# Post-Quantum Proof Techniques Part 1:

# Introduction to Quantum Rewinding

Fermi Ma

(Simons & Berkeley)

Based on:

- "Quantum Proofs of Knowledge" by **Dominique Unruh** (2012)
- "Computationally Binding Quantum Commitments" by **Dominique Unruh** (2016)
- "Zero Knowledge Against Quantum Attacks" by **John Watrous** (2005)
- "Quantum Arthur Merlin Games" by **Chris Marriott** and **John Watrous** (2005)
- "Traité des substitutions et des équations algébriques" by **Camille Jordan** (1870)

# Today's Goal:

We want *classical* cryptography secure against *quantum* attacks (post-quantum cryptography)

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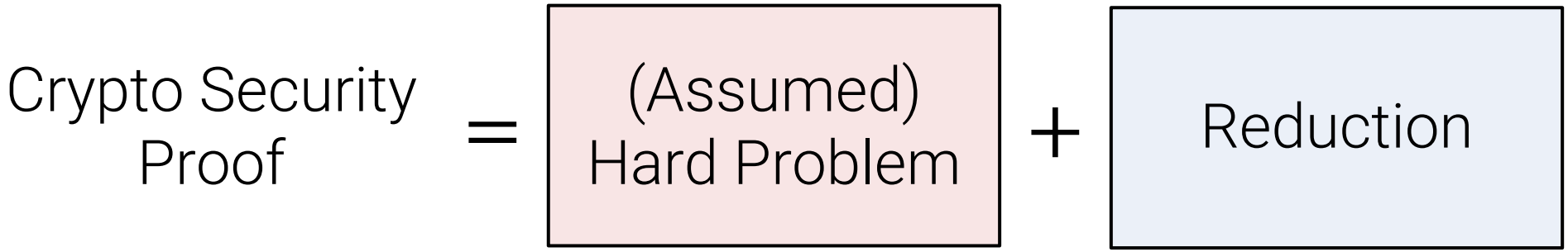

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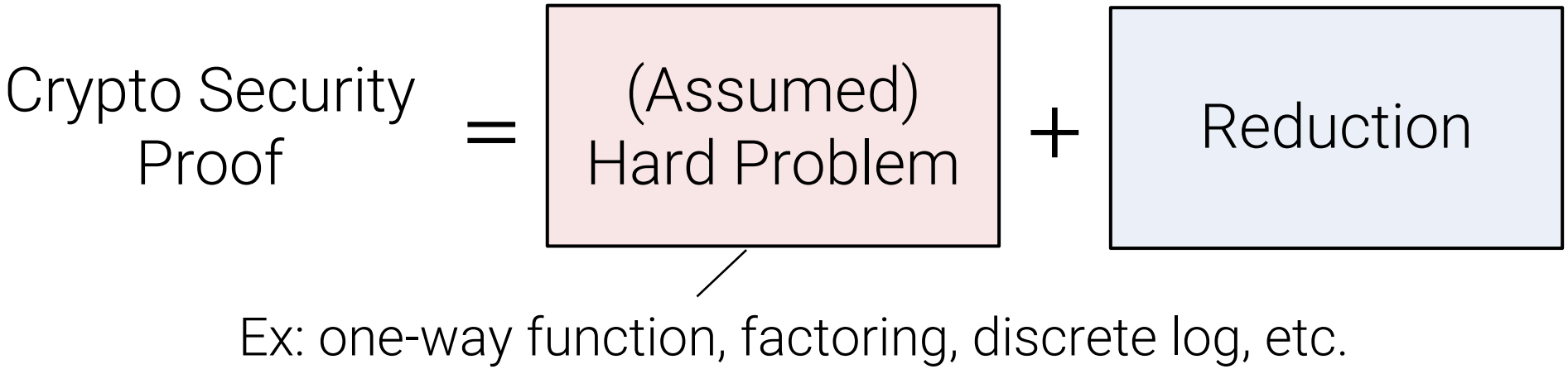

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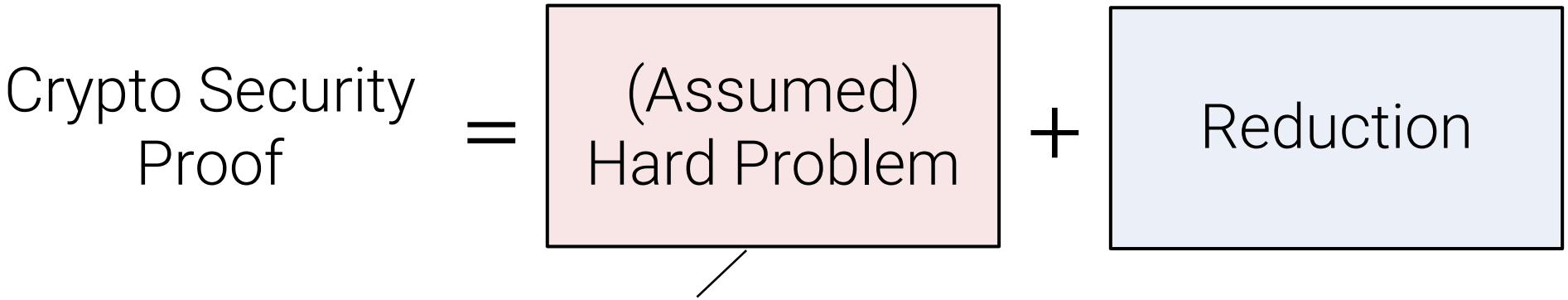

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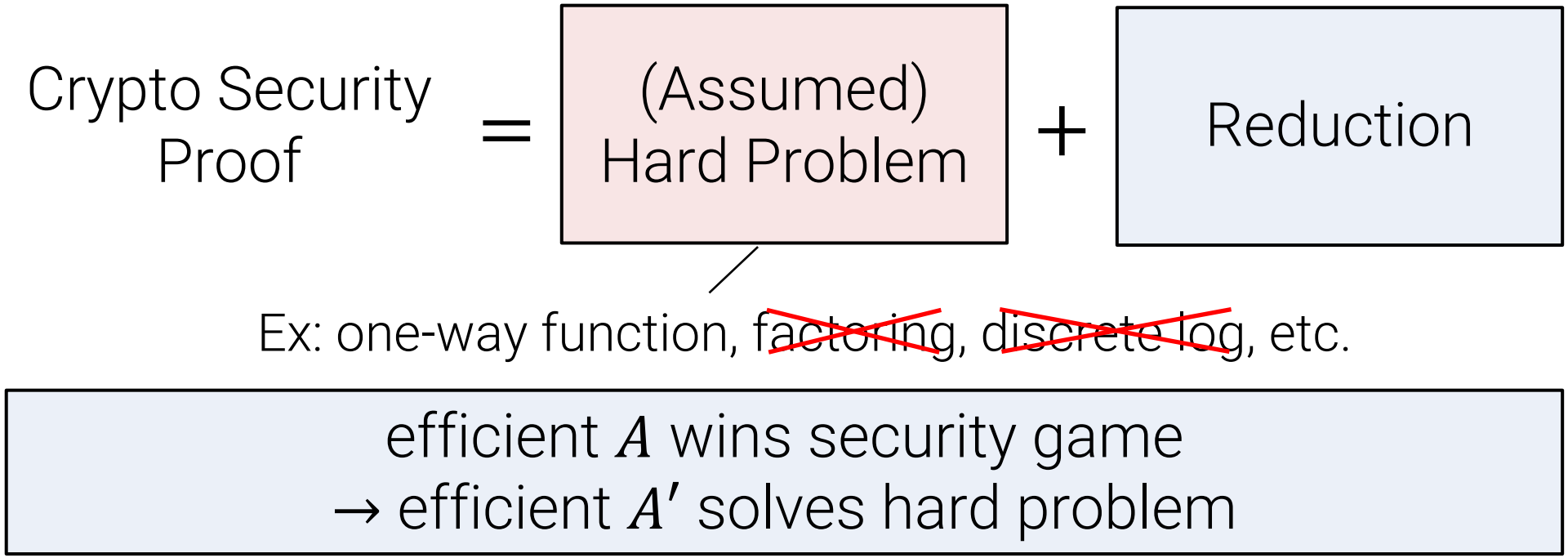


**Key point:** problem must be hard for quantum computers!

# How do cryptographers prove security?

Crypto Security Proof = *Quantum-Hard* Problem + Reduction

efficient $A$ wins security game
$\rightarrow$ efficient $A'$ solves hard problem

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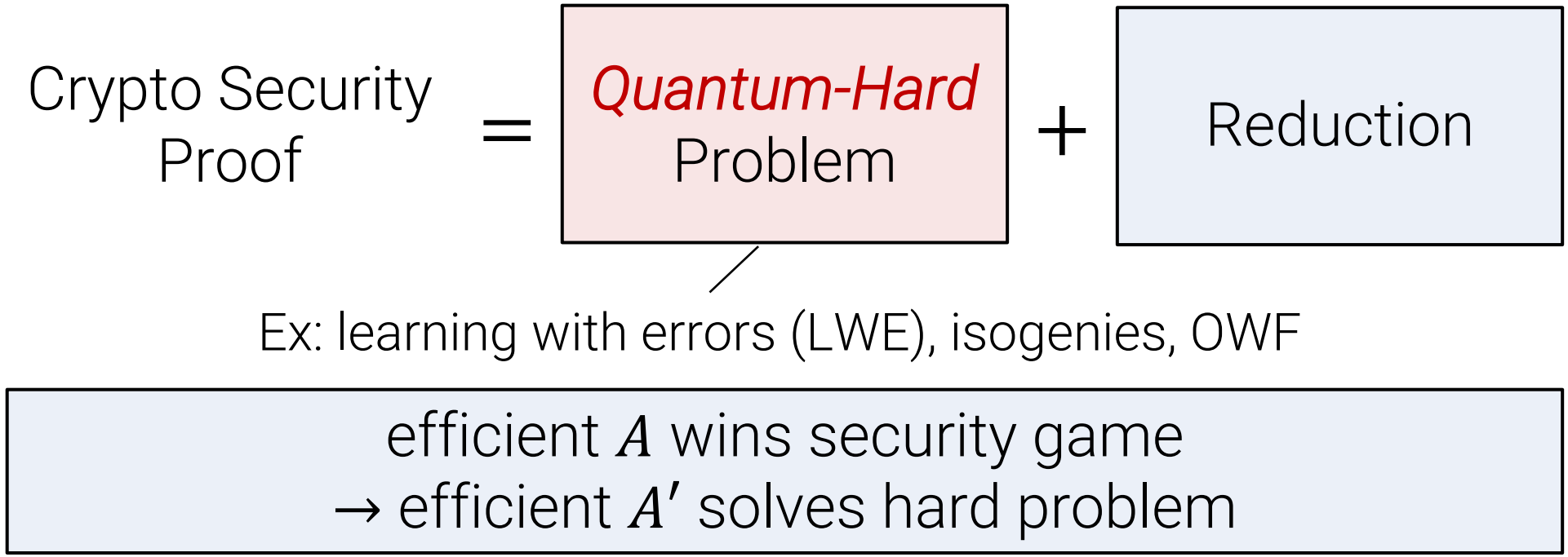


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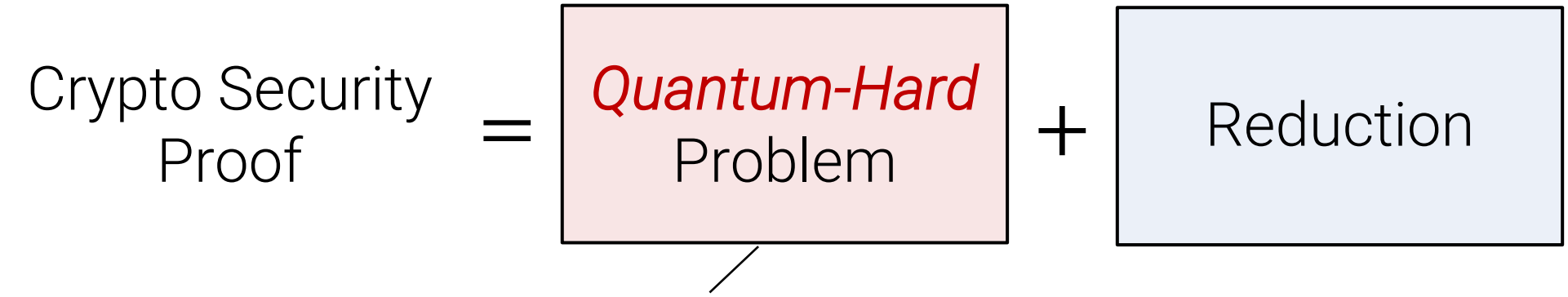


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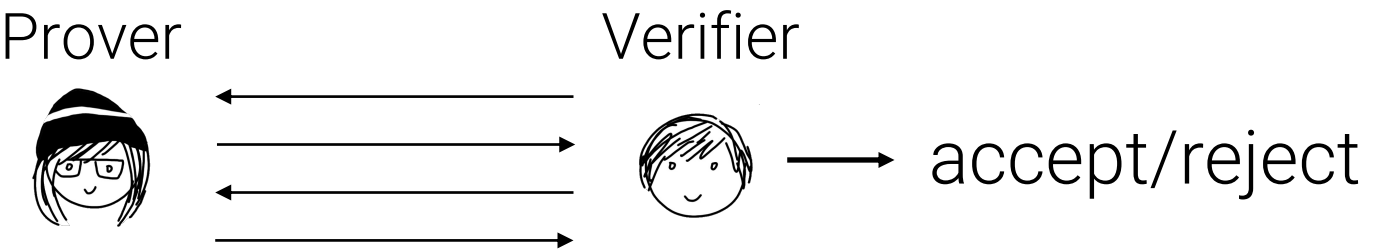

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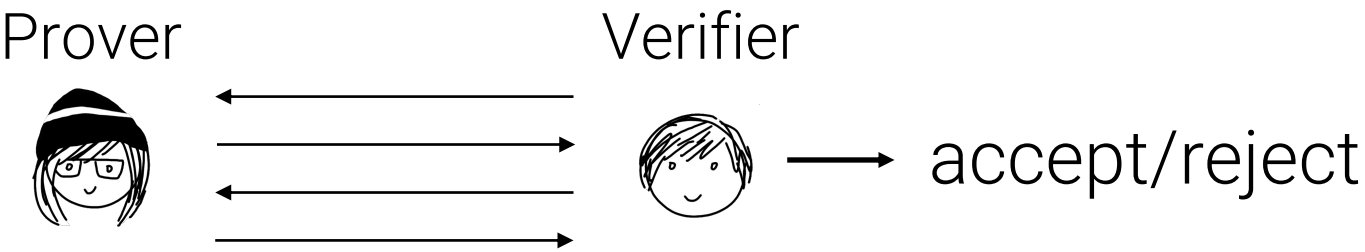


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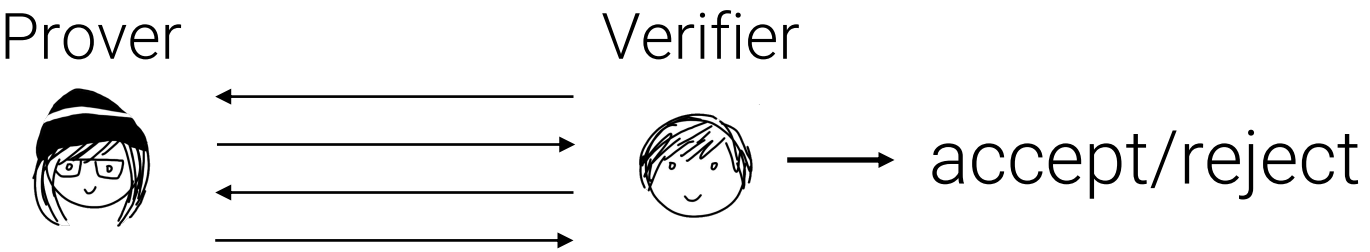


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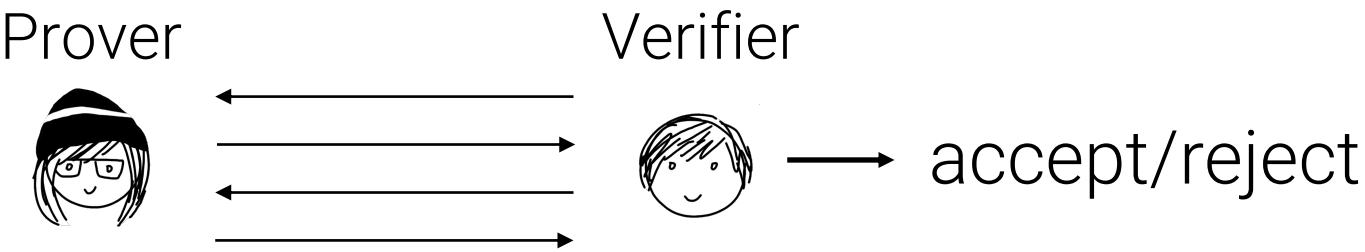


- Efficient classical P *cannot* make V accept assuming LWE.
- Efficient quantum P *can* convince V to accept.

In [BCMVV18] this is presented as a proof of quantumness.

## Takeaway

Quantum computers can break classically secure crypto *without* solving the underlying hard problem!

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| Classical reduction: | efficient *classical* $A$ wins security game → efficient *classical* $A'$ solves hard problem |
| Quantum reduction: | efficient *quantum* $A$ wins security game → efficient *quantum* $A'$ solves hard problem |

Crucially, the classical security reduction for the [BCMVV18] protocol ***does not handle quantum attacks***.

