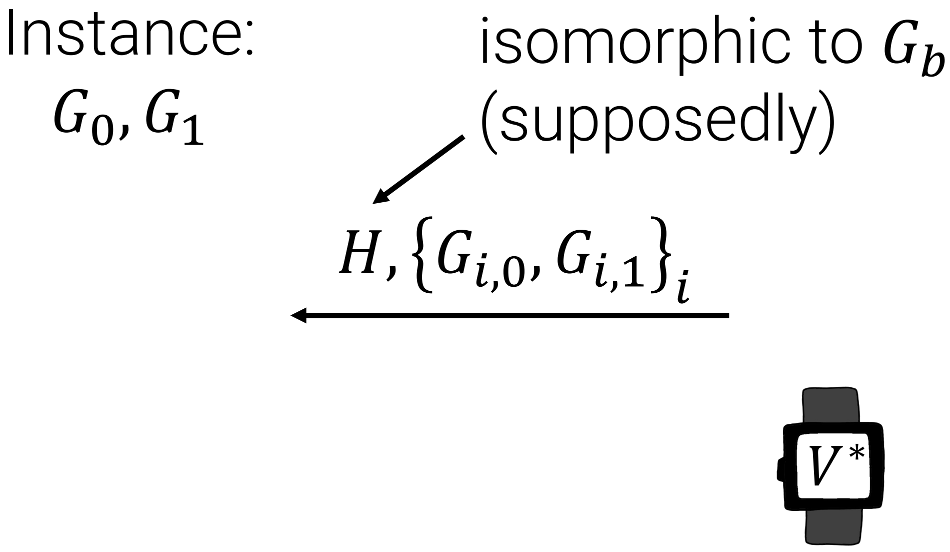

Instance:
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isomorphic to G_b
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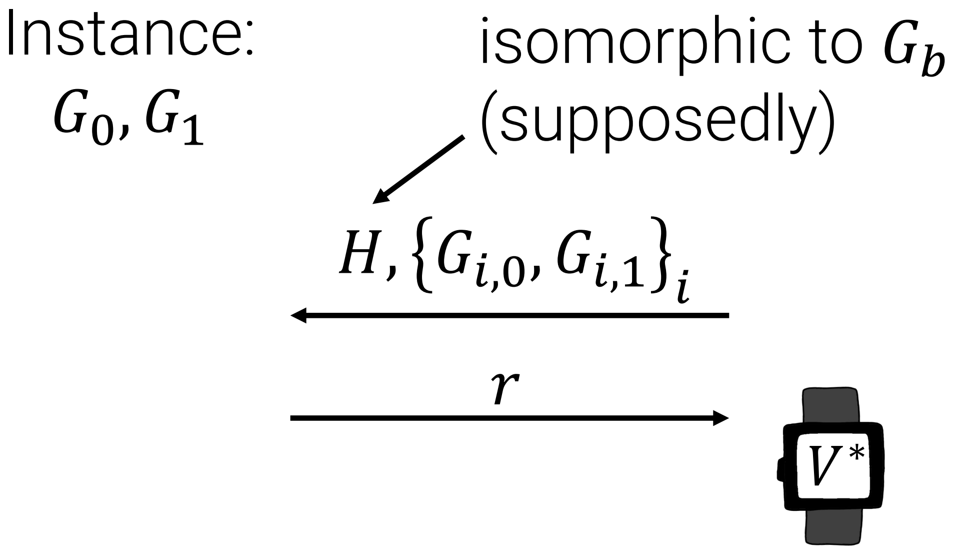

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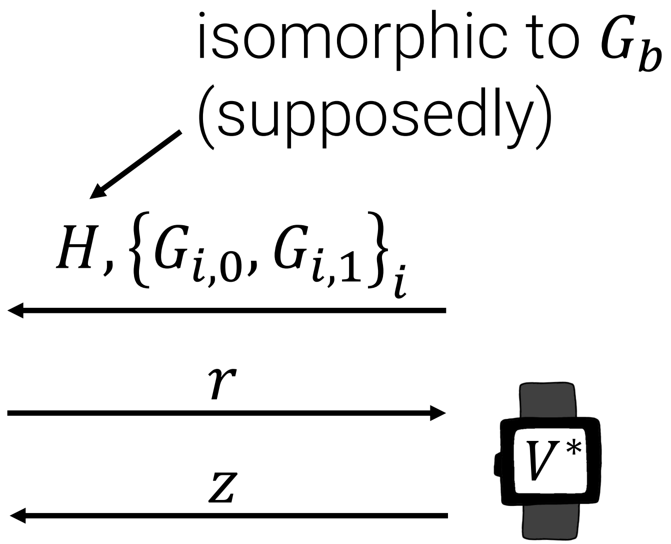

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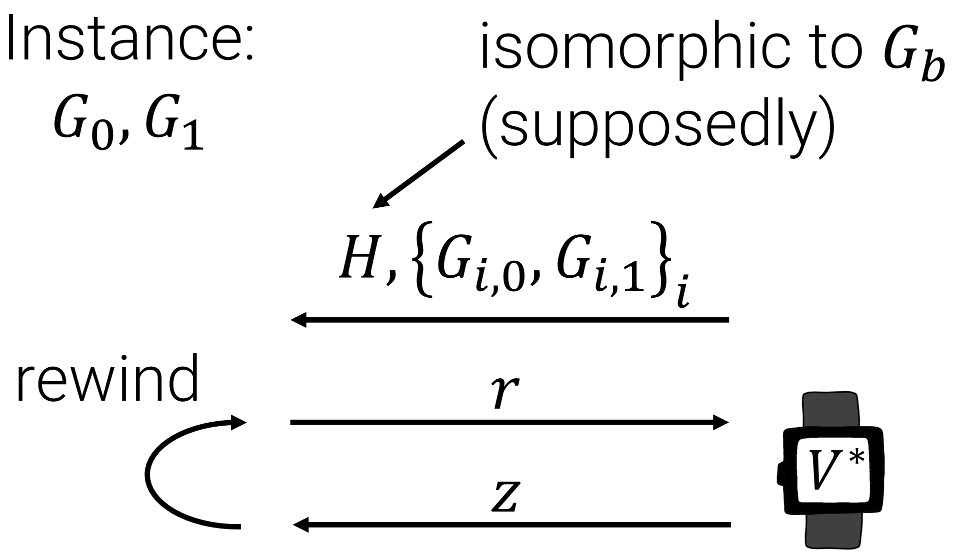

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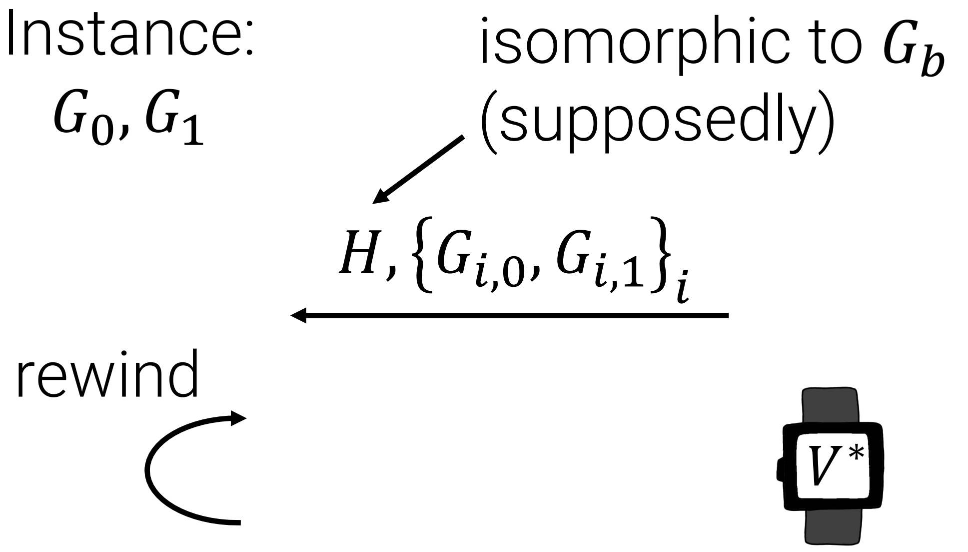