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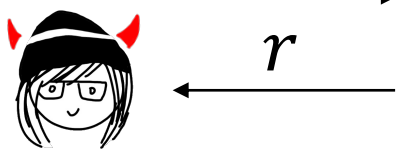


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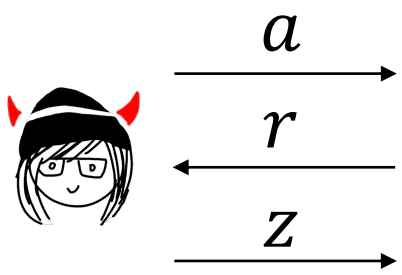


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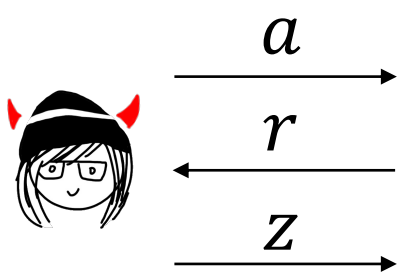


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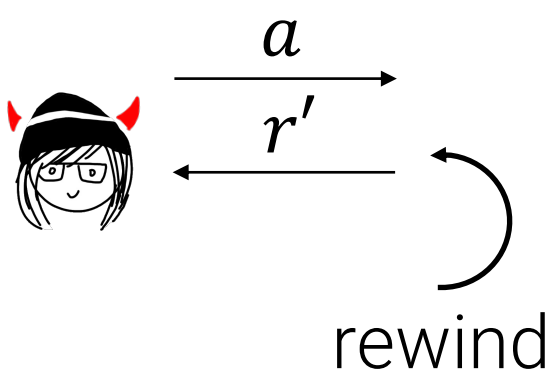


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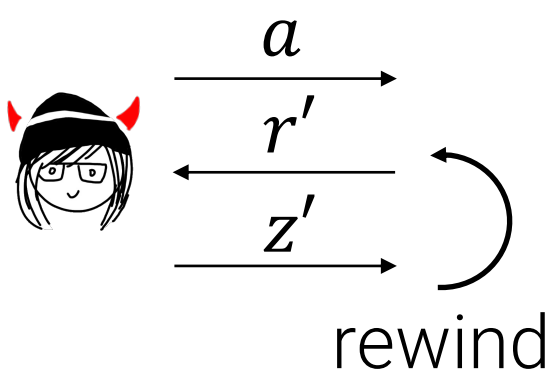


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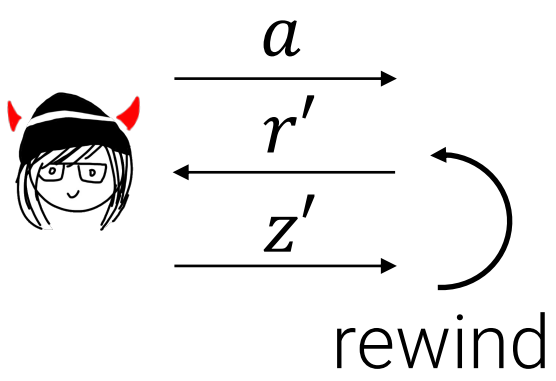


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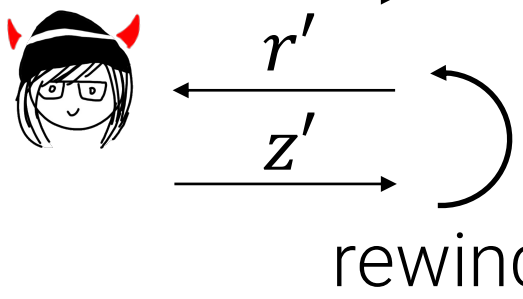


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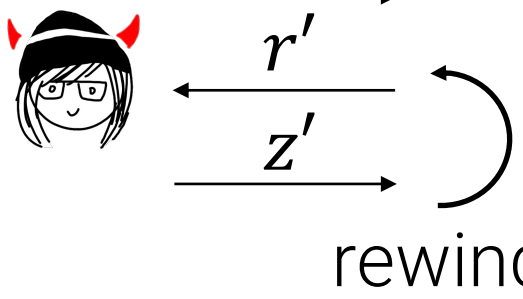


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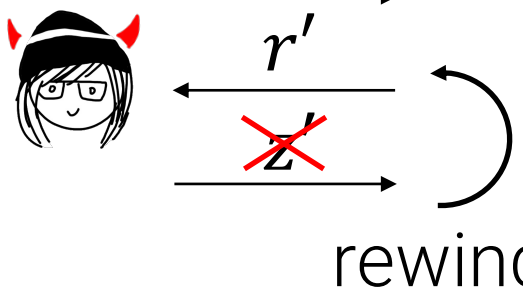


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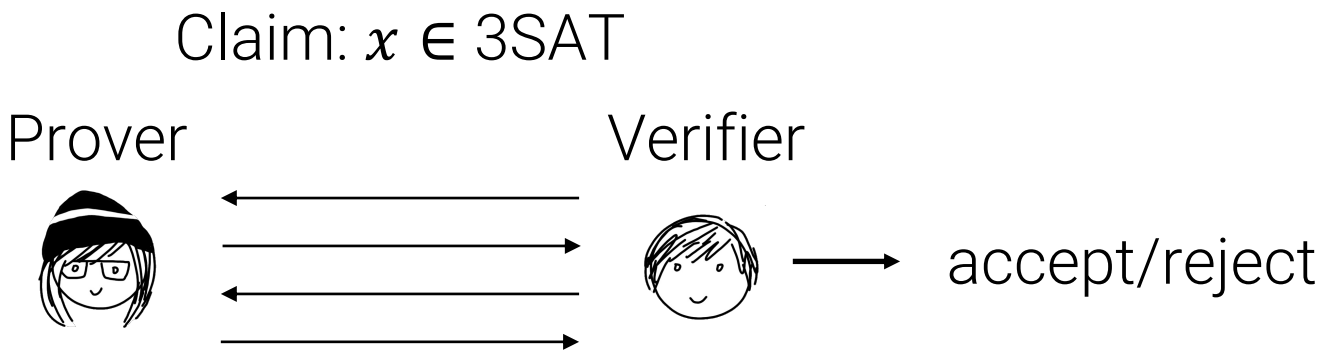

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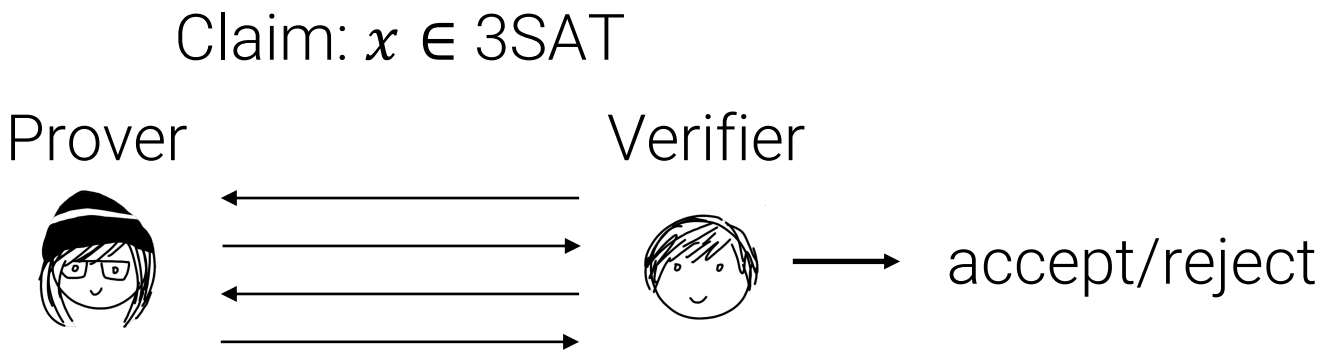


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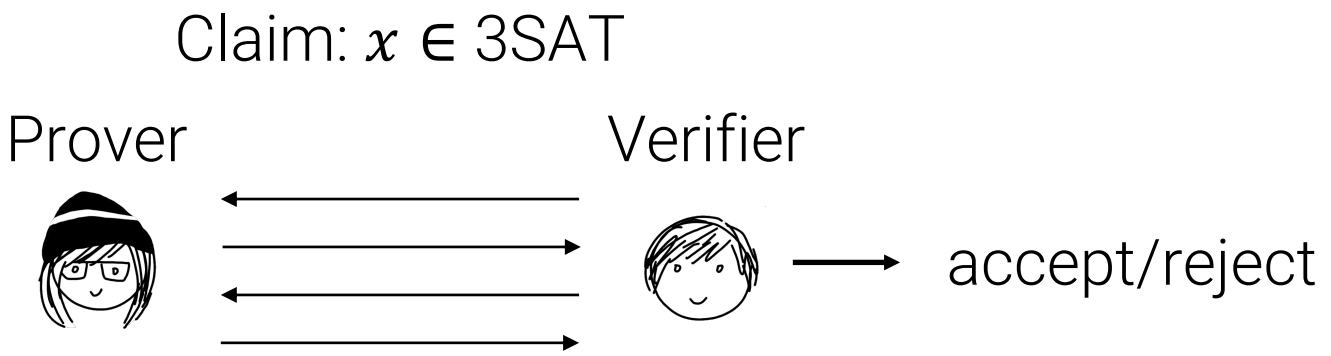


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**Zero Knowledge [GMR85]:**
View of malicious $V^*$ can be efficiently ***simulated*** without $P$.

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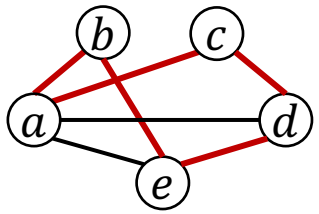
$G =$ 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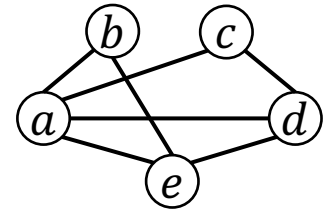
$G =$ 


Verifier 

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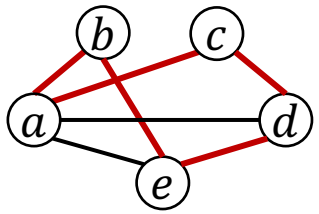
$G =$ 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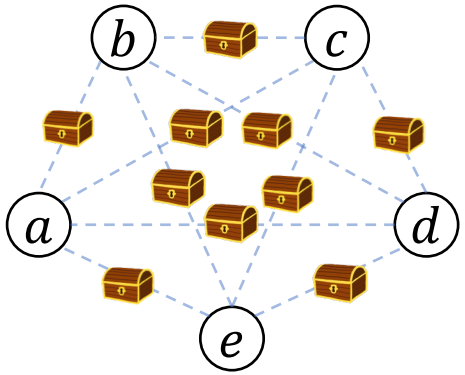
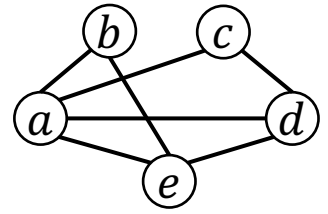



$G =$ 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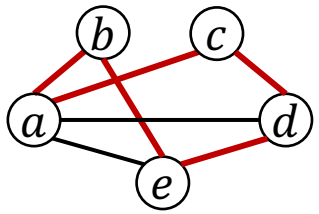
$G =$ 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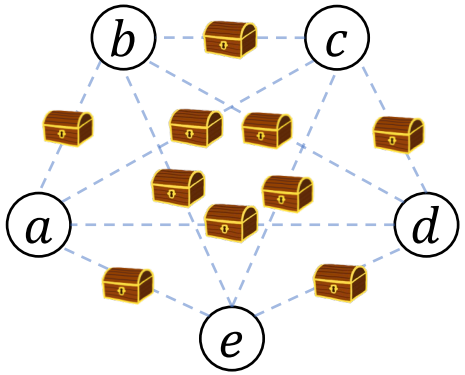
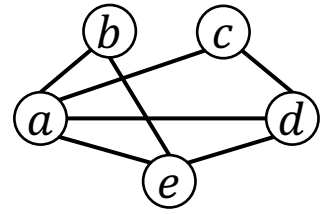



$G =$ 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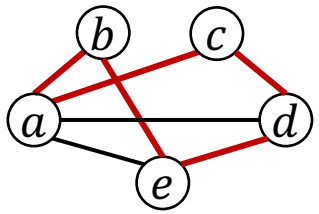
$G =$ 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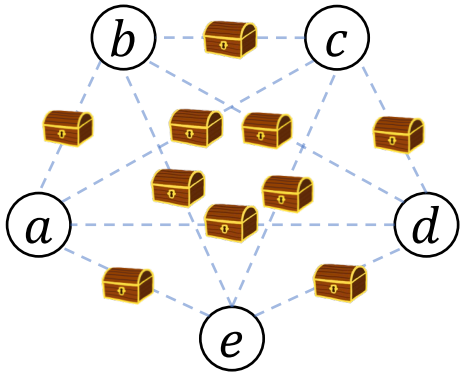
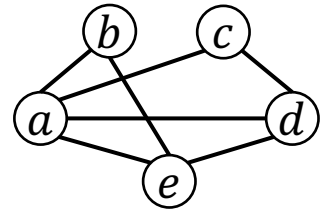



$G =$ 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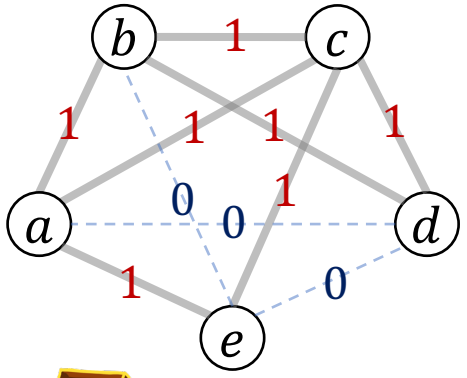
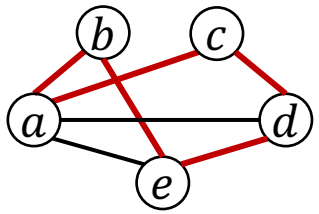
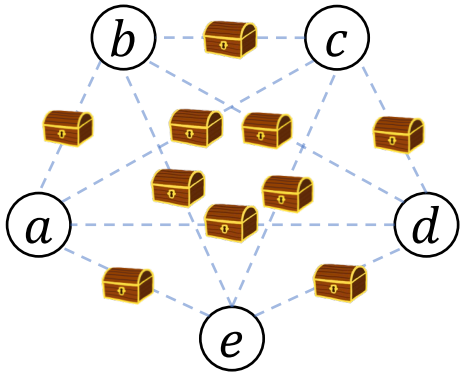
$\pi,$ 

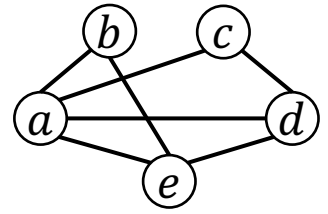

 (if $r = 0$)

# Blum's Protocol for Hamiltonian Cycles

$G =$ 


Prover 




$G =$ 

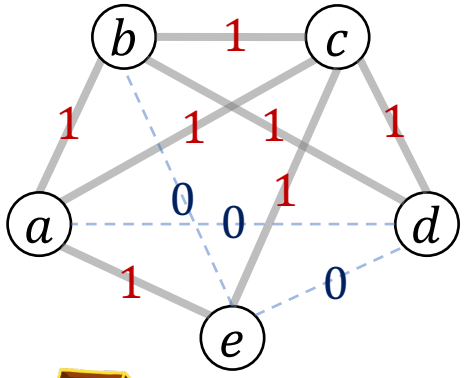

Verifier 

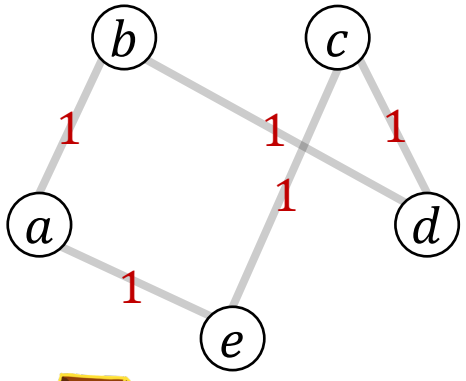
Sample $\pi \leftarrow S_V$.

Commit to the adjacency matrix of $\pi(G)$

$r$

Sample random $r \leftarrow \{0,1\}$

$\pi$, 


or 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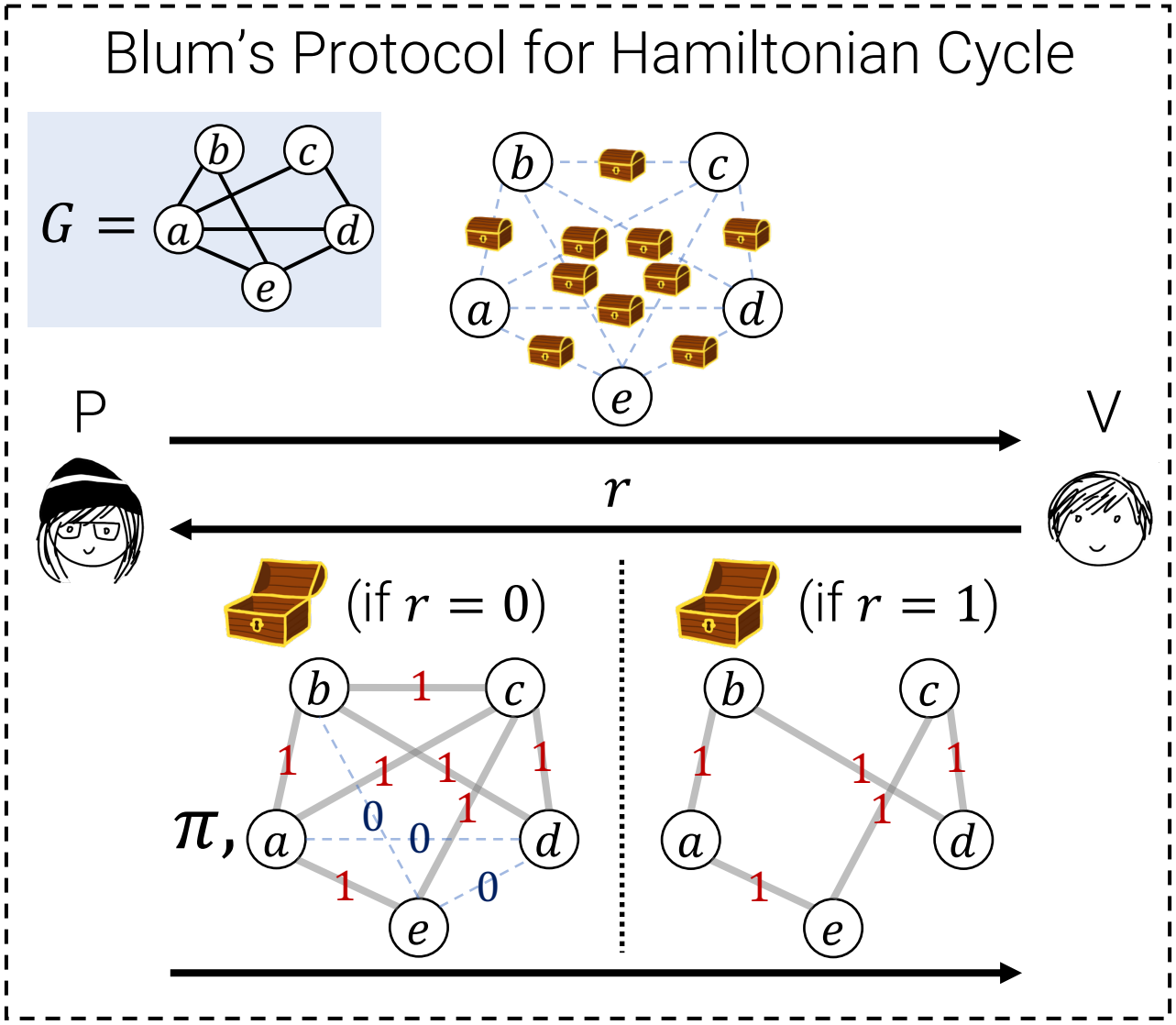


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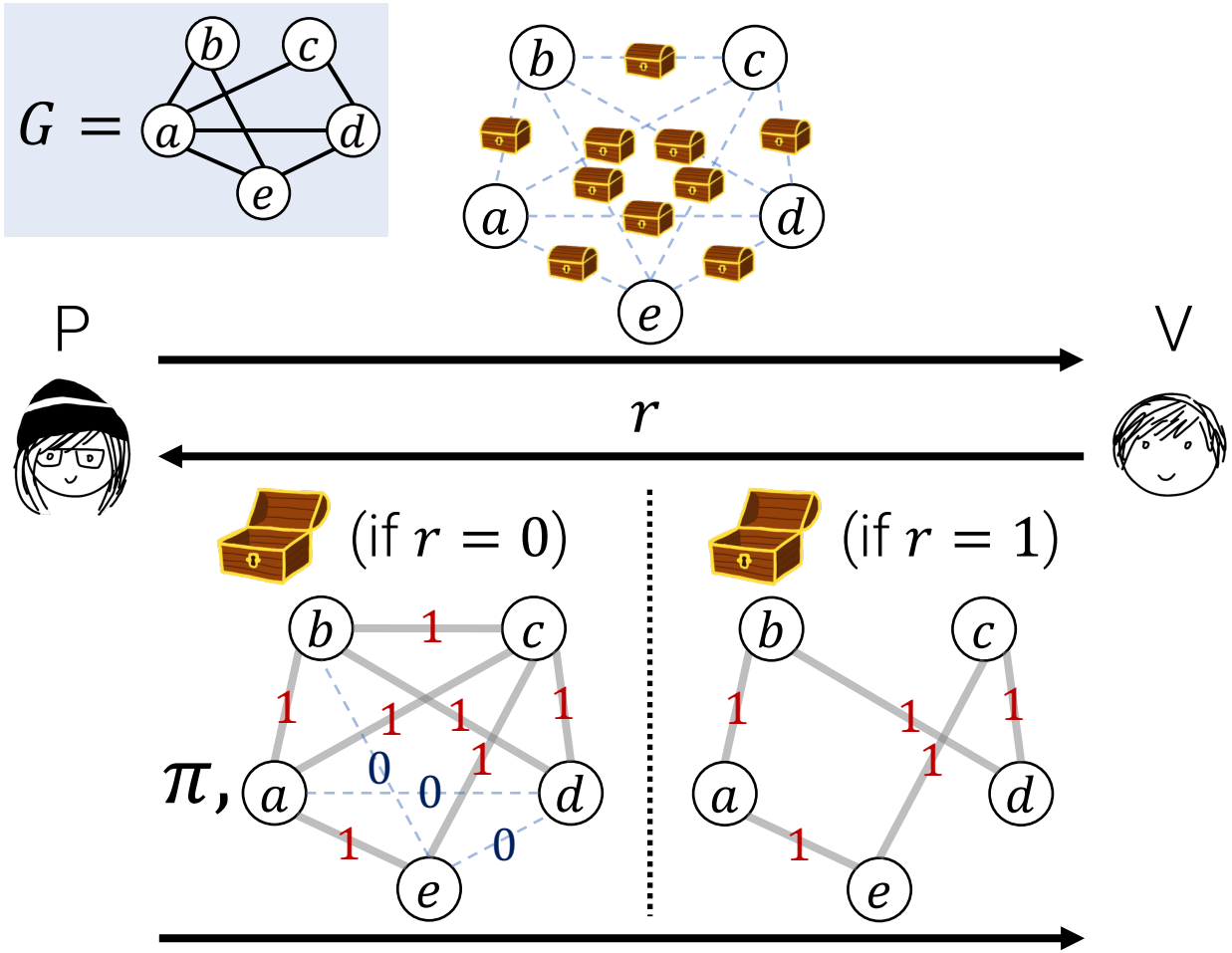


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## Zero knowledge (intuition)

First message reveals nothing since commitments are ***hiding***.