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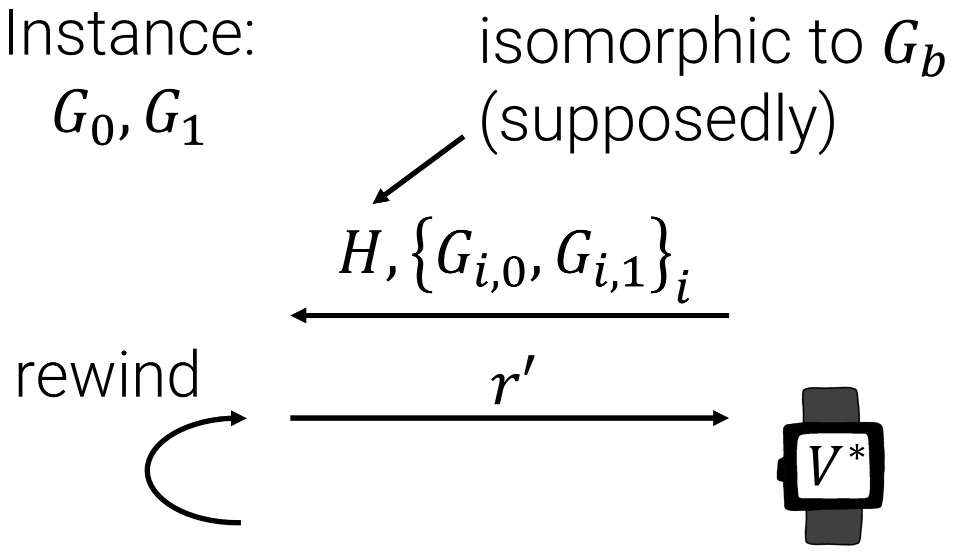

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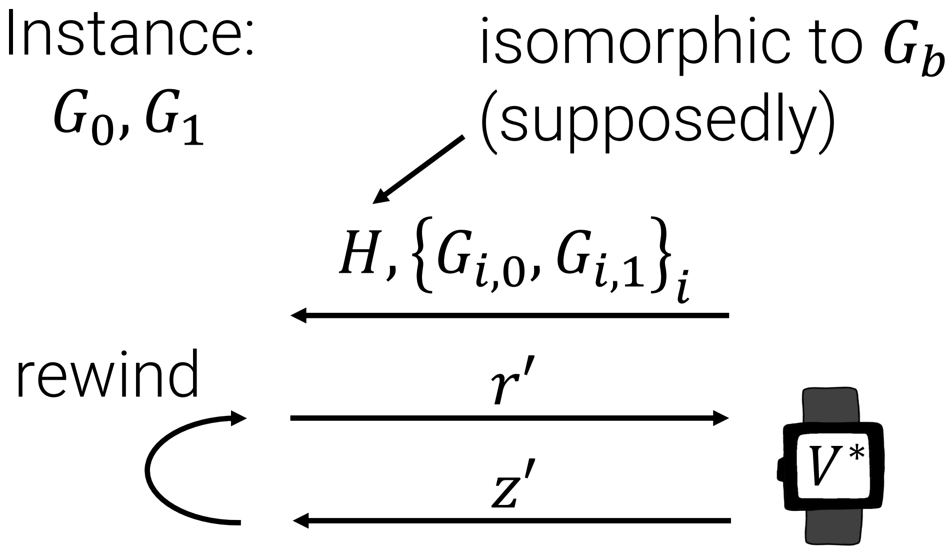

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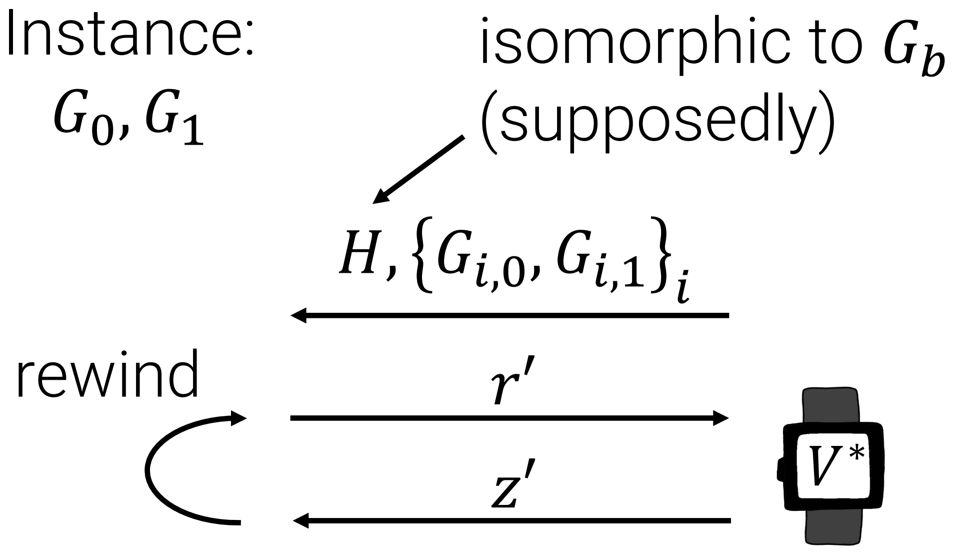

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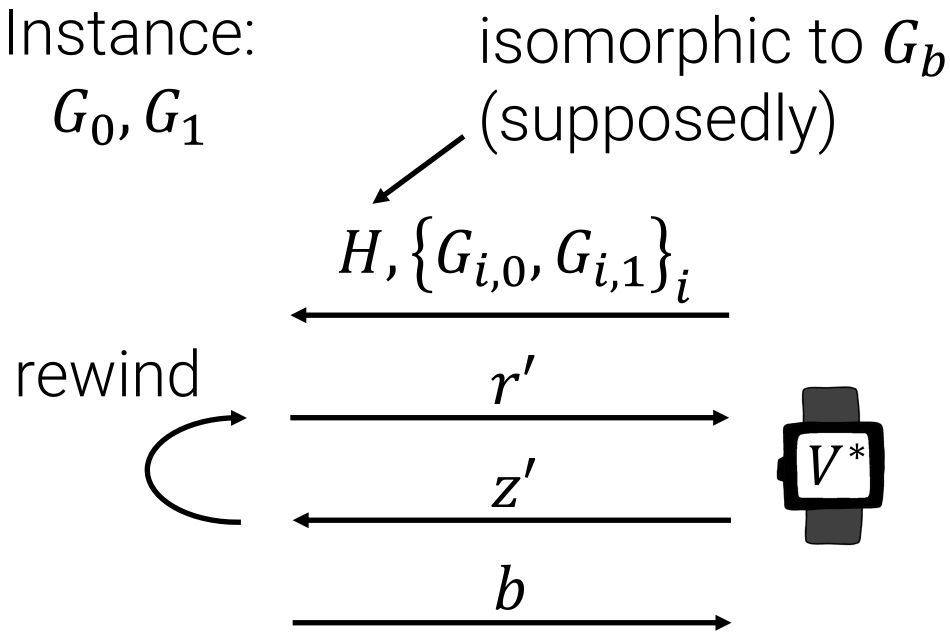

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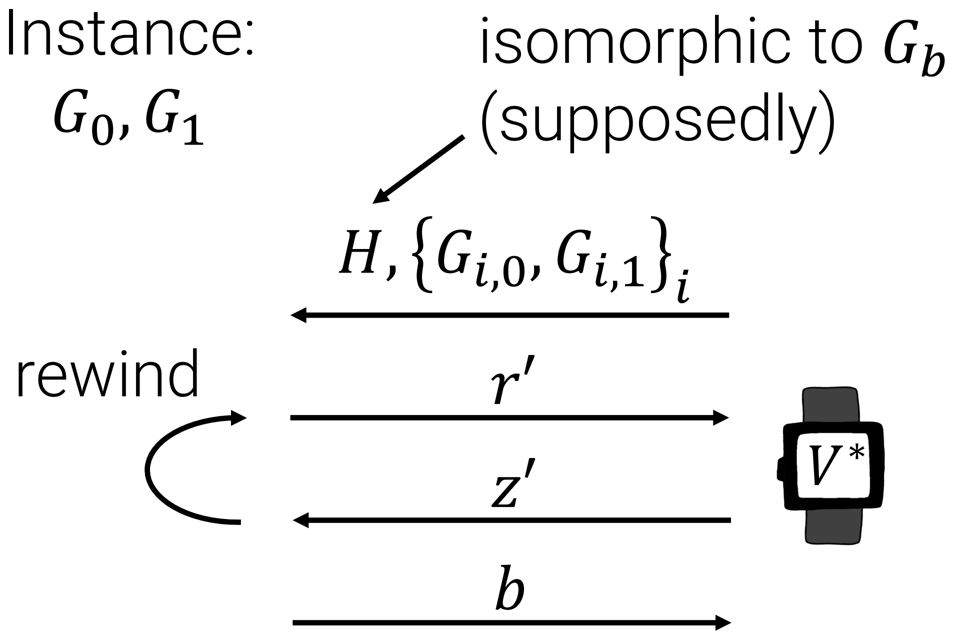

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This "extract and simulate" approach appears in many textbook ZK protocols [GMR85,GMW86,FS90]

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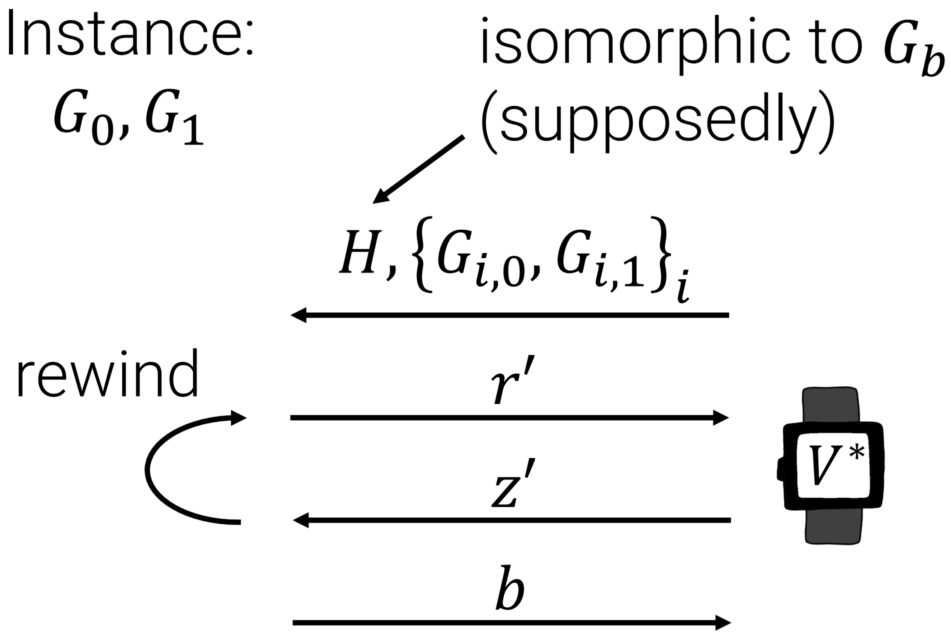

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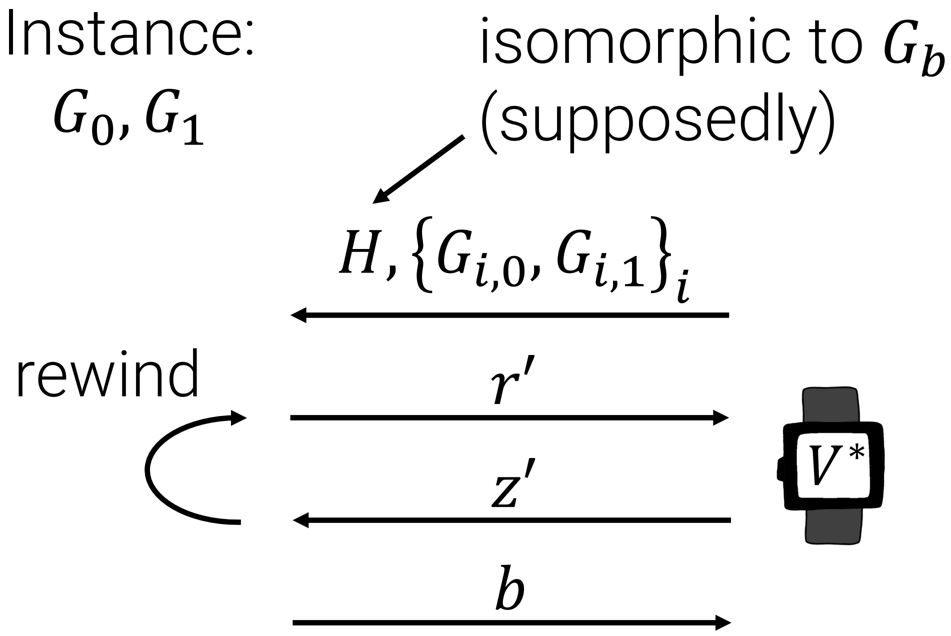

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ZK def allows *expected poly-time* simulation $(\varepsilon \cdot \frac{1}{\varepsilon} = 1)$

Plan for Today

1. Recap: GNI protocol + “extract-and-simulate” ✓
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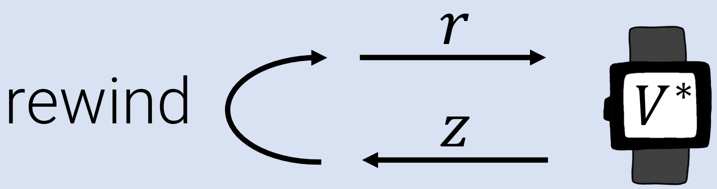