Last message also reveals nothing:

- $(r = 0)$ random permutation of $G$
- $(r = 1)$ random cycle

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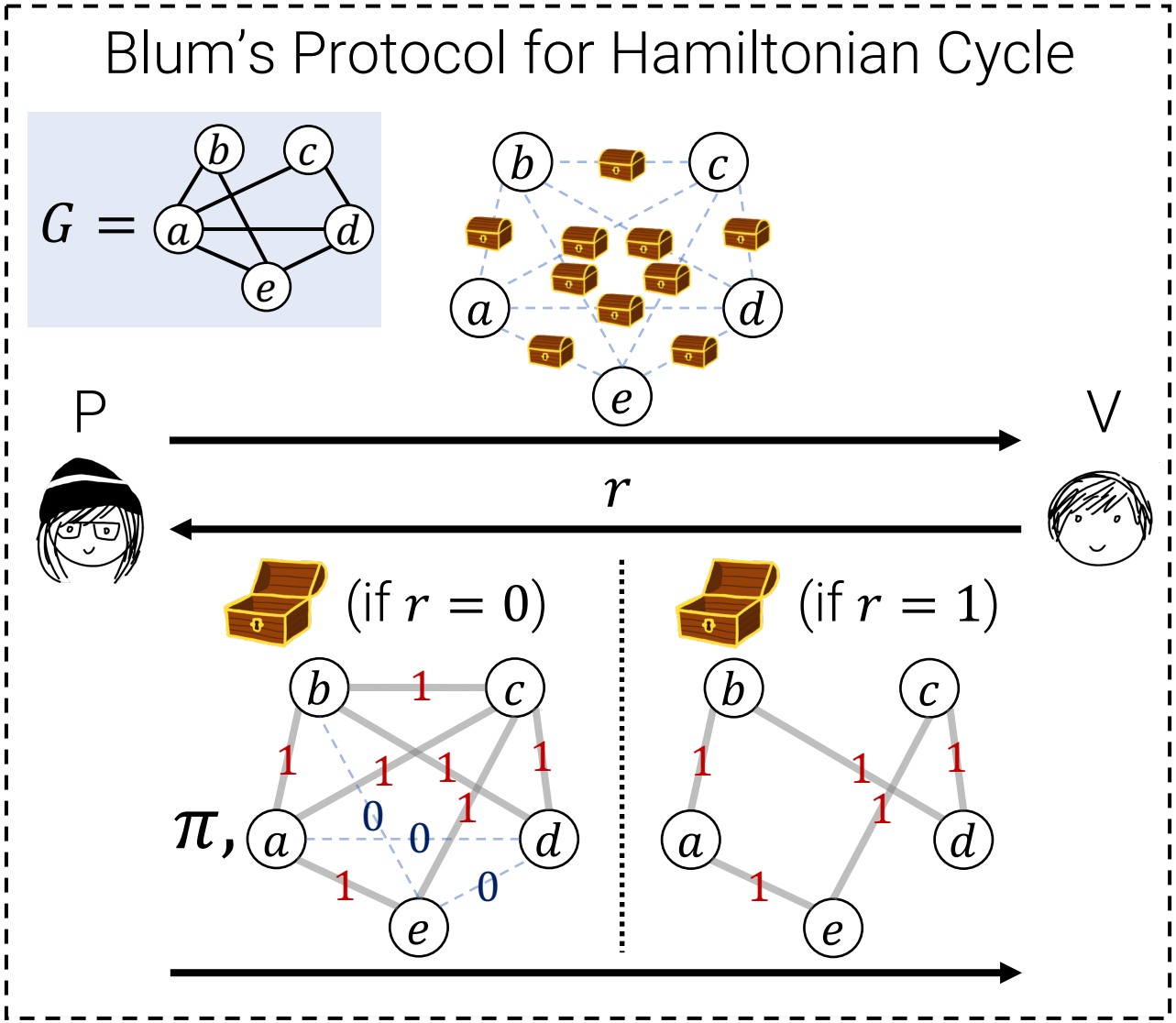
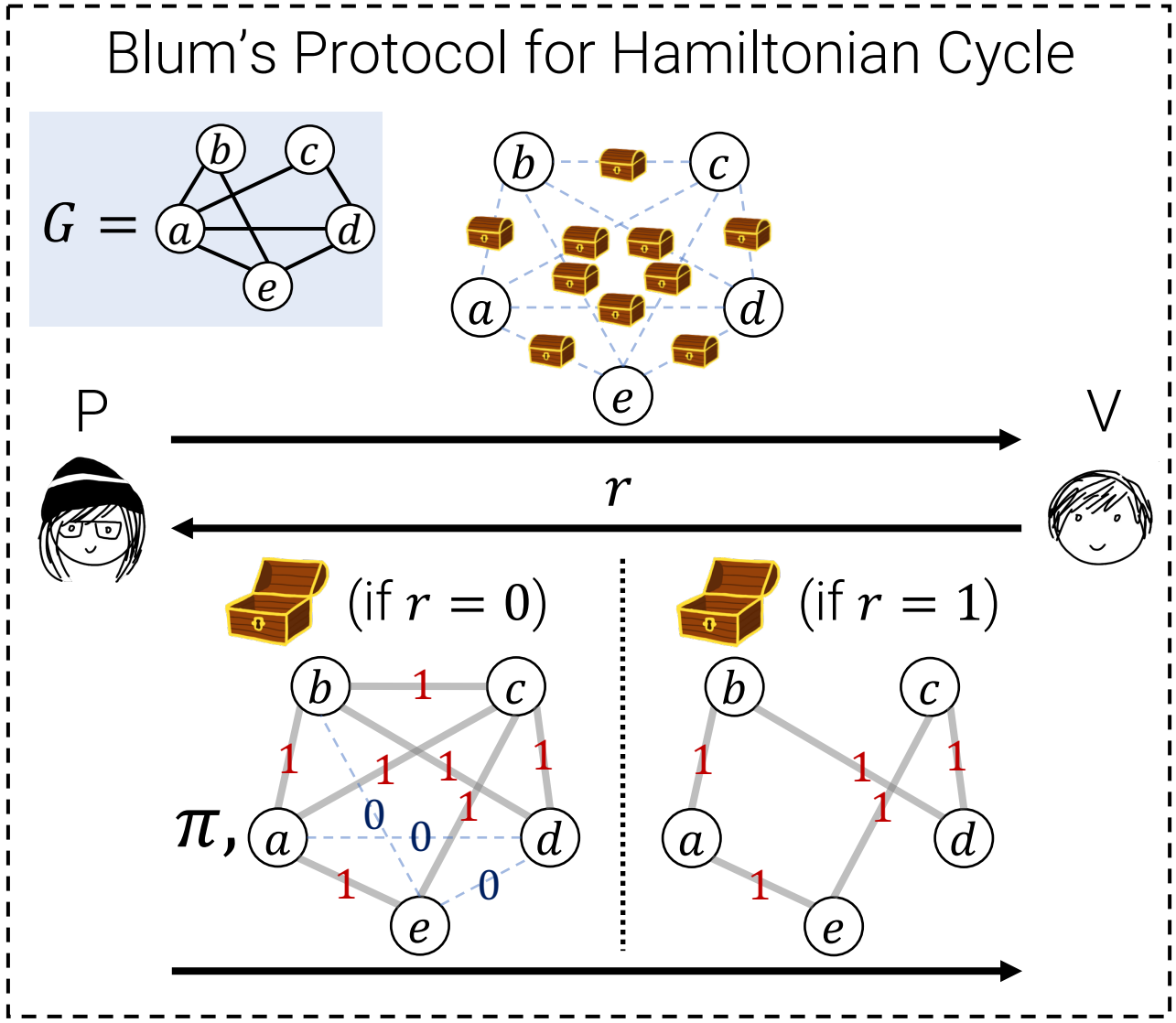



**PoK:** If efficient classical $P^*$ convinces $V$ with prob $\frac{1}{2} + \varepsilon$, then we can extract a witness from $P^*$.

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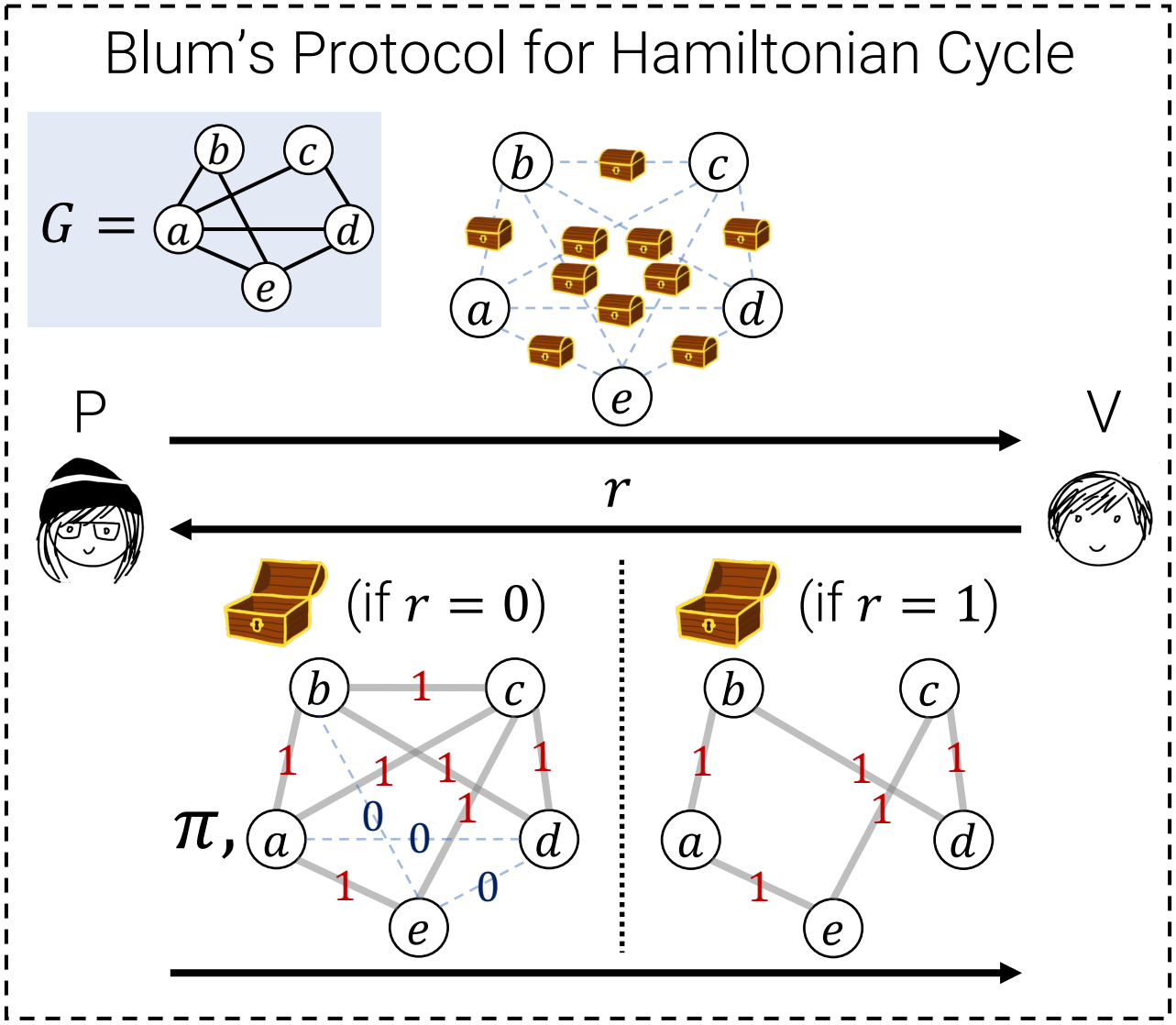


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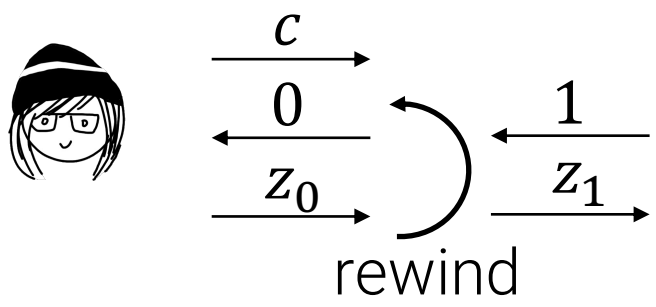

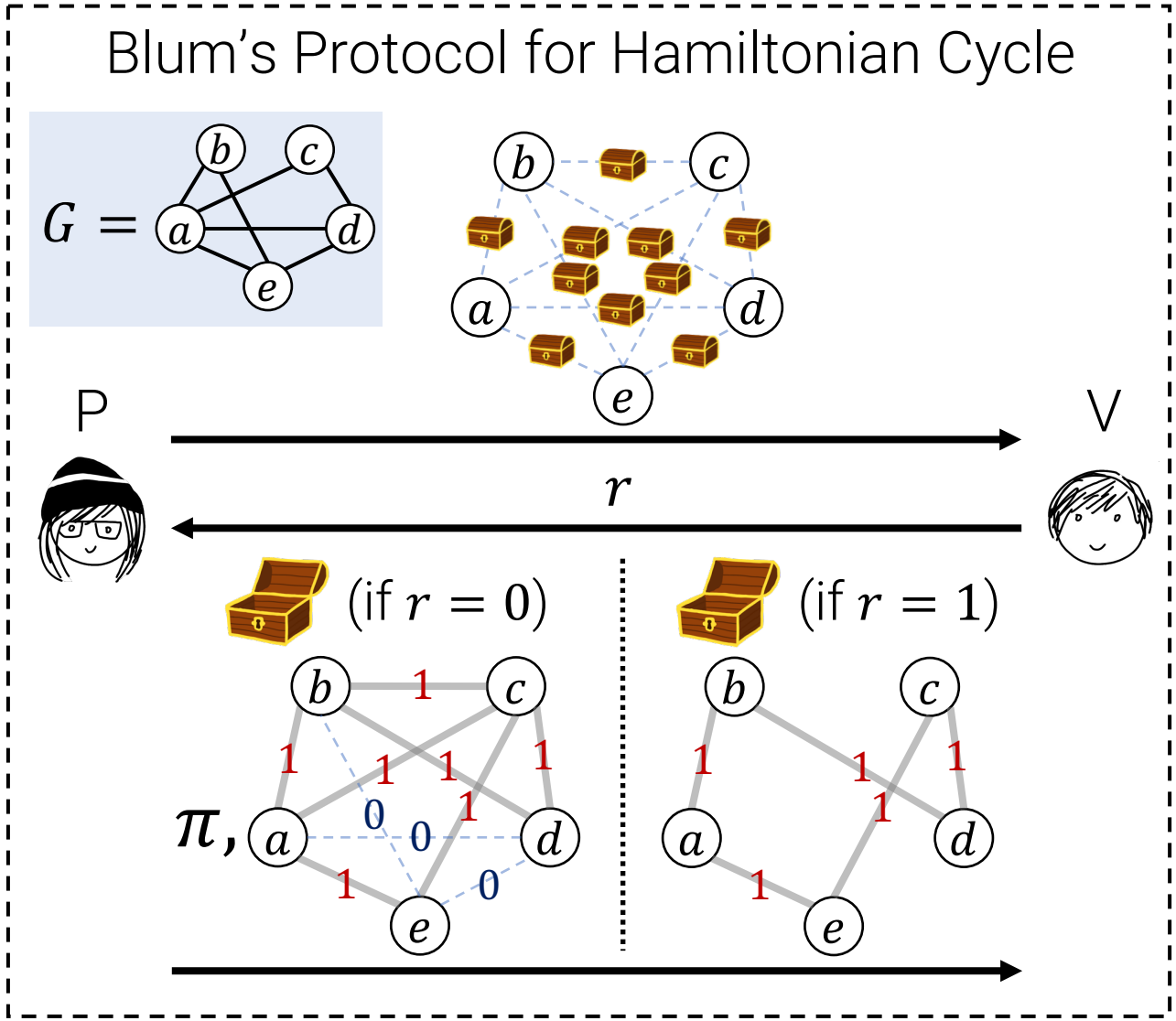


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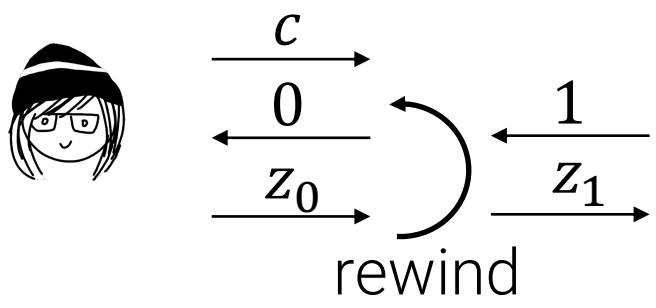
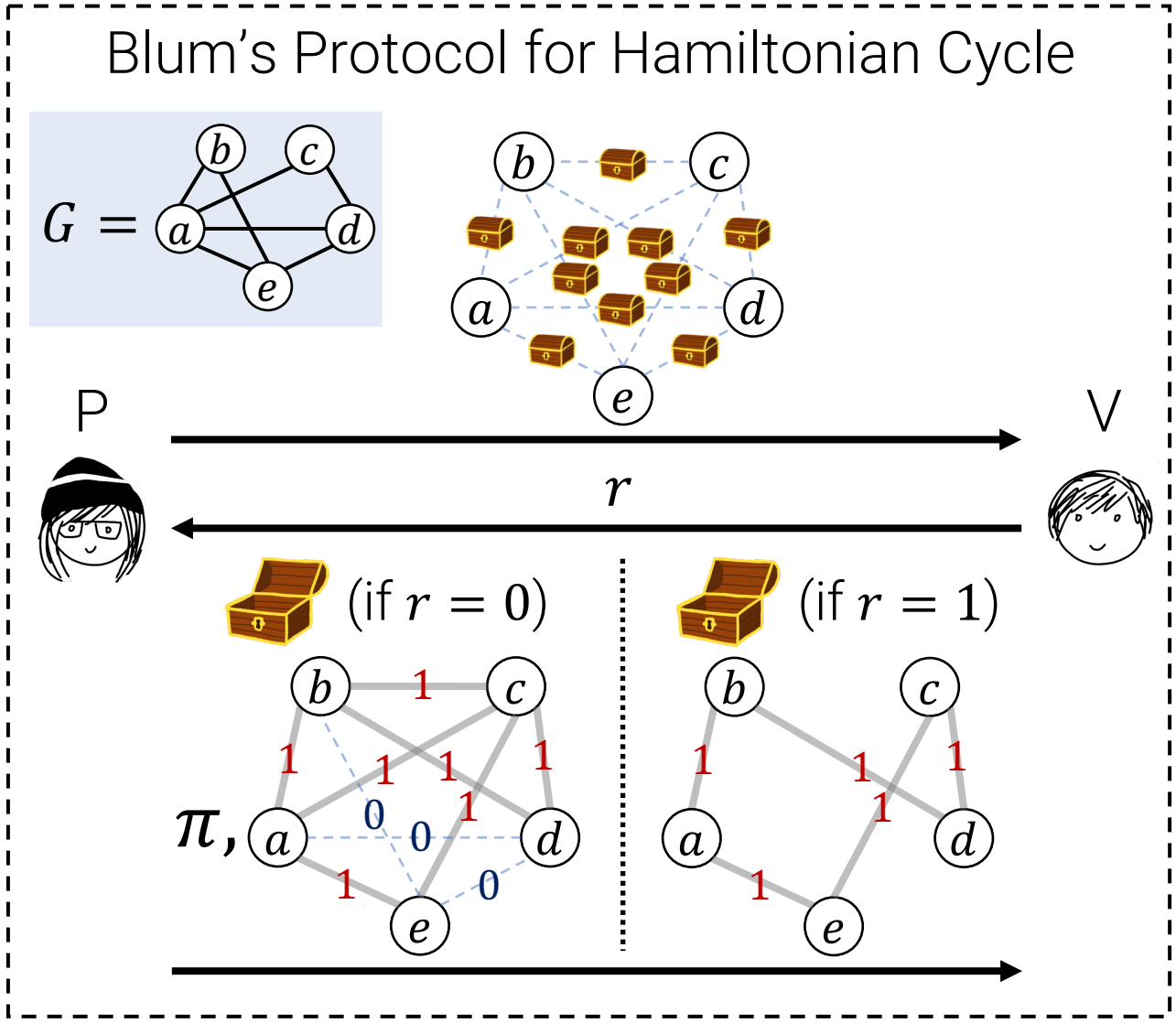



Get two accepting transcripts after $O\left(\frac{1}{\varepsilon}\right)$ rewinds.