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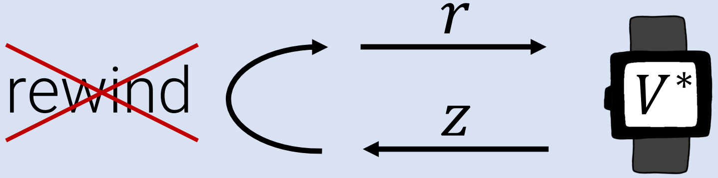

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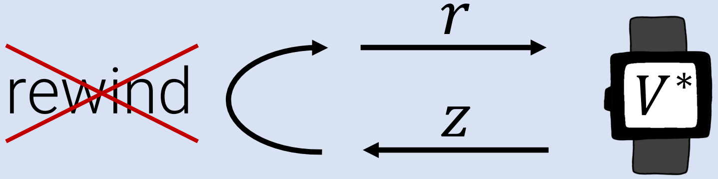

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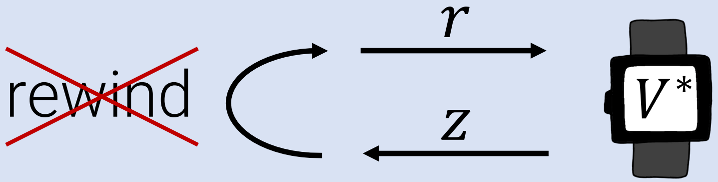

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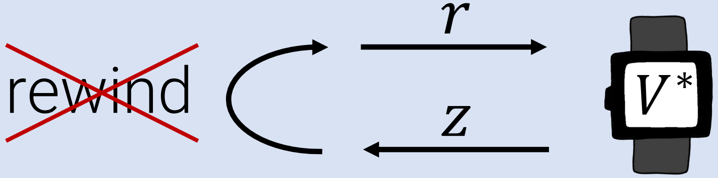

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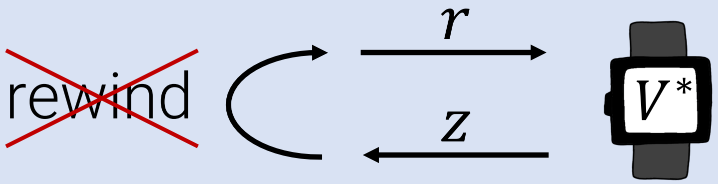

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- Defining variable-time quantum computation is subtle. [M97,O98,LP98]

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This work: new ZK def that circumvents CCLY21 barrier.

This Work: Measured vs. Coherent EQPT Simulation

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Measured EQPT

halt qubit $|0\rangle$ —

input $|\psi\rangle$ —

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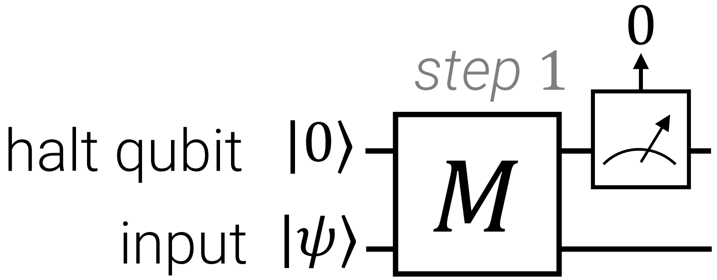

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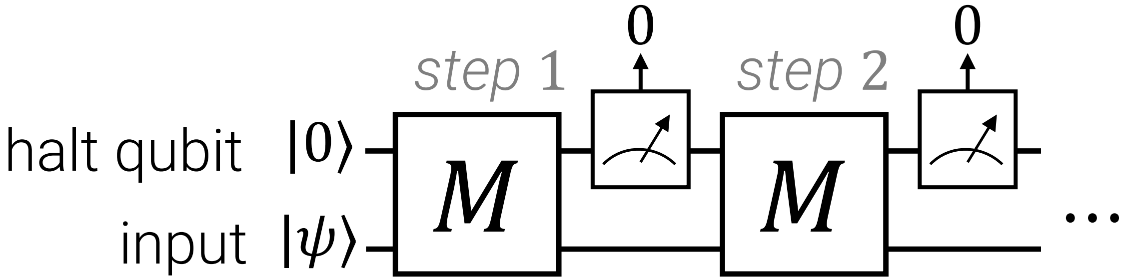

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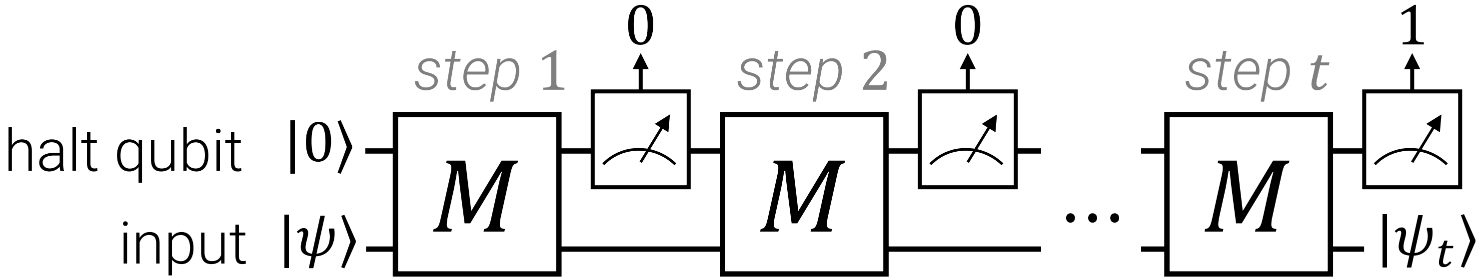

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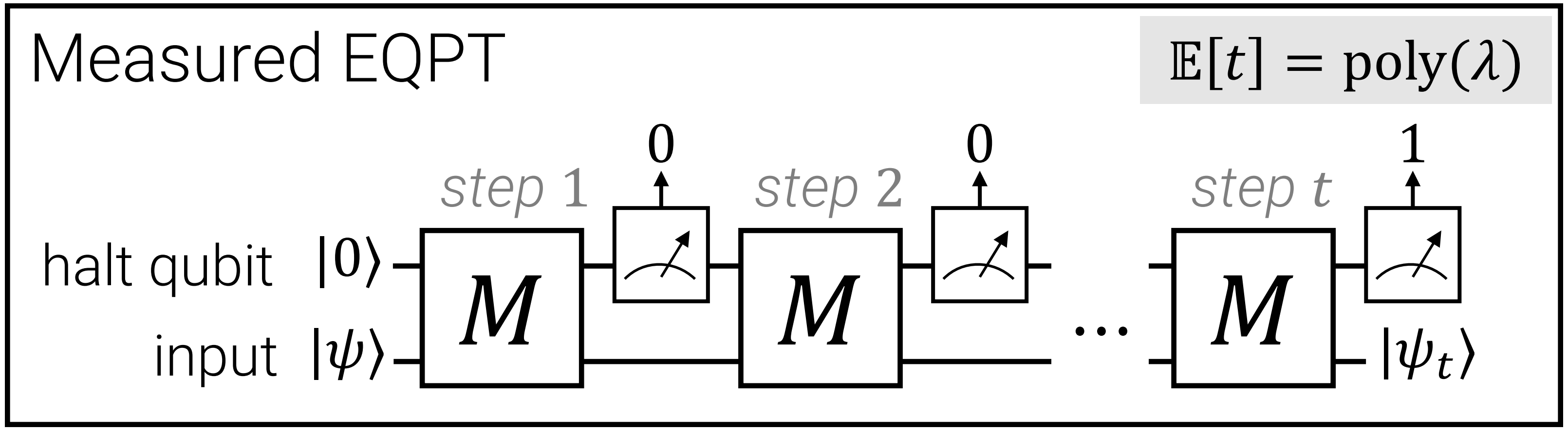

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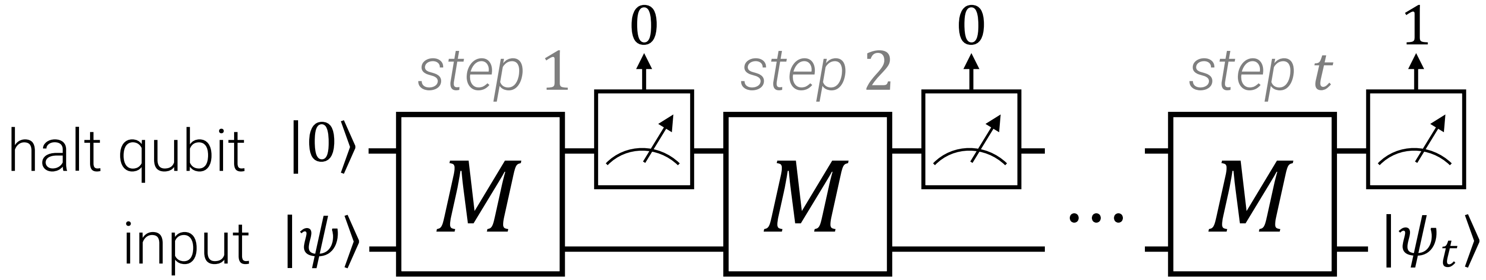
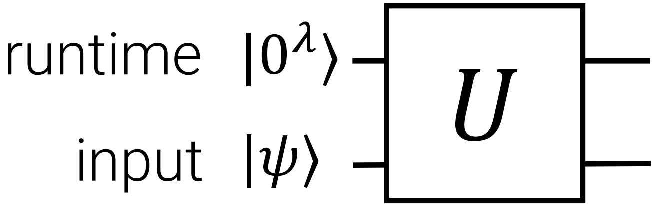


Coherent EQPT



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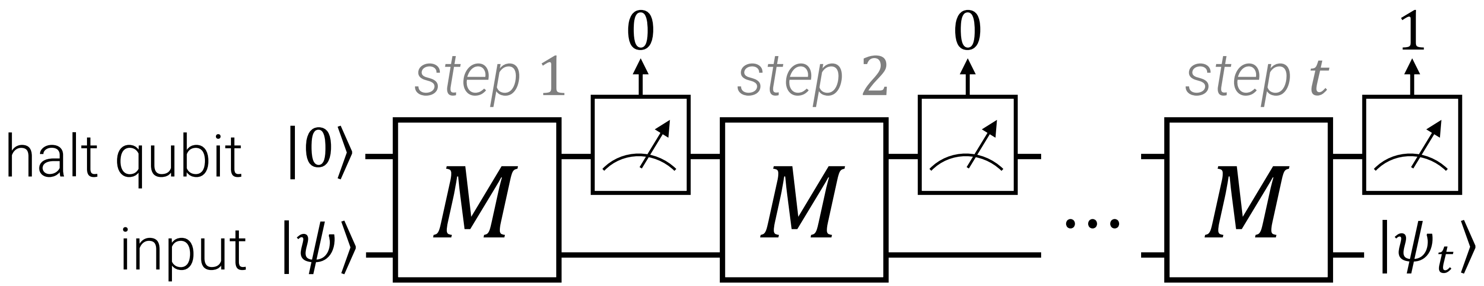

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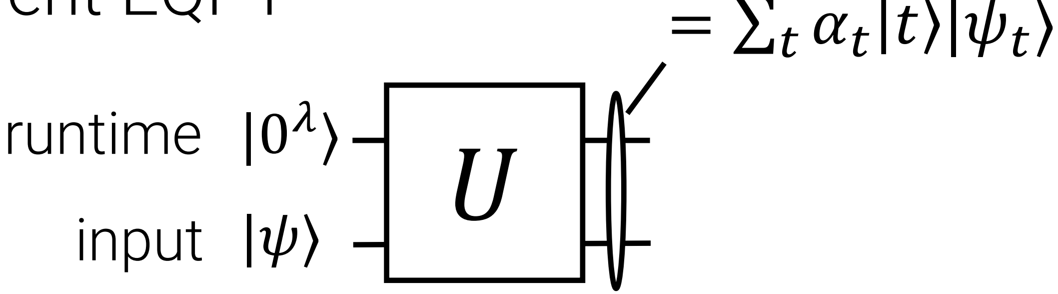

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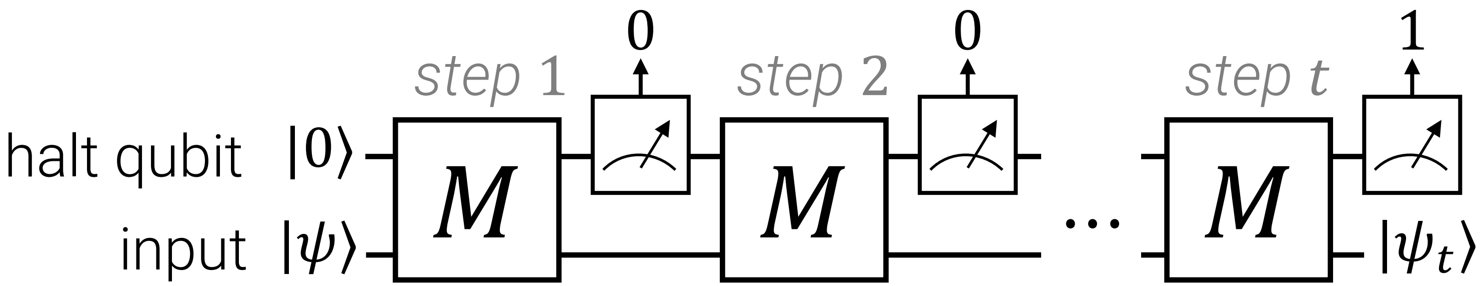

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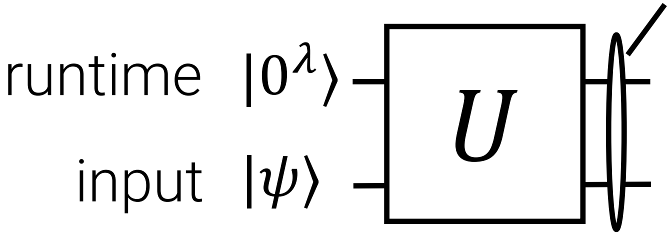