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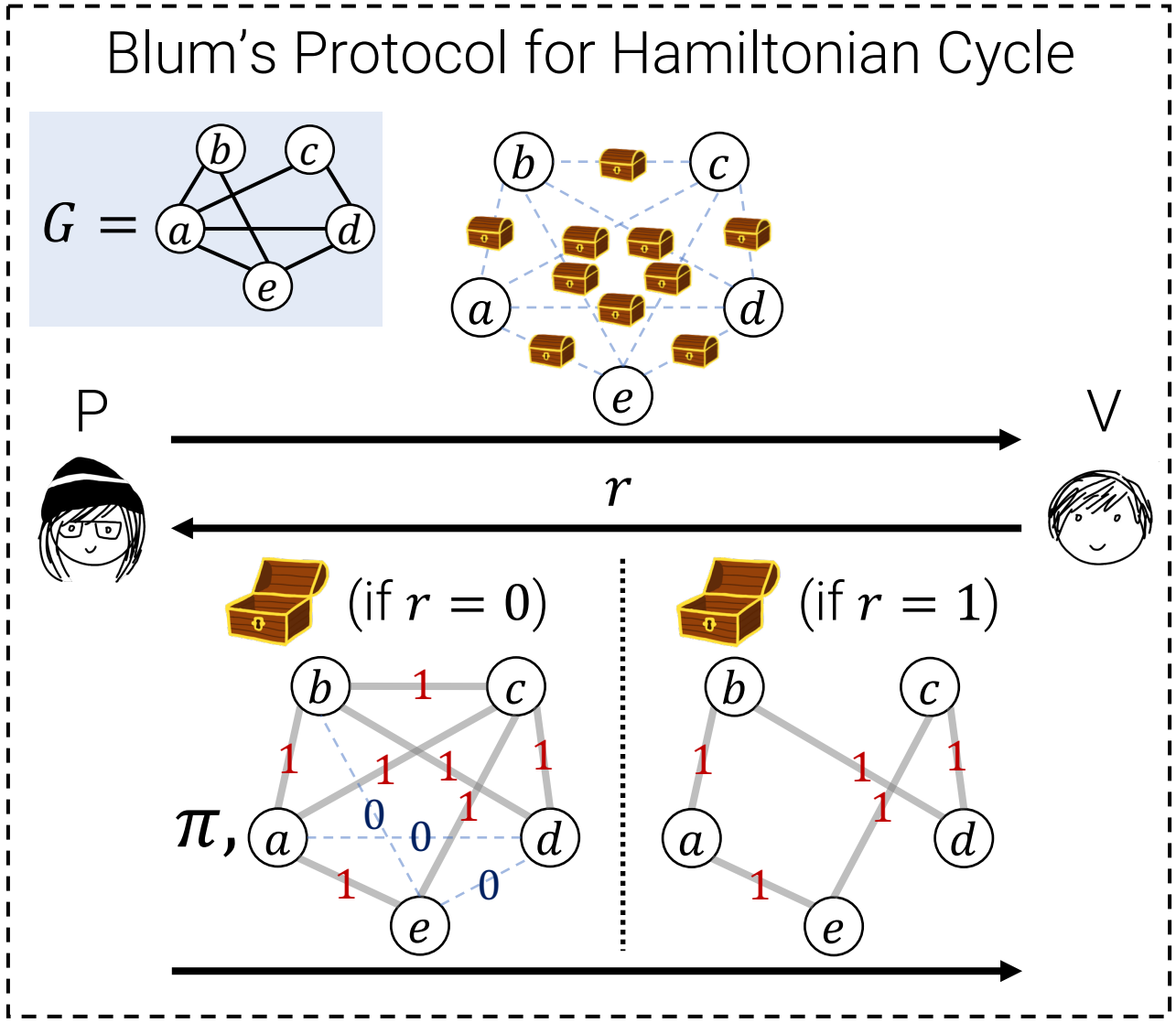


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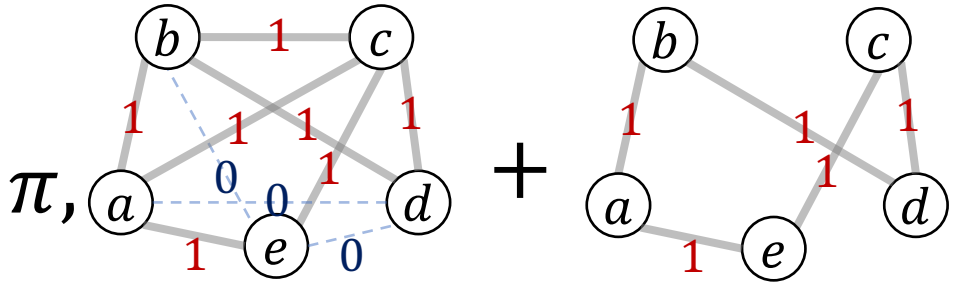

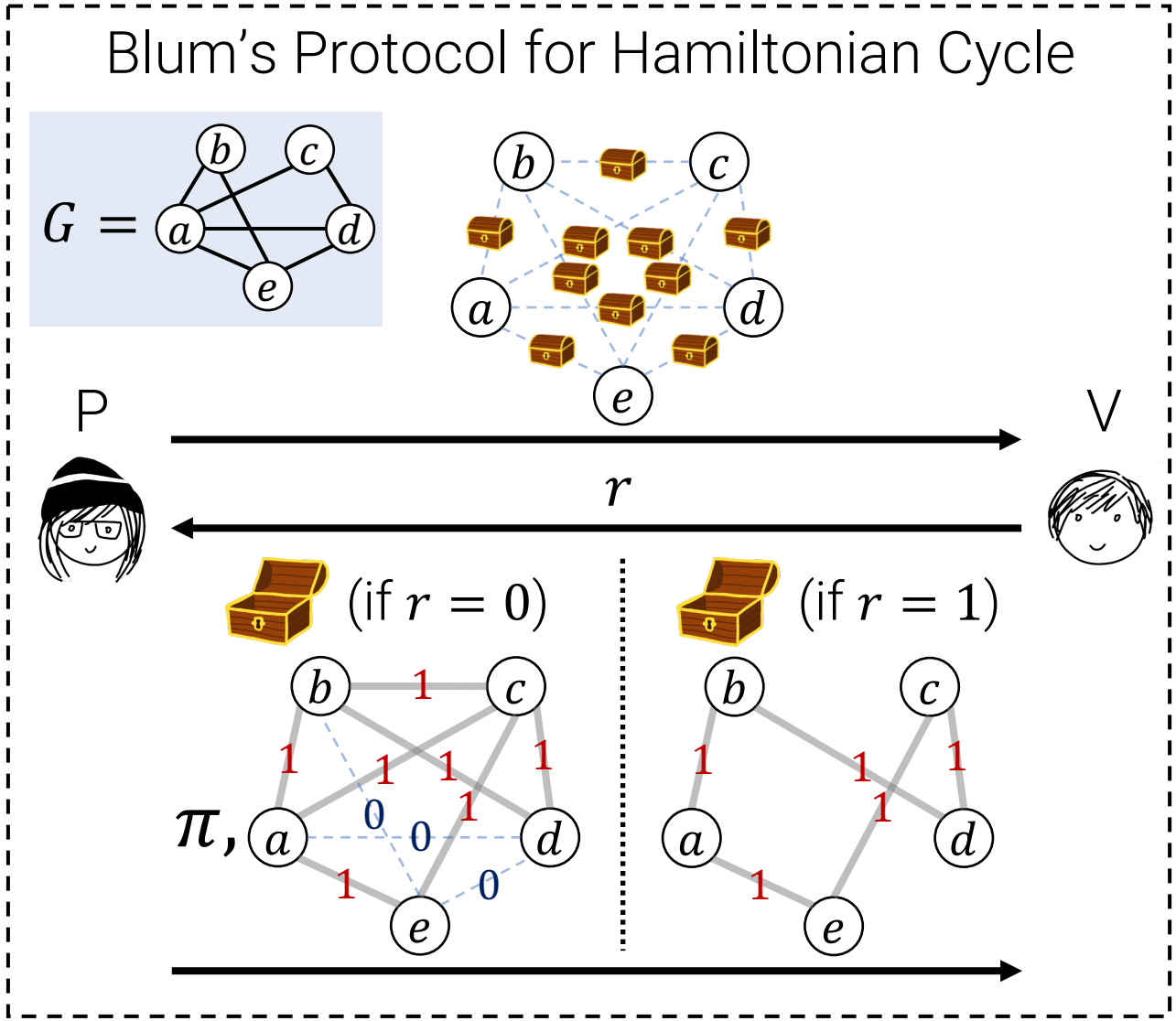


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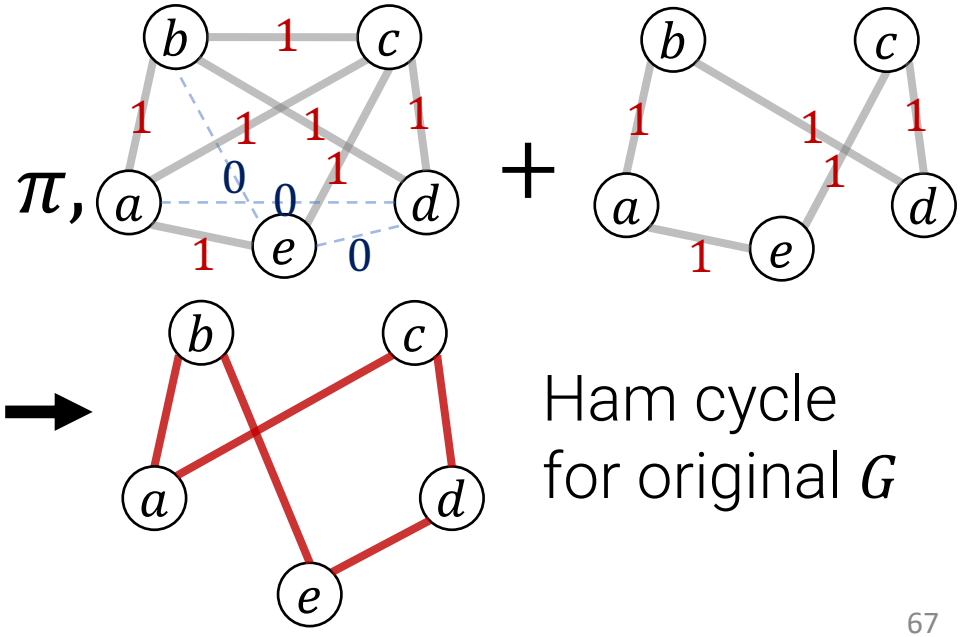

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[ARU14]: No!

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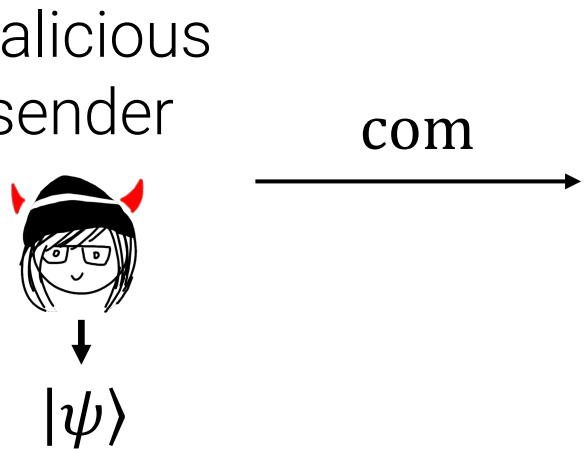

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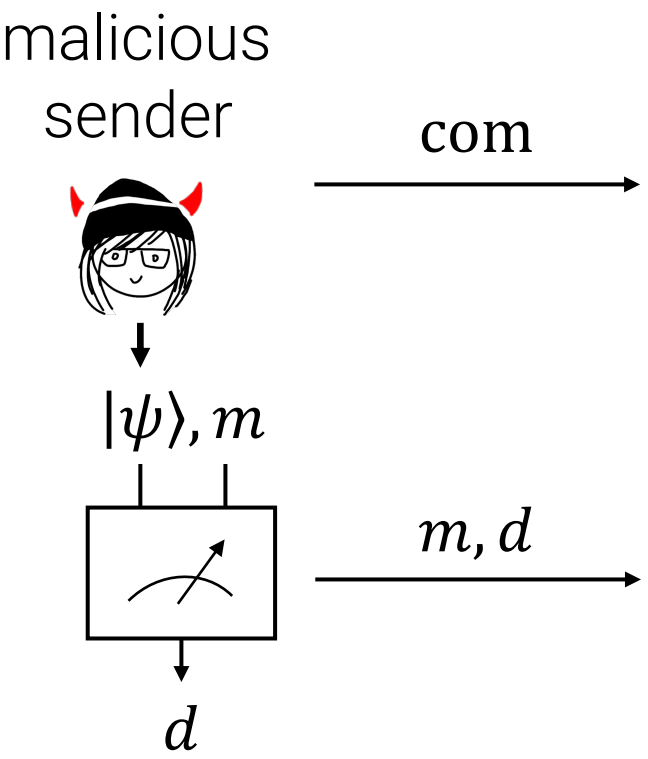

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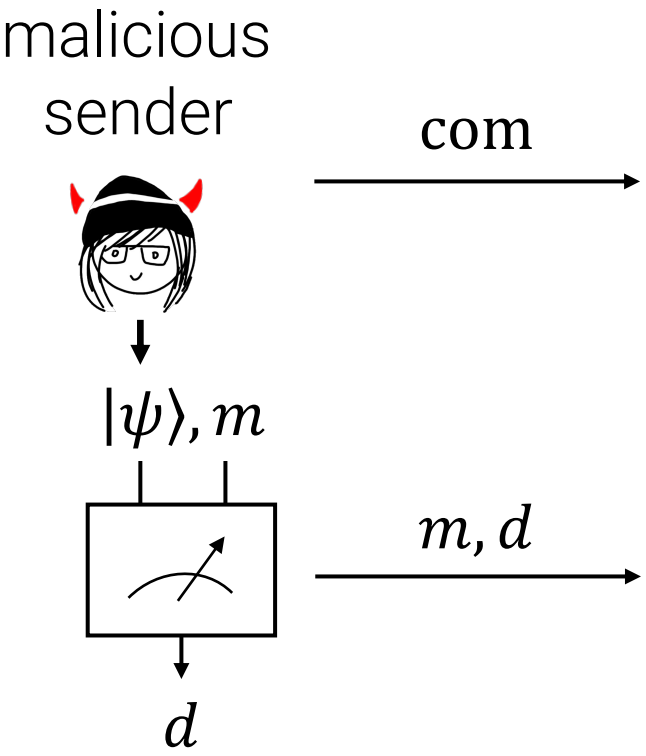

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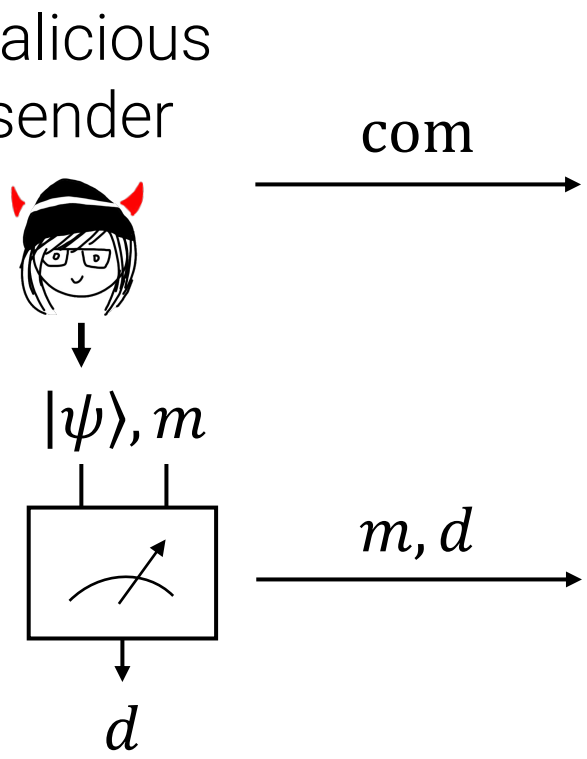


*Caveat: assuming a quantum oracle

**Open: construct example without oracles

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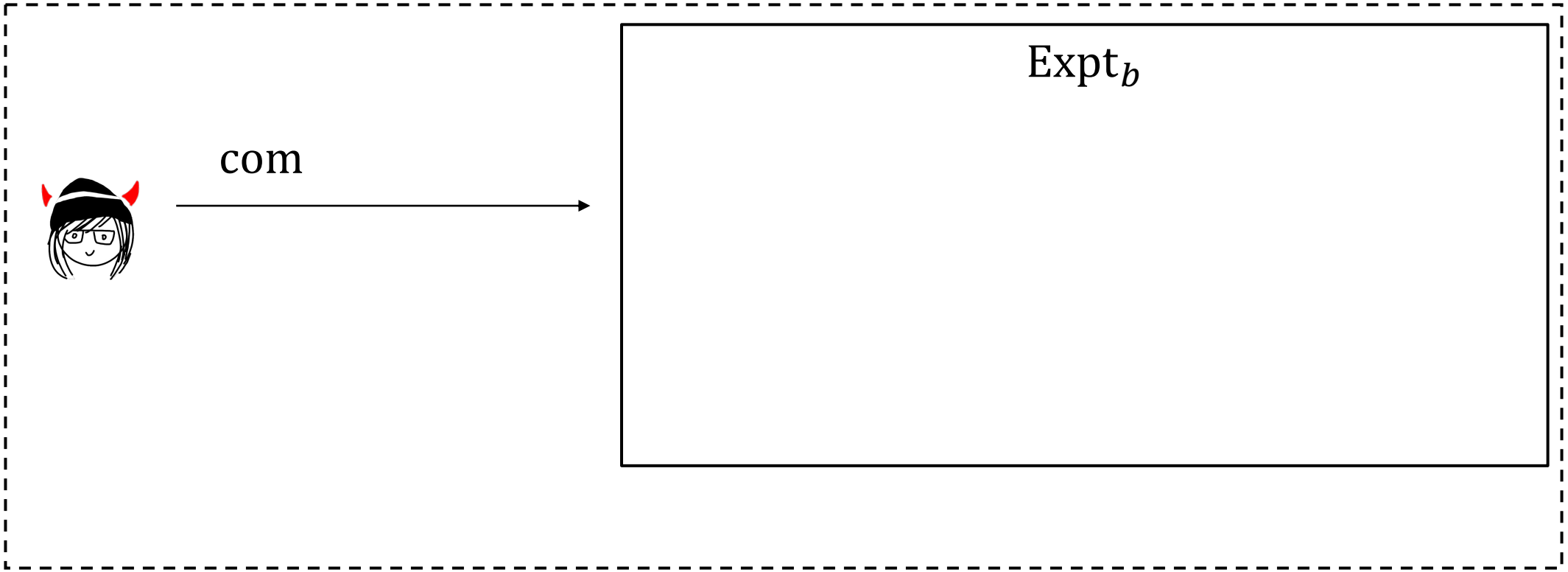

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*[U12,U16] analyze a slightly modified version of Blum's protocol, but later on [LMS22] showed the original Blum protocol is post-quantum secure.

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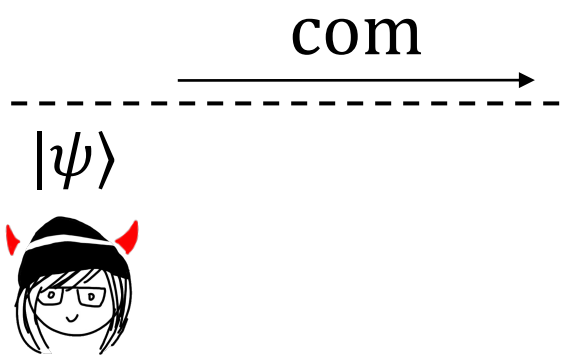


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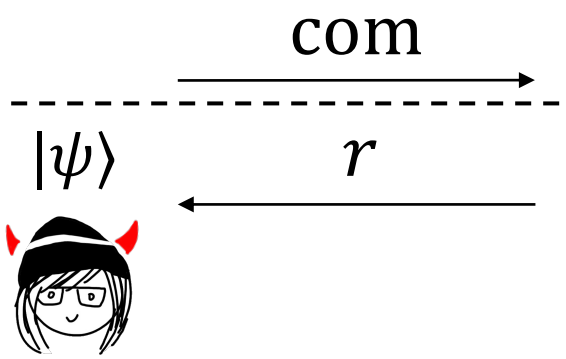


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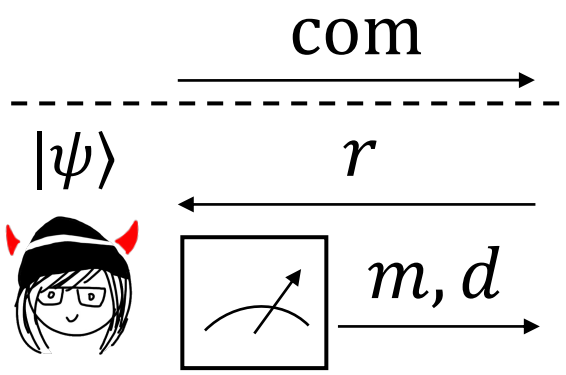


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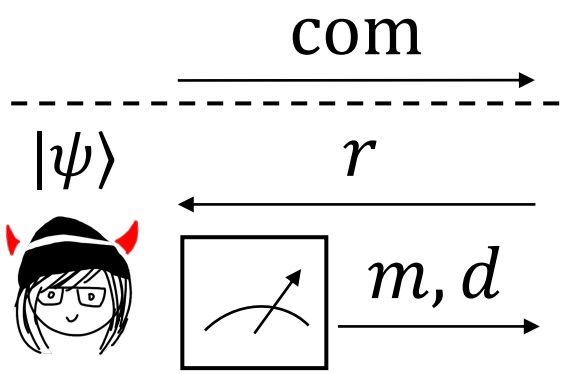


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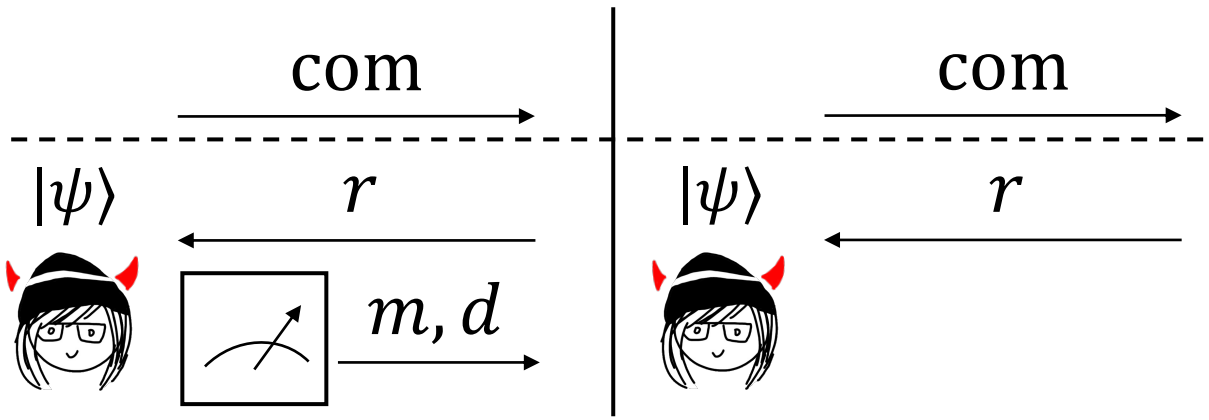


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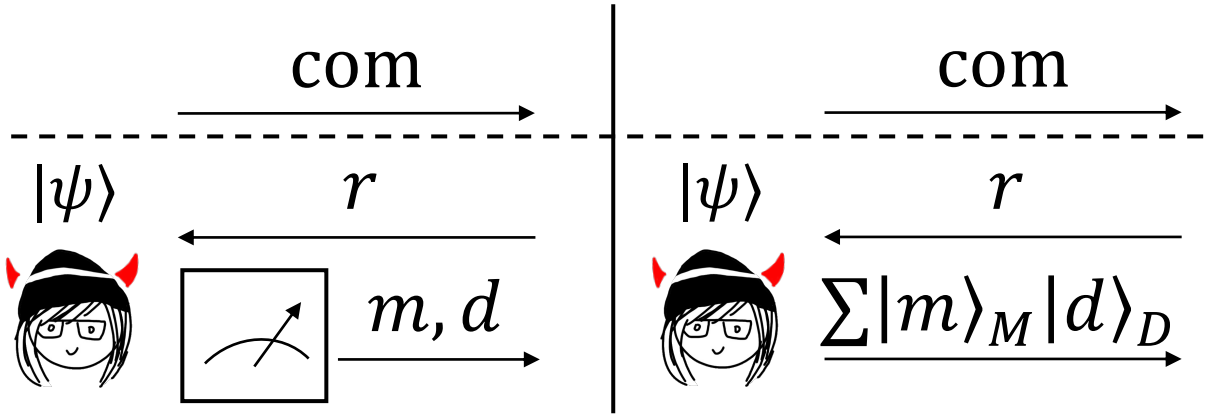


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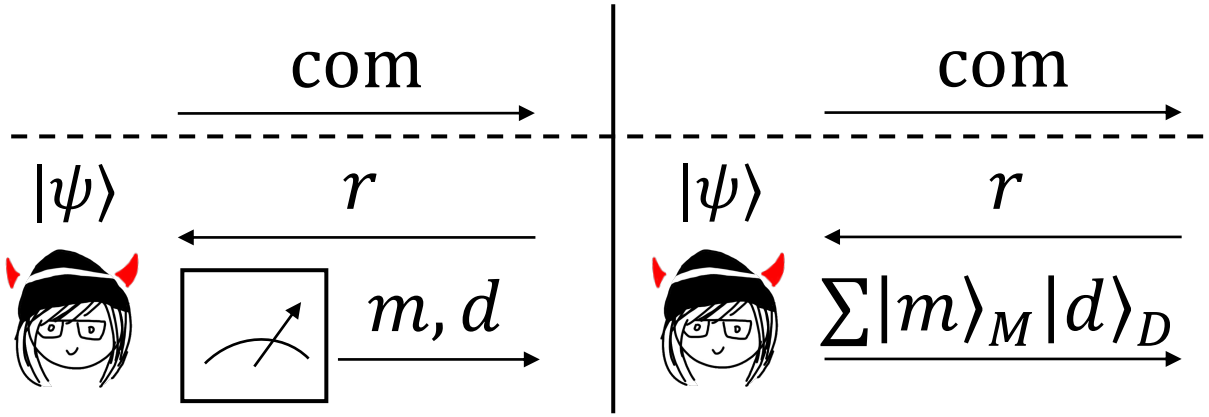


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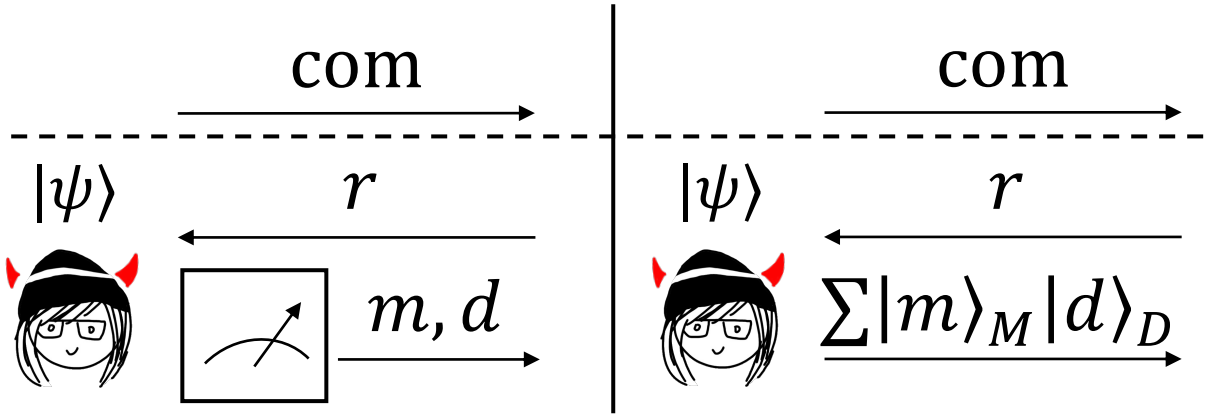


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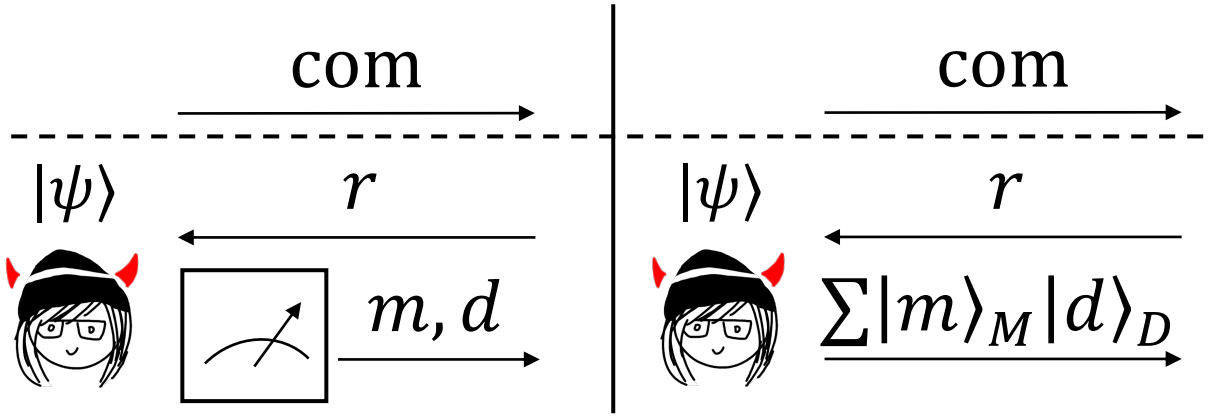


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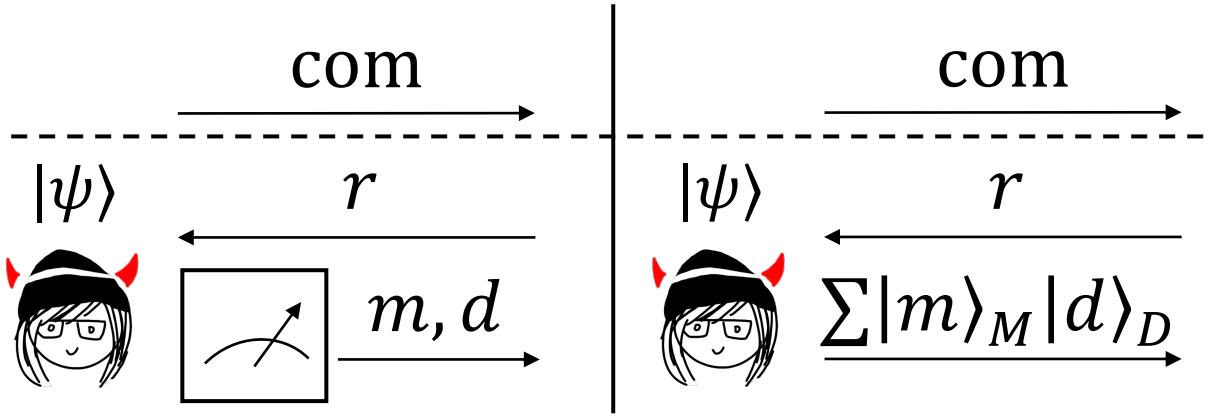


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**Key point:** with collapse-binding commitments, we just need to consider measuring the 1-bit decision (accept/reject).

1-bit Rewinding Lemma [Unruh12]: For any state $|\psi\rangle$ and set of projectors $\{\Pi_r\}_{r\in R}$, if $p = \mathbb{E}_{r\leftarrow R}\||\Pi_r|\psi\rangle\|^2$, then $\mathbb{E}_{r\leftarrow R,s\leftarrow R}\|\Pi_s\Pi_r|\psi\rangle\|^2 \geq p^3$

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Aside: $\Pi_r = U_r^\dagger\left(\sum_{\mathrm{Ver}(m,d)=1}|m,d\rangle\langle m,d|\right)U_r$ where $U_r$ is the adversary's unitary for challenge $r$.

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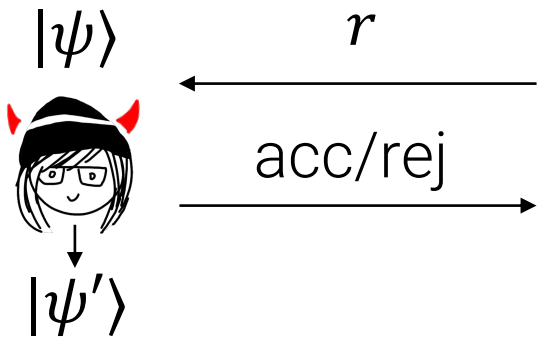

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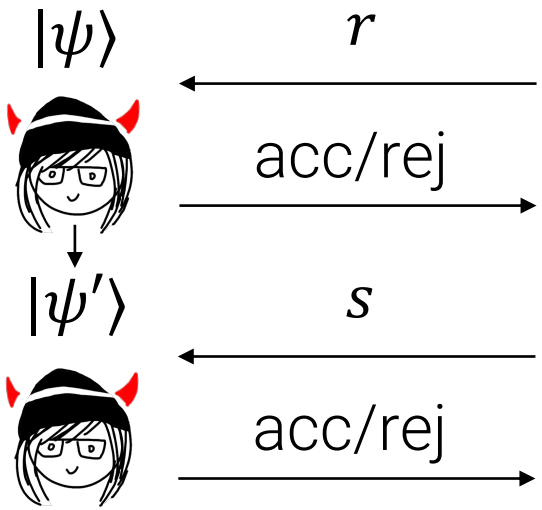

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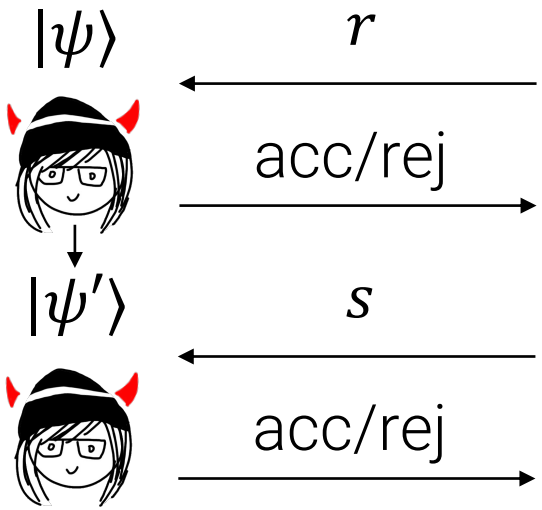

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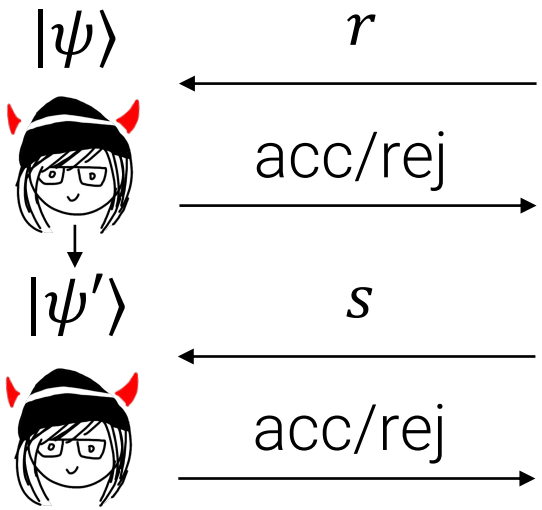


**Annoying detail:** this is only useful if $r \neq s$. For Blum ($R = \{0,1\}$), we only extract a witness with $\Omega(\varepsilon)$ probability when $p \geq 1/\sqrt{2} + \varepsilon$.

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(However, this bound is much stronger than a gentle measurement bound)

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**Step 1: Collapsing commitments** [U16]:

recording adversary's response $\approx_c$ recording 1-bit decision

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**Step 2: 1-bit-rewinding lemma** [U12]:

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In the next talk, we'll see a different quantum rewinding technique that achieves the original classical guarantees.