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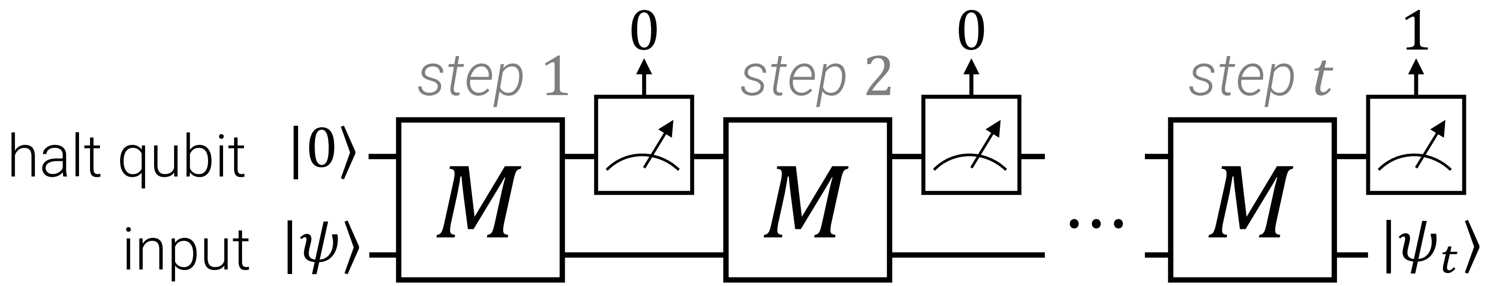

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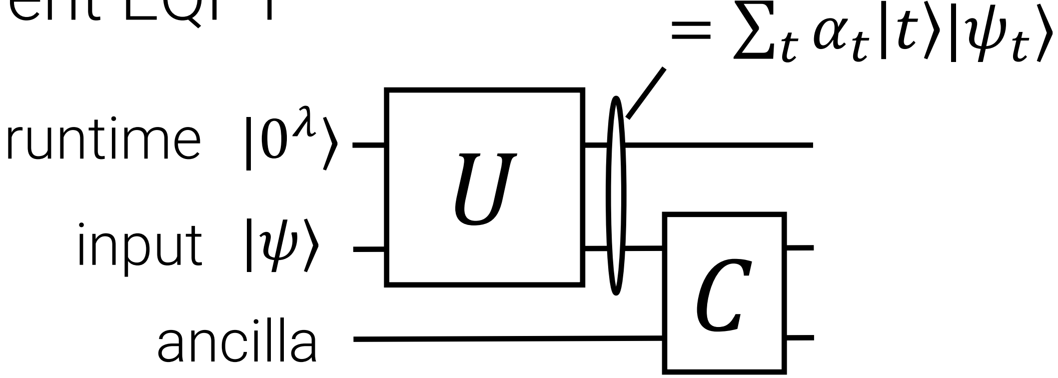

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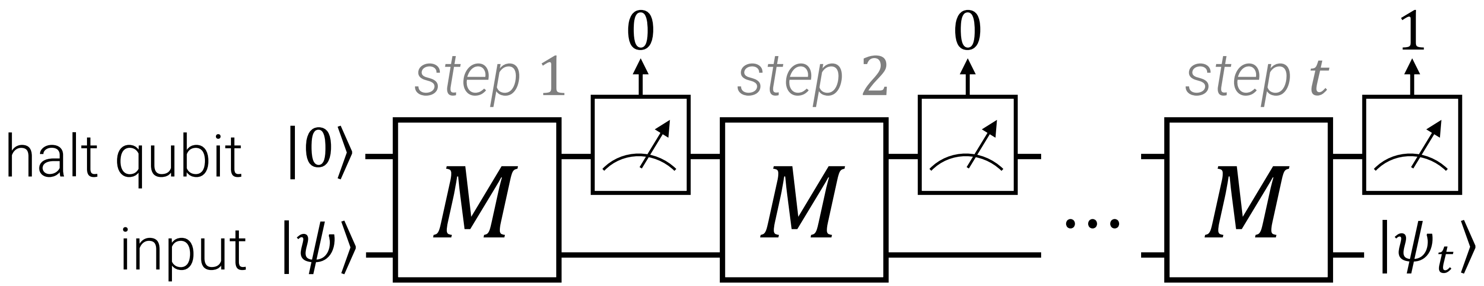

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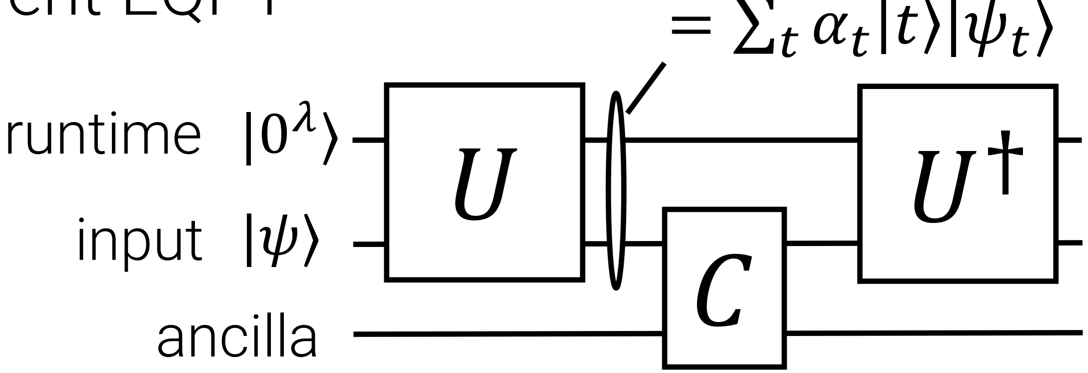

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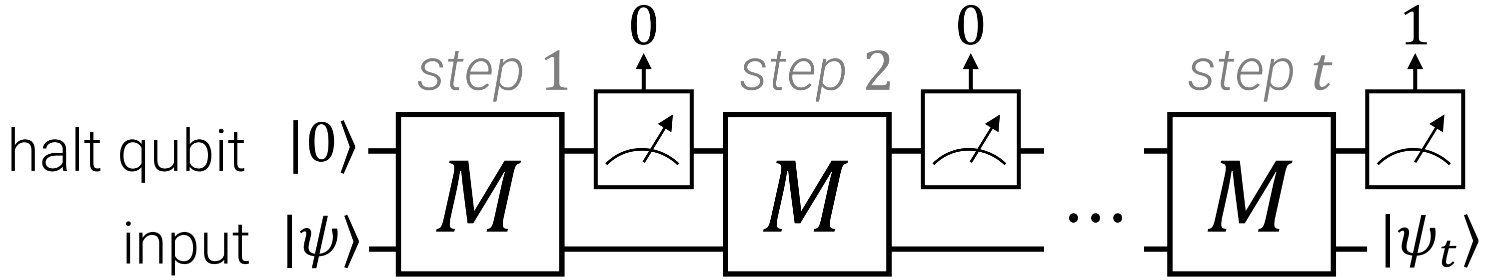

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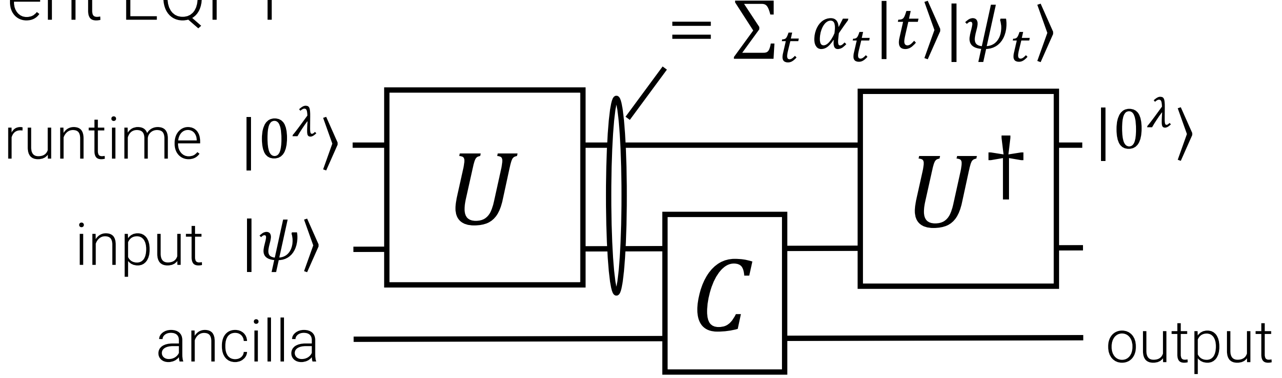