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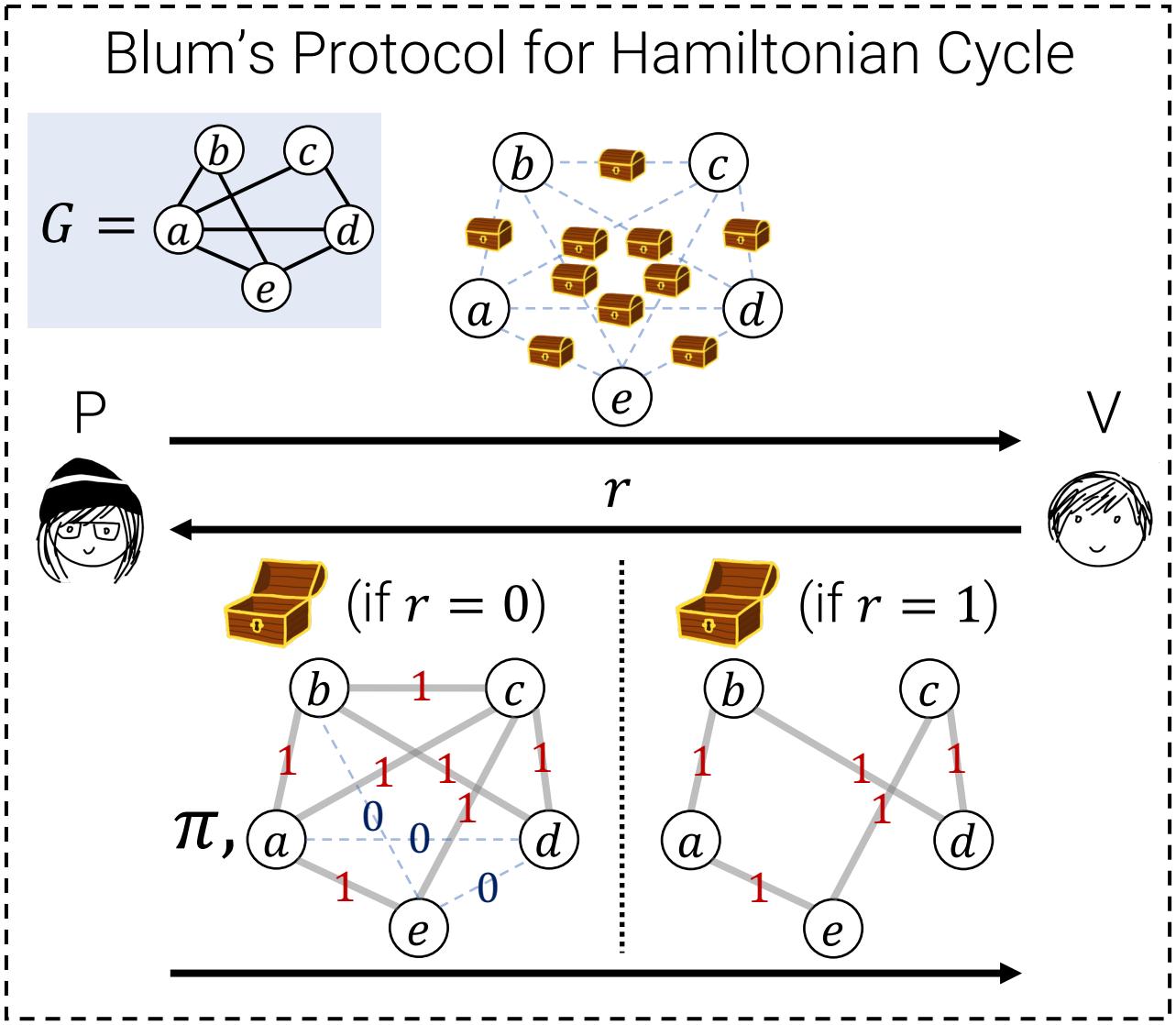


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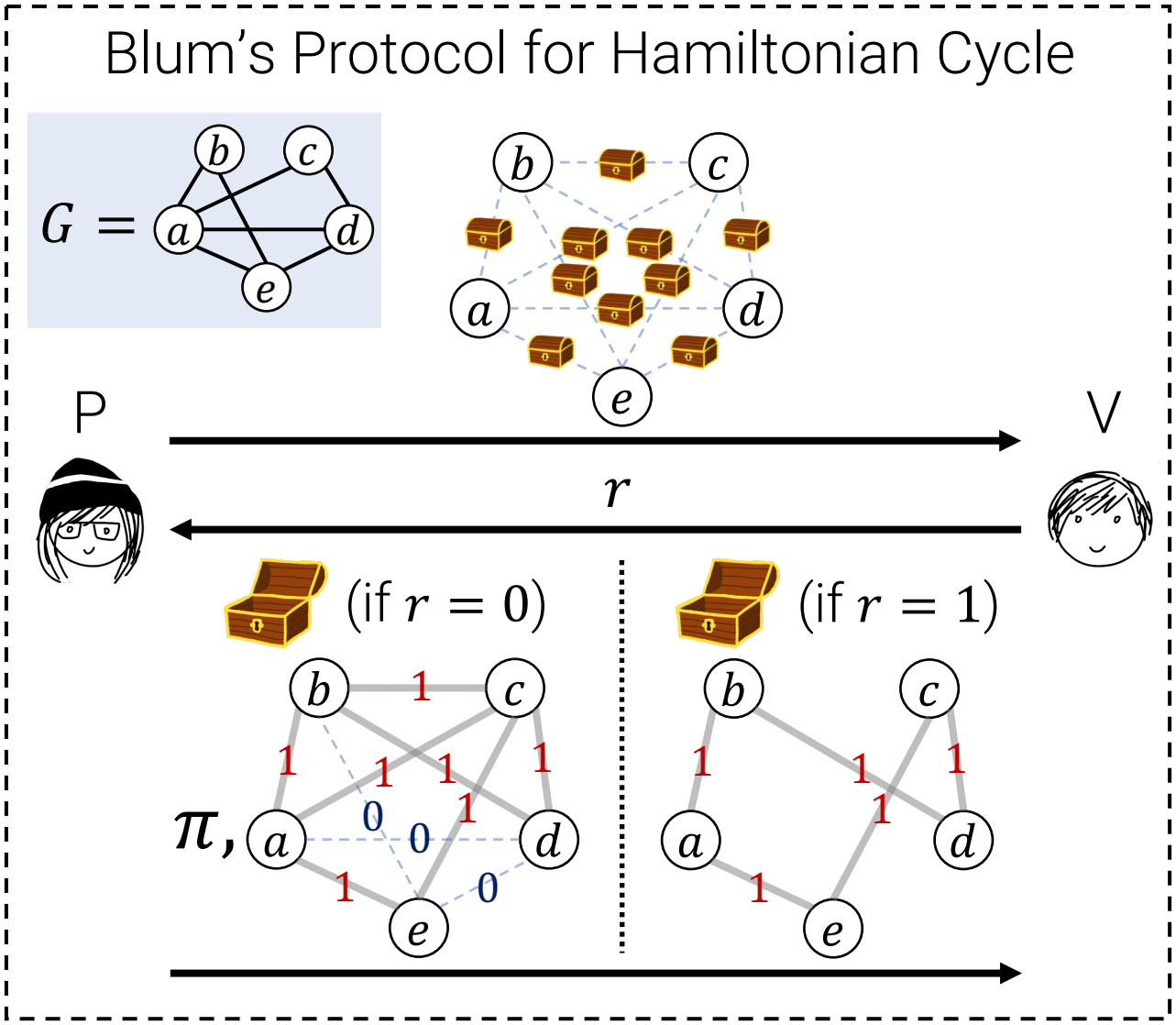


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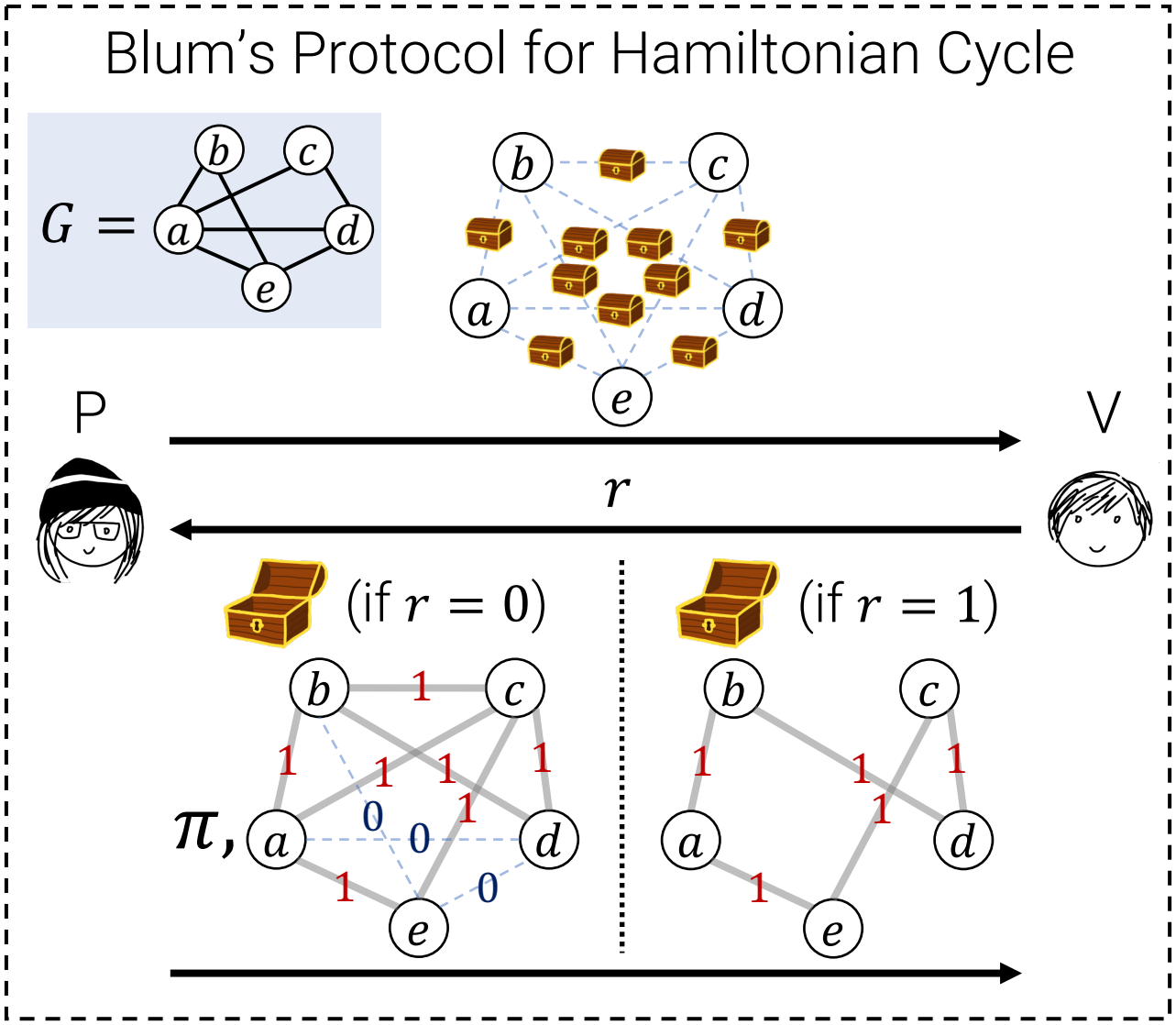


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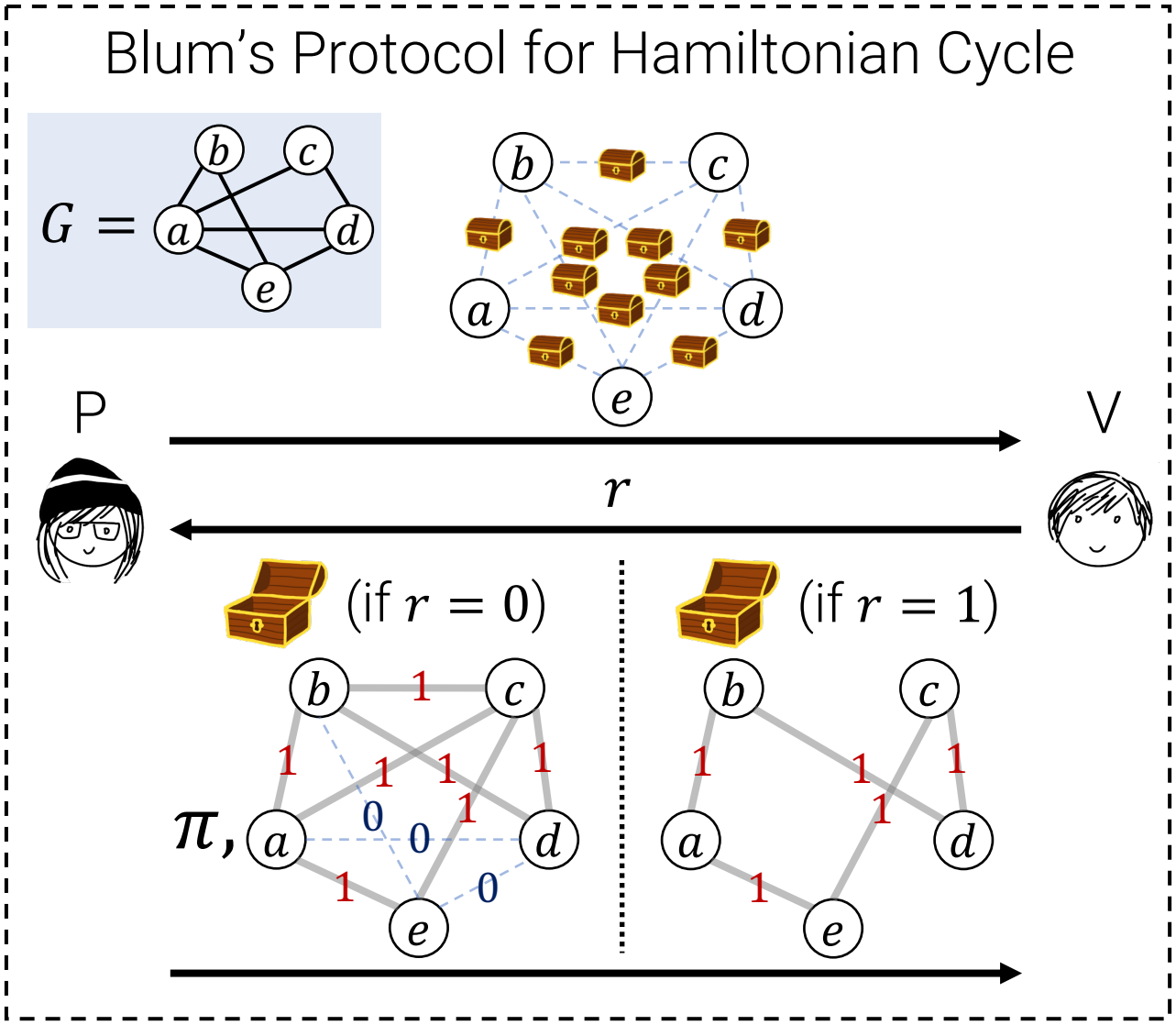


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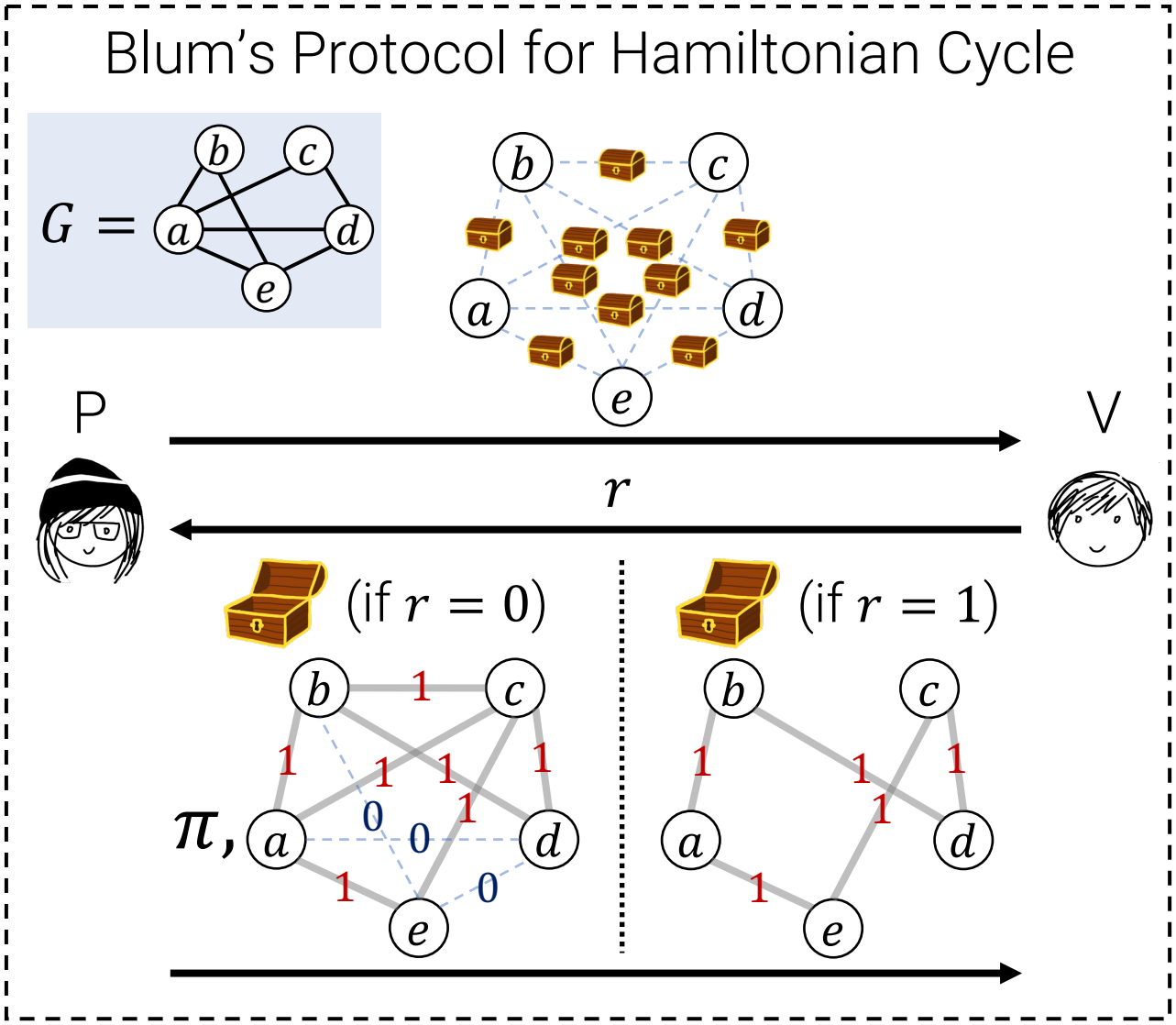


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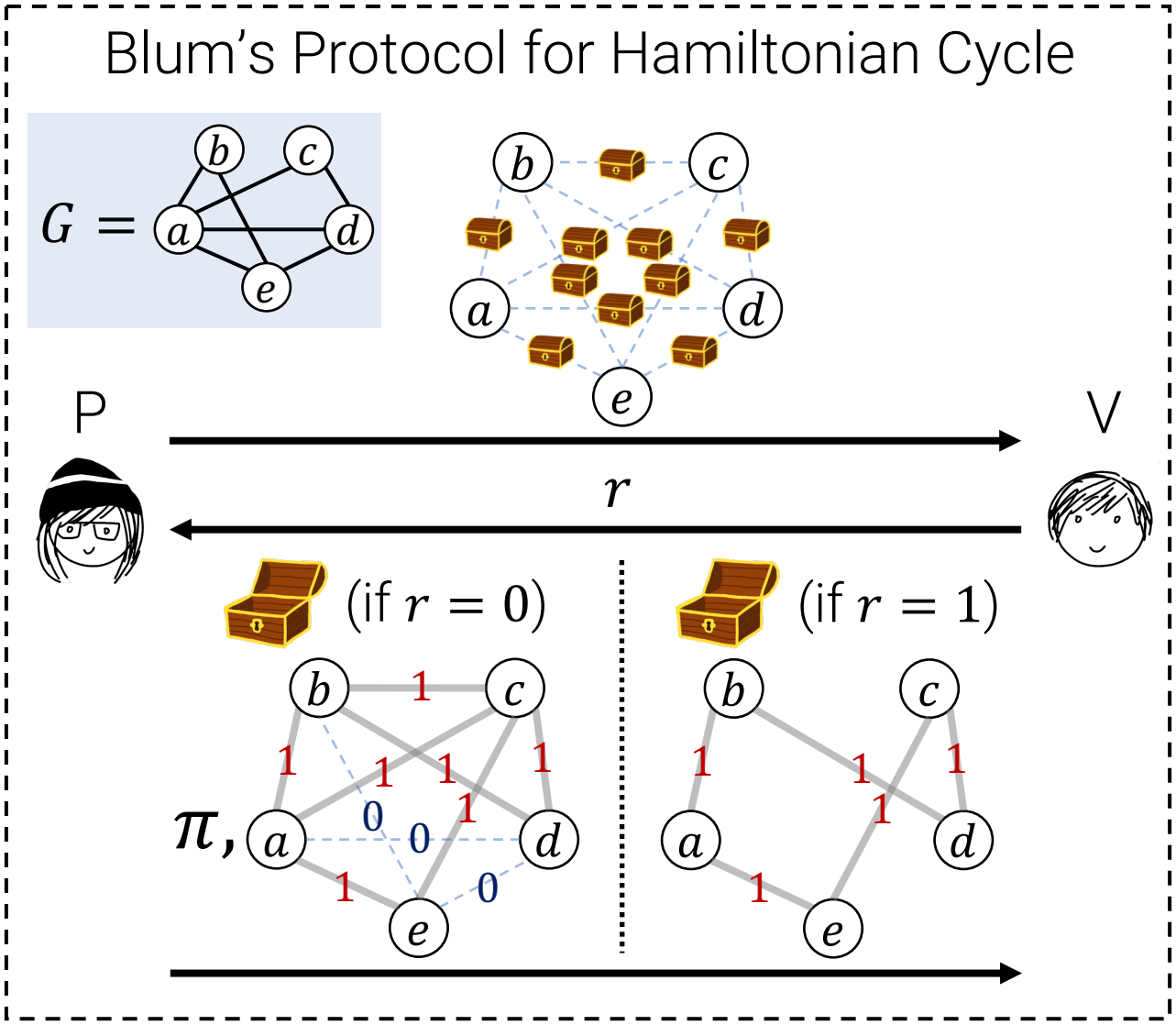


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By hiding, $(c, r', z) \leftarrow \mathbf{HVSim}$ looks like honest-verifier view.