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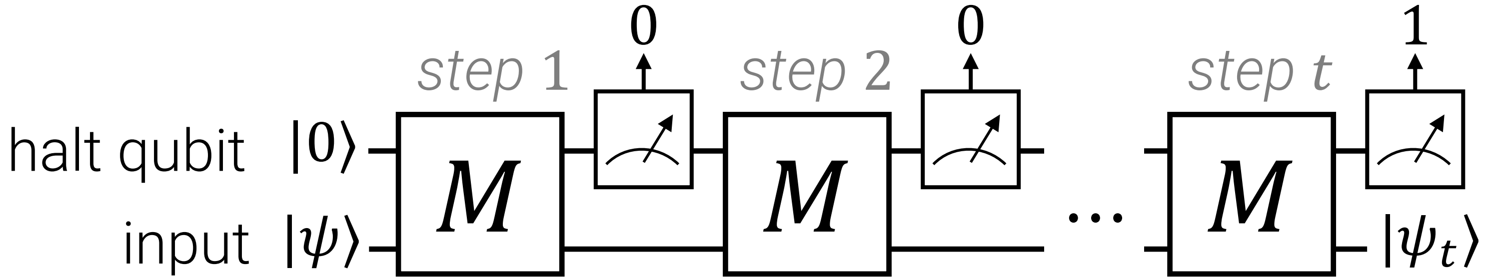

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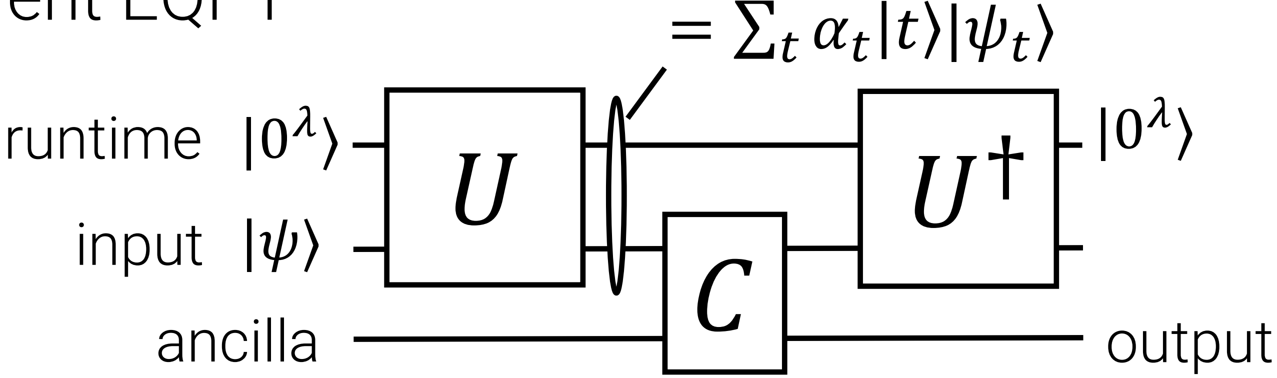

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CCLY21 does not rule out coherent EQPT simulation!

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Related: why is EPT simulation reasonable in classical ZK?

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Ex: if assumption A is broken in EPT, it's also broken in poly time.

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Related: why is EPT simulation reasonable in classical ZK?

1) **EPT is no stronger than poly-time.**

Ex: if assumption A is broken in EPT, it's also broken in poly time.

2) **EPT appears to be the weakest model that makes ZK possible.***

[BL02]: poly-time sim is impossible.*

*for constant-round protocols
w/ black-box simulation

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We show an analogous state of affairs for coherent EQPT.

1) Coherent EQPT is no stronger than QPT.

Ex: if assumption A is broken in coherent EQPT, it's also broken in QPT.

2) Coherent EQPT appears to be the weakest model that makes post-quantum ZK possible.*

[CCLY21]: measured EQPT sim impossible.*

*for constant-round protocols
w/ black-box simulation

Plan for Today

1. Recap: GNI protocol + “extract-and-simulate” ✓
2. Challenges in the post-quantum setting ✓
3. This work: defining post-quantum ZK ✓
4. This work: quantum extract-and-simulate

Goal: build coherent EQPT simulator that extracts b from V^* without disturbing its internal state $|\psi\rangle$.

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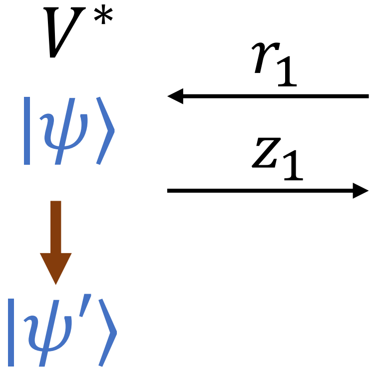

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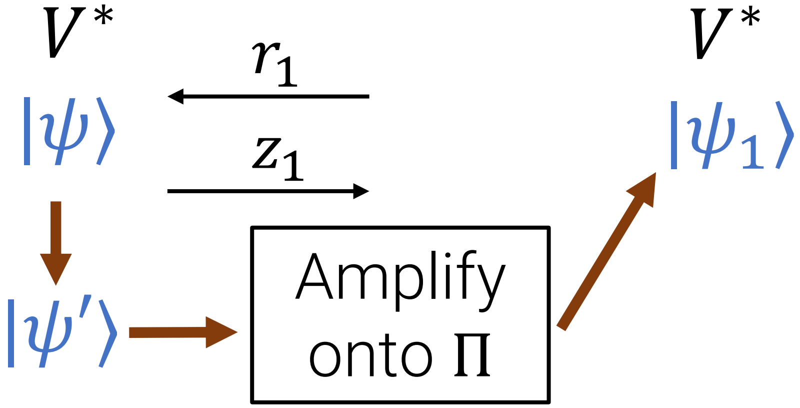

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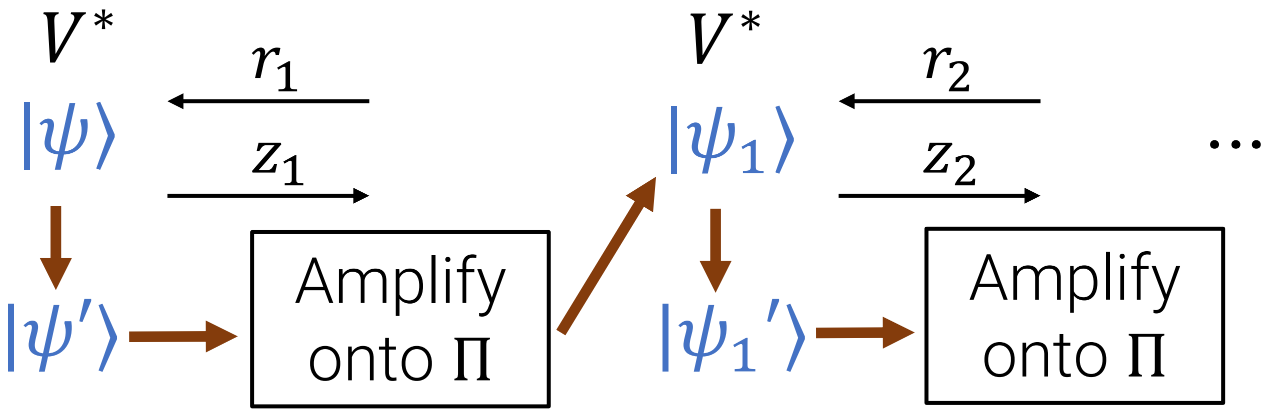