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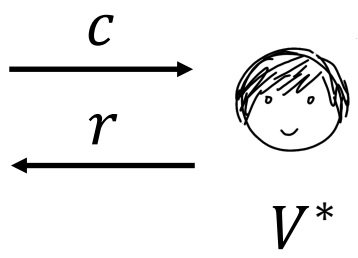

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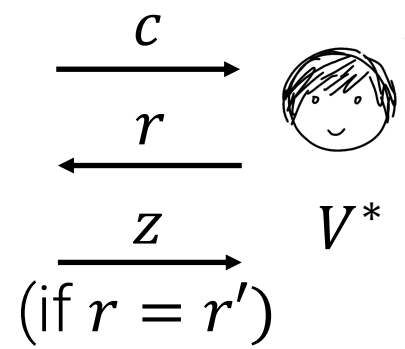

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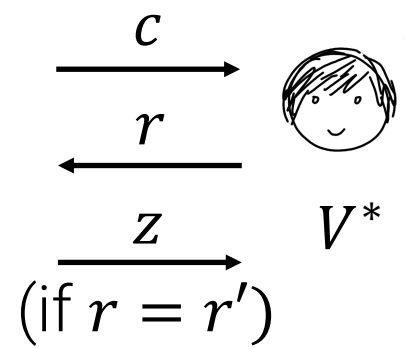


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We combine this with rewinding to get the full ZK simulator:

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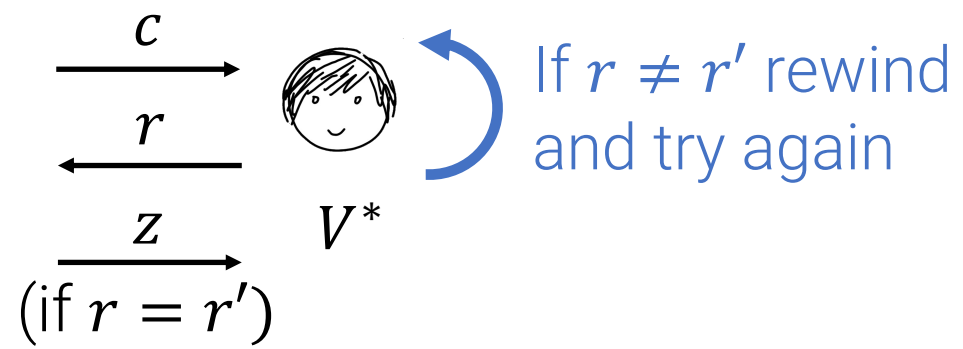


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Unfortunately, this simulator won't suffice for post-quantum ZK! If a malicious $V^*$ has an unknown initial state $|\psi\rangle$ running $\mathbf{Guess}(V^*, |\psi\rangle)$ may irreversibly disturb it.

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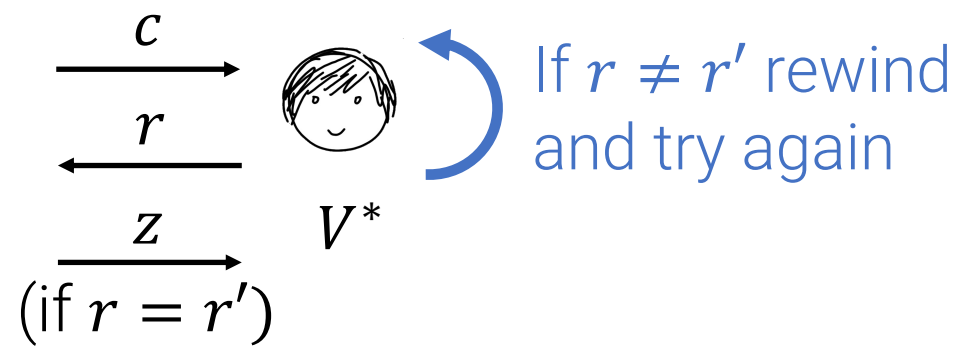


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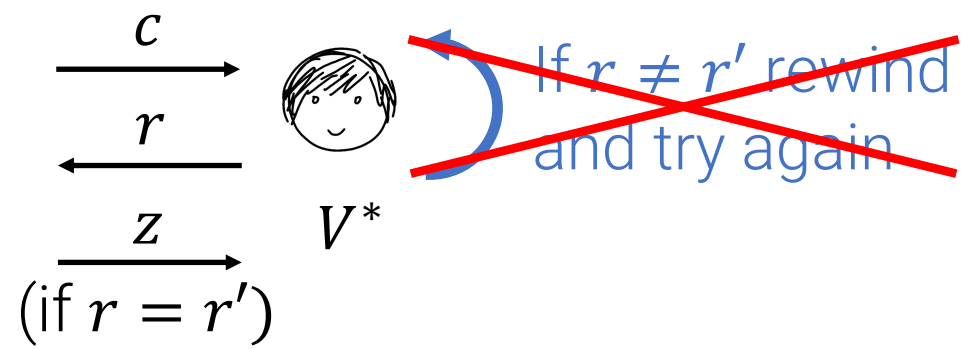


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But there is a different simulator due to [Watrous05] that works.

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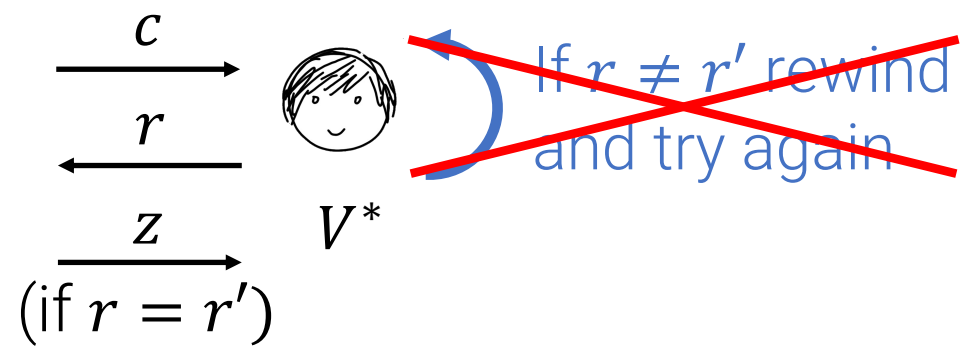


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[Watrous05]: If commitment scheme is hiding, then the Blum protocol is post-quantum ZK.

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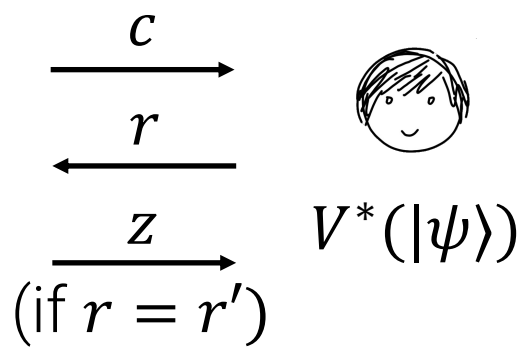


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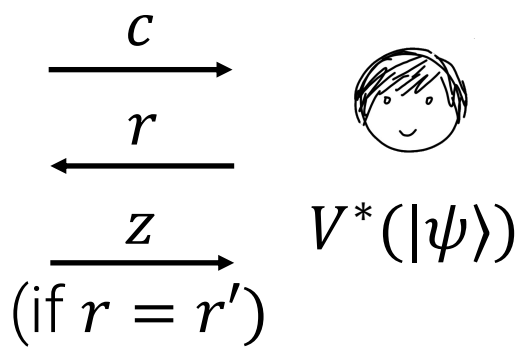


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We'll write this process as a quantum circuit on $|\psi\rangle$.

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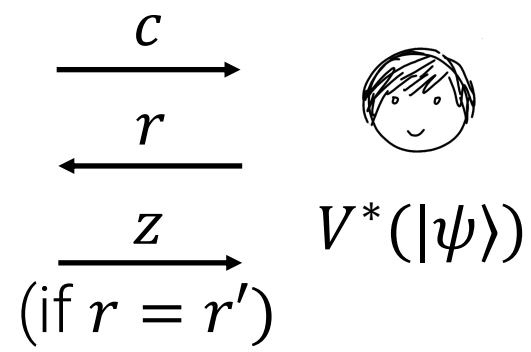


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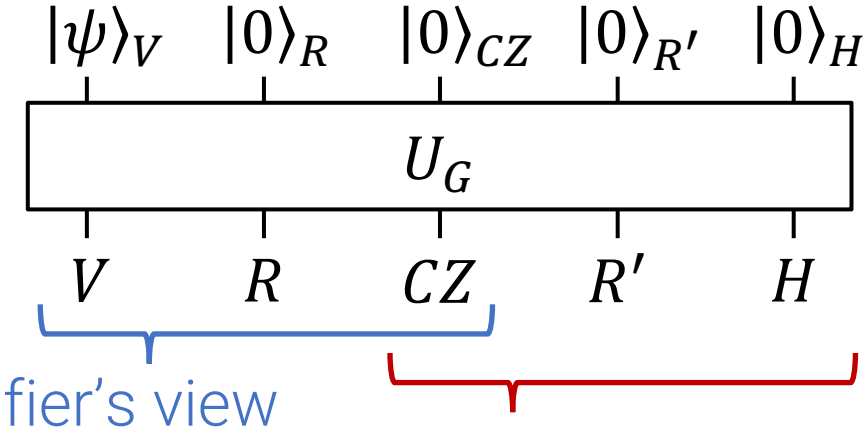


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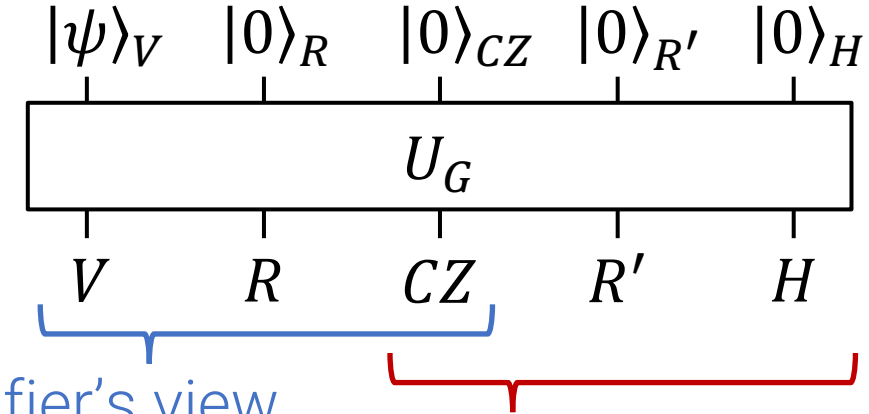


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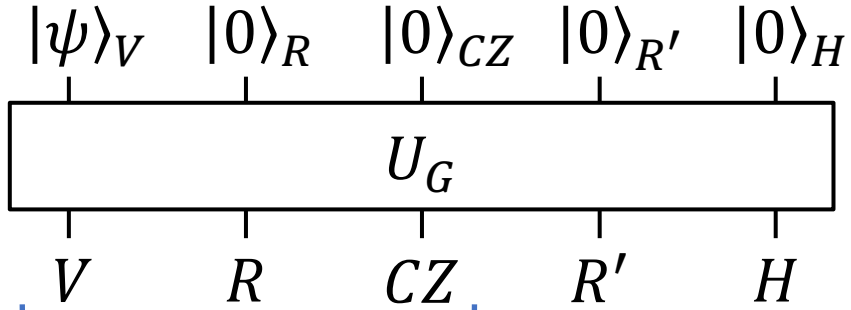


Verifier's view (state + transcript)

**HVSim** (workspace + output)

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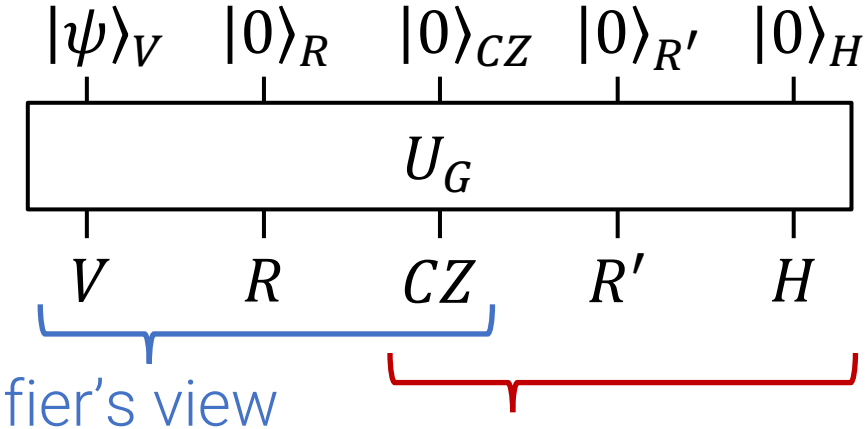


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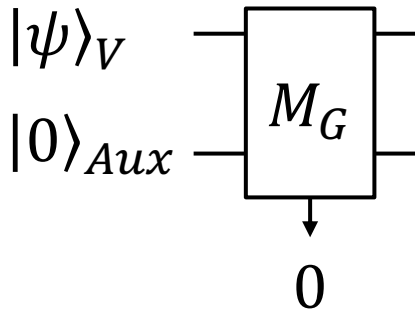

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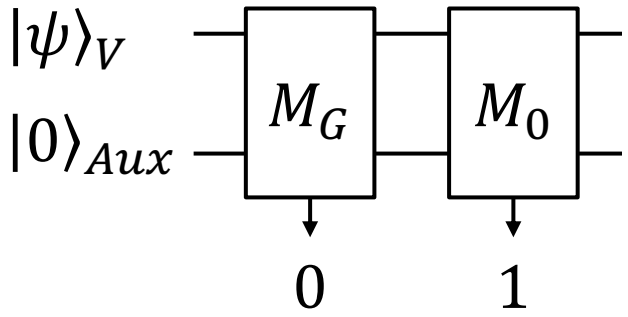

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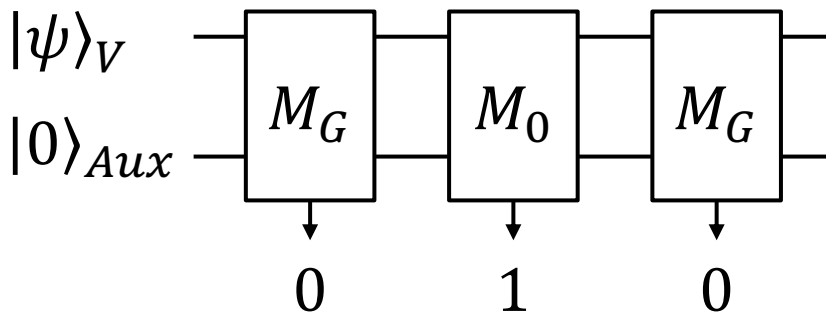

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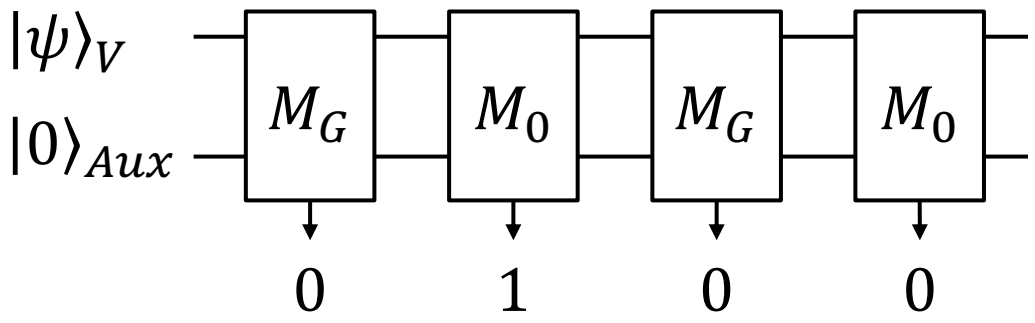

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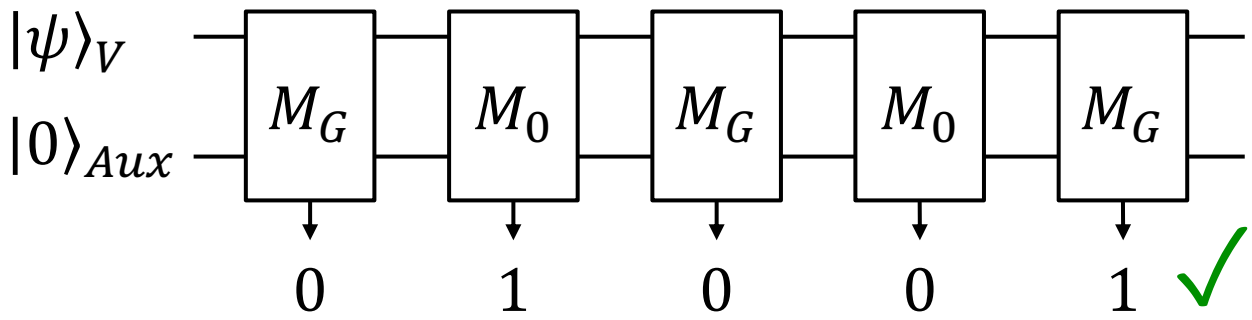

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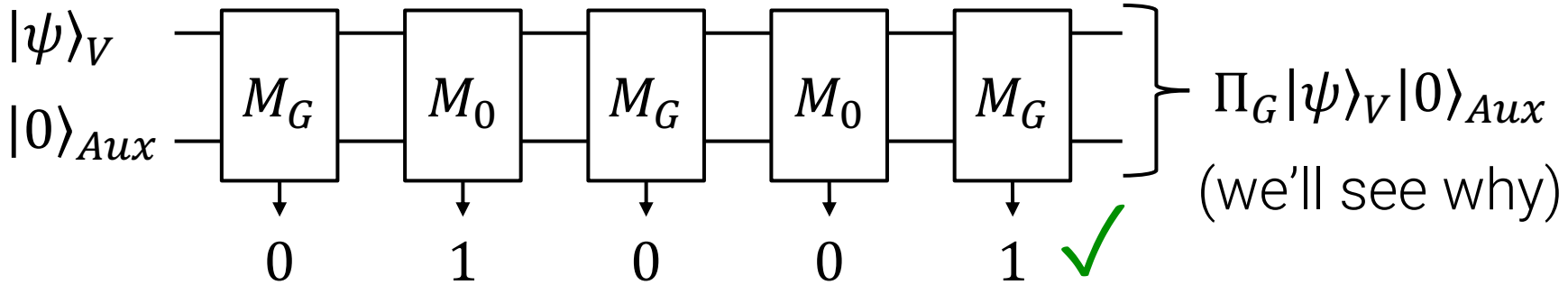