V^*

$|\psi\rangle$

r_1

z_1

$|\psi'\rangle$

Amplify onto Π

V^*

$|\psi_1\rangle$

r_2

z_2

$|\psi_1{}'\rangle$

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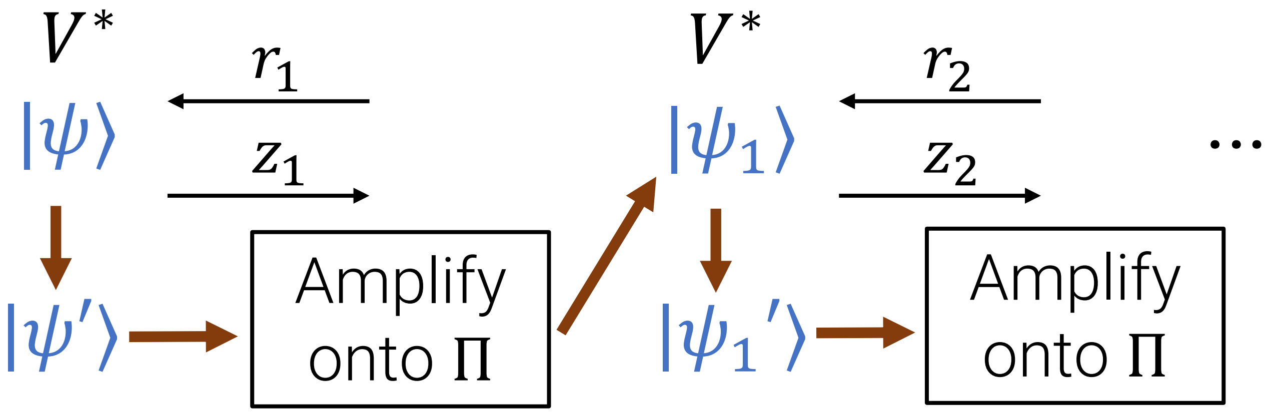

Idea: define variable-time procedure

$\mathrm{Ext} =$"Run CMSZ to get valid $(r, z), (r', z')$"

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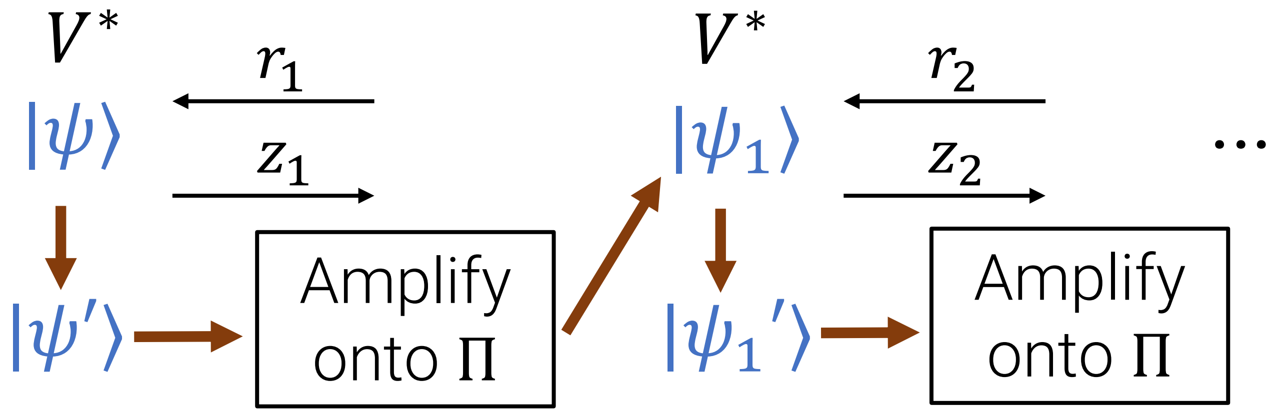

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1) Run U_{Ext} (coherent version of Ext)

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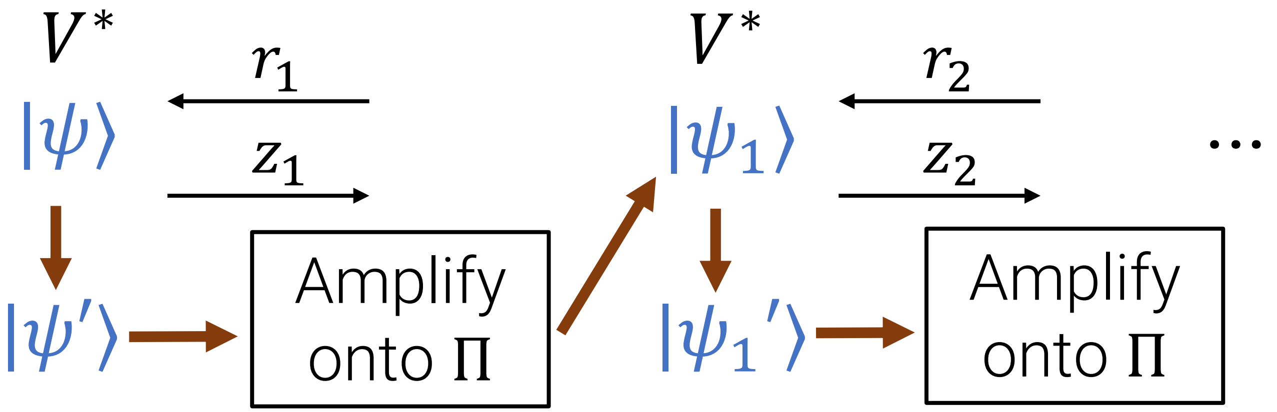

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This works if:

- Ext is measured-EQPT
- Ext succeeds with probability ≈ 1
- b is unique.

Big problem: CMSZ-based $\mathbf{Ext}$ doesn’t run in expected poly-time!

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$$\text{Expected runtime is } \varepsilon \underbrace{\left(\frac{1}{\varepsilon}\right)}_{\text{Classical runtime}} \underset{\text{CMSZ repair runtime}}{\left(\frac{1}{\varepsilon^4}\right)} = \mathbf{unbounded}$$

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- New rewinding template where accepting (r, z) is generated by amplifying adversary onto *guaranteed accepting executions* instead of querying on random r.
  - Not obvious: why does repair still work?
- Further speed up CMSZ-style repair with new, variable-time algorithms based on the quantum singular value transform [GSLW19].

Conclusions

- Definitions and reductions for post-quantum ZK are subtle!
- We define post-quantum ZK using **coherent** EQPT simulation.
- We build coherent EQPT simulators for textbook ZK protocols by combining a new rewinding template with variable-time algorithms based on [CMSZ21,GLSW19].

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Future directions

- Develop general theory of post-quantum ZK.
- Quantum-communication ZK? Natural starting point: [BG20, GJMZ22]

Thanks for listening!