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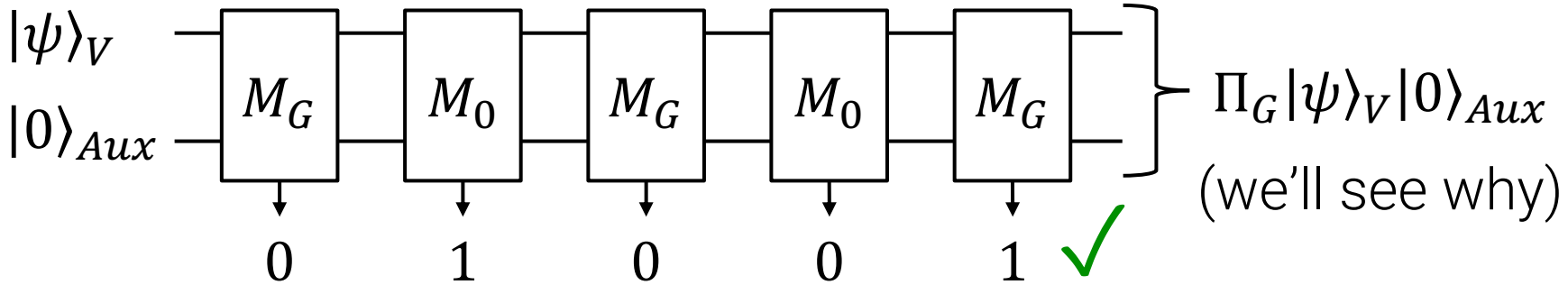

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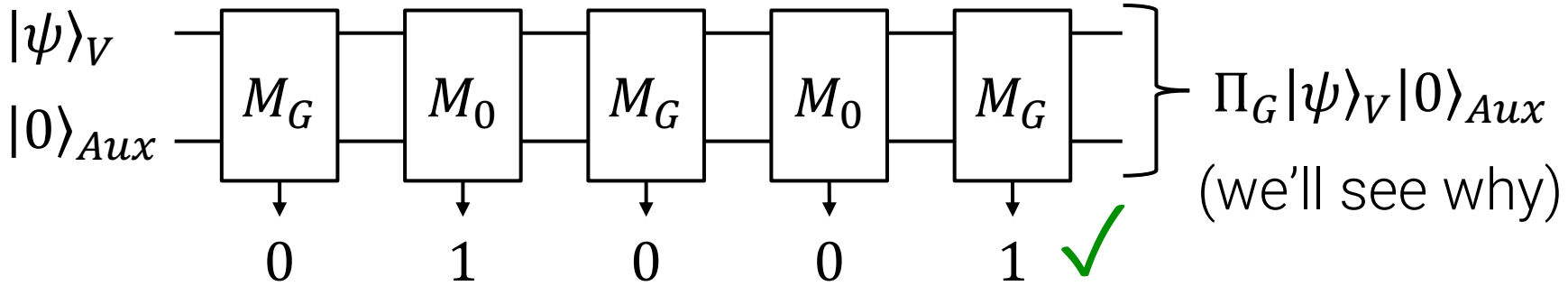


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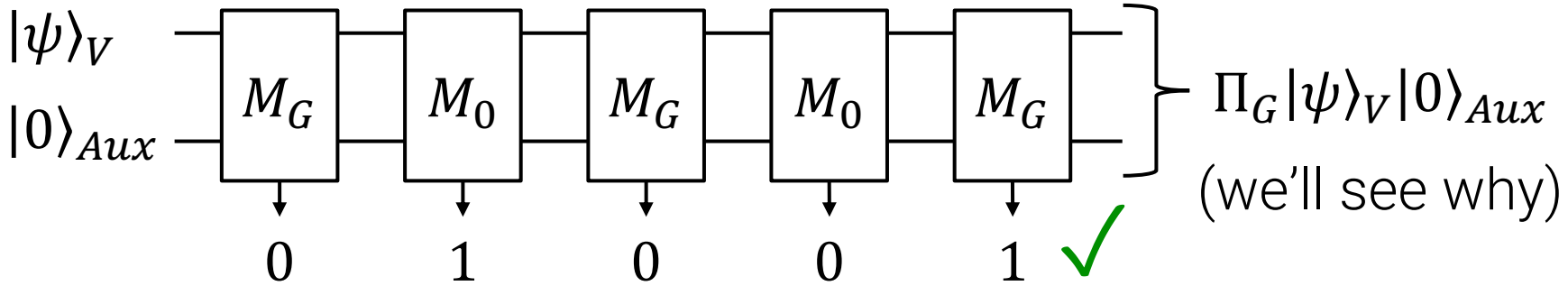


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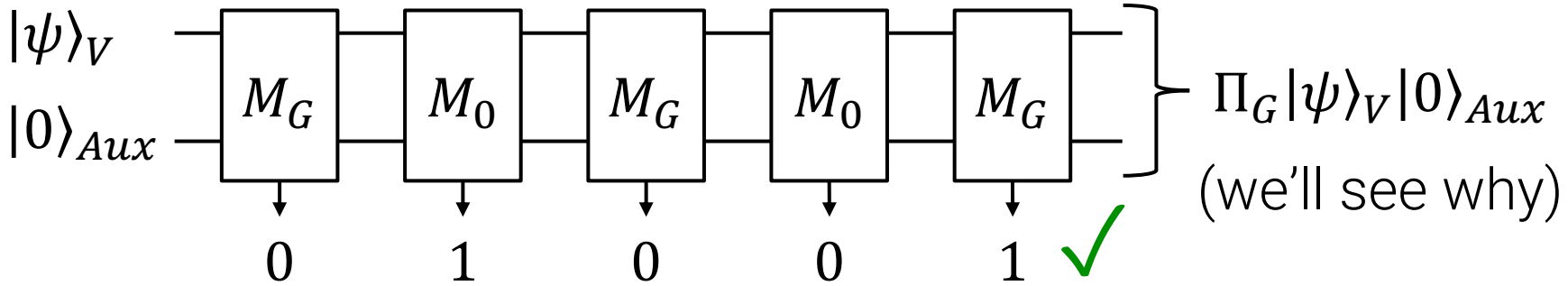


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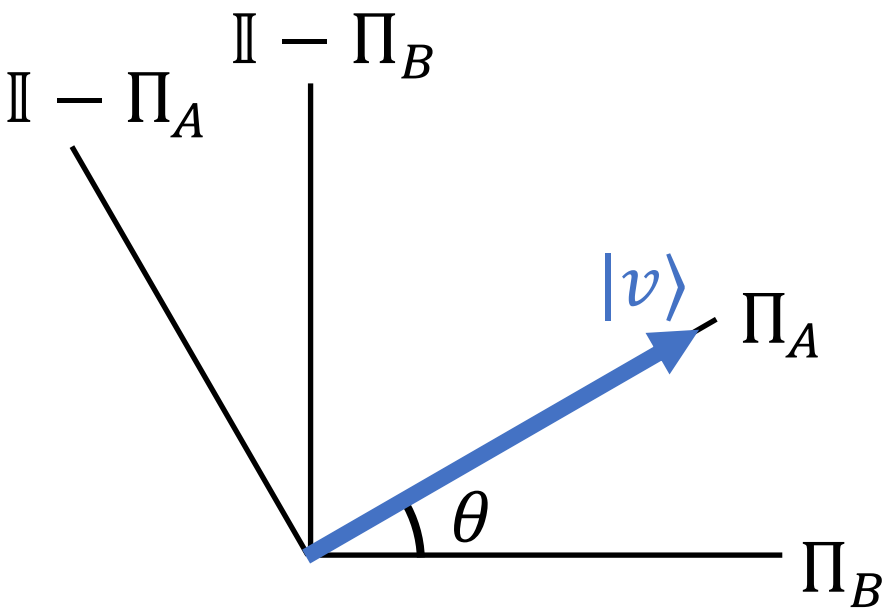

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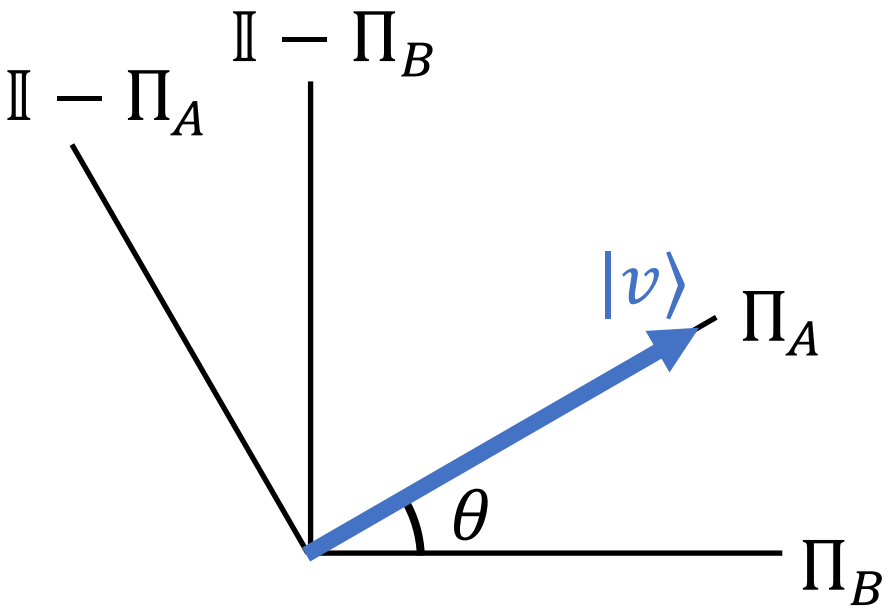


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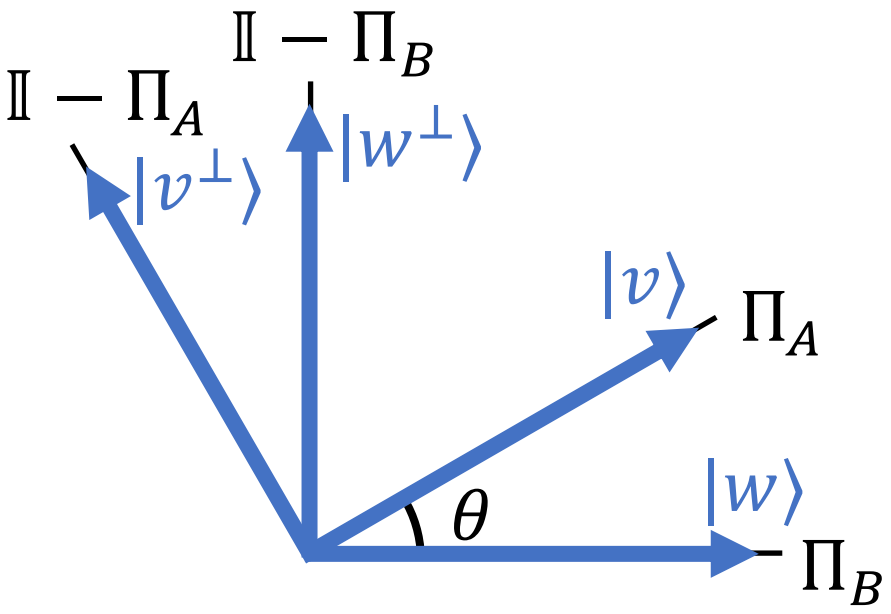


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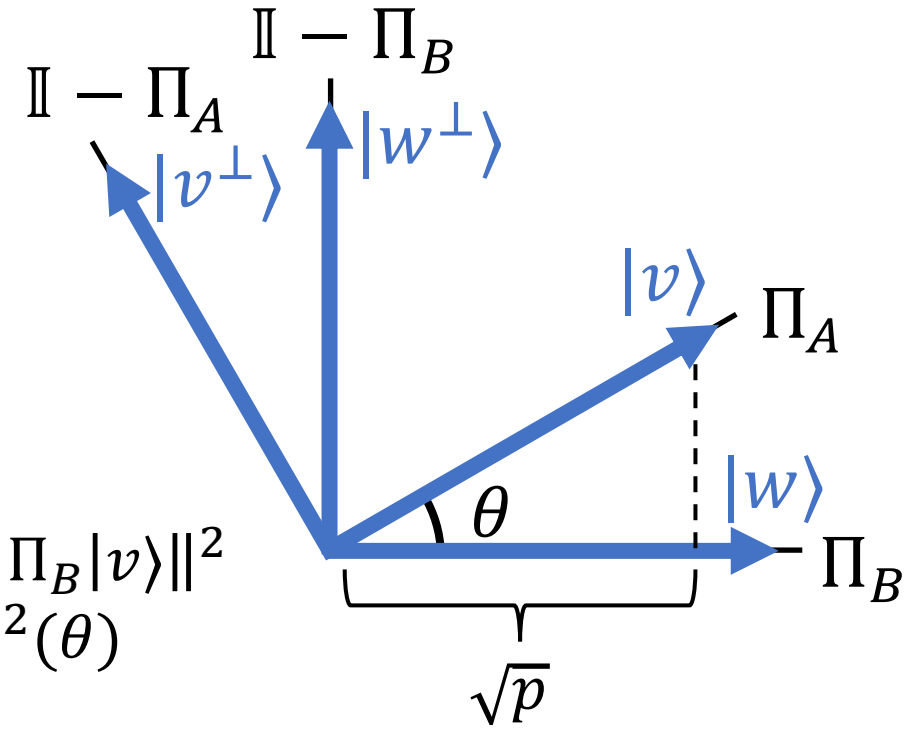


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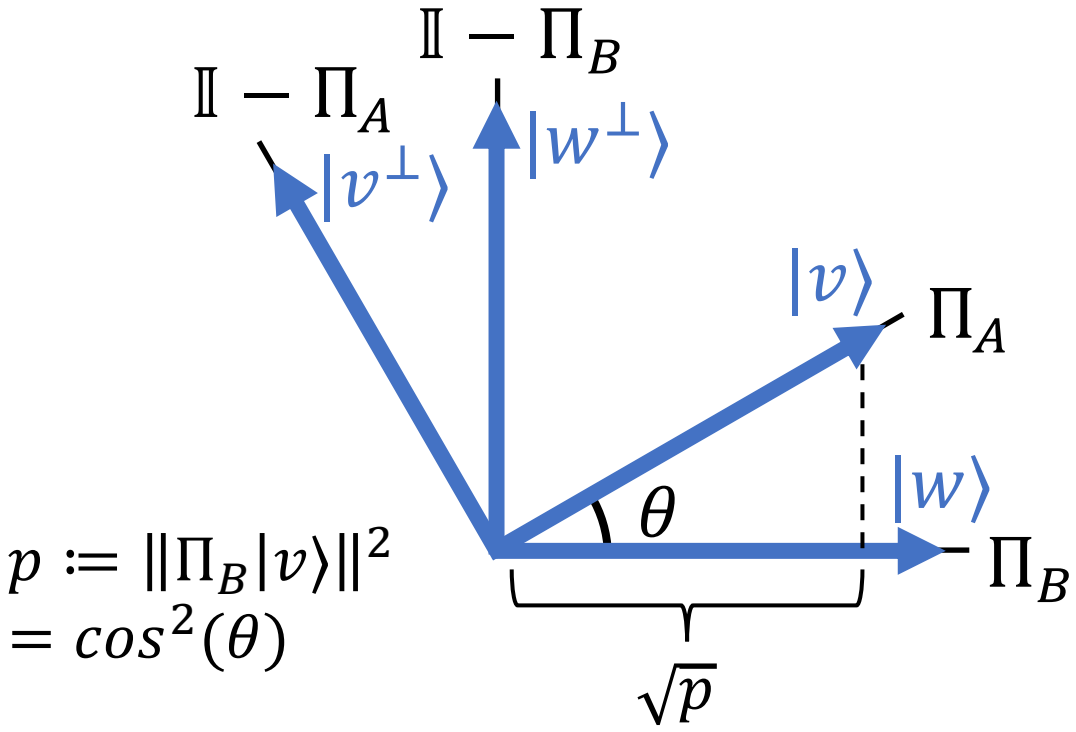


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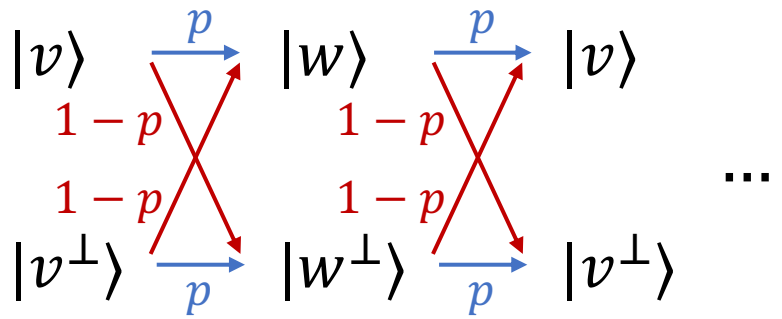

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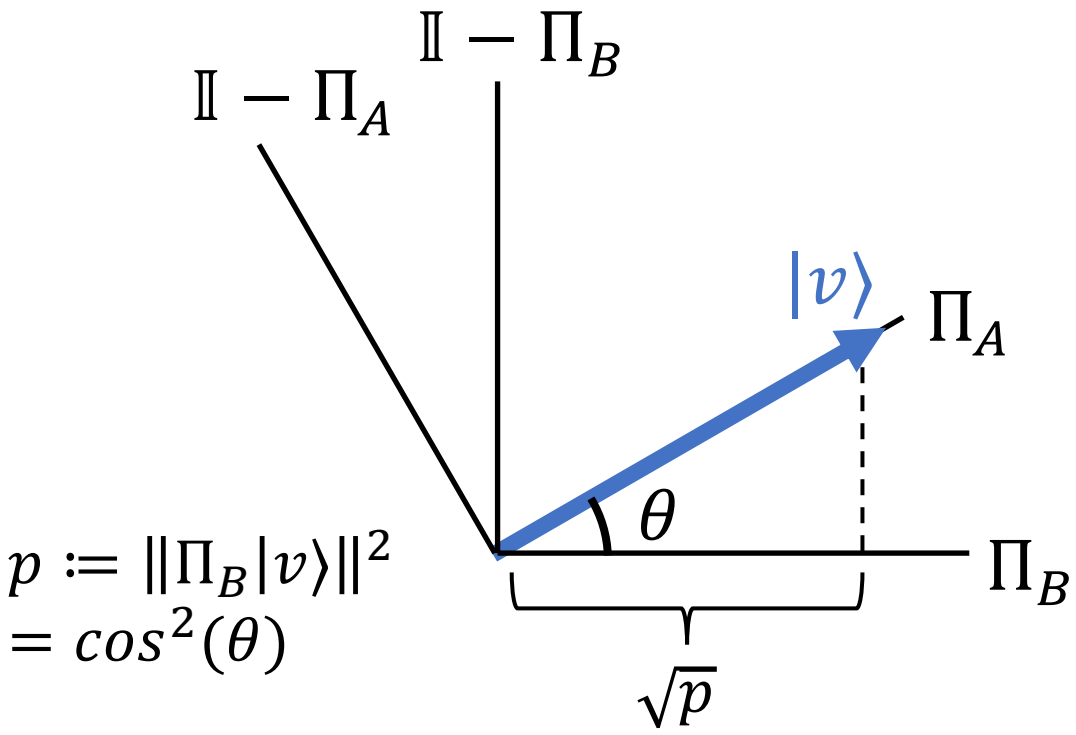


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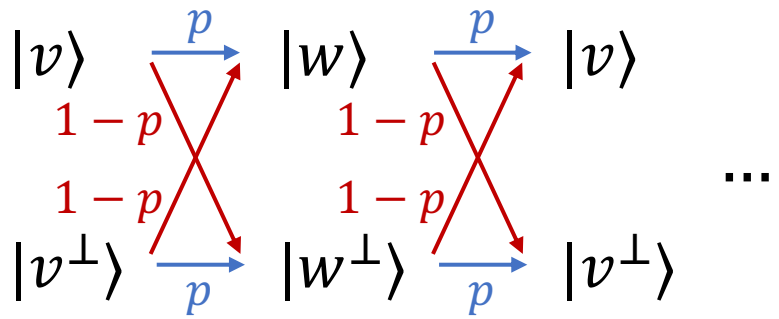

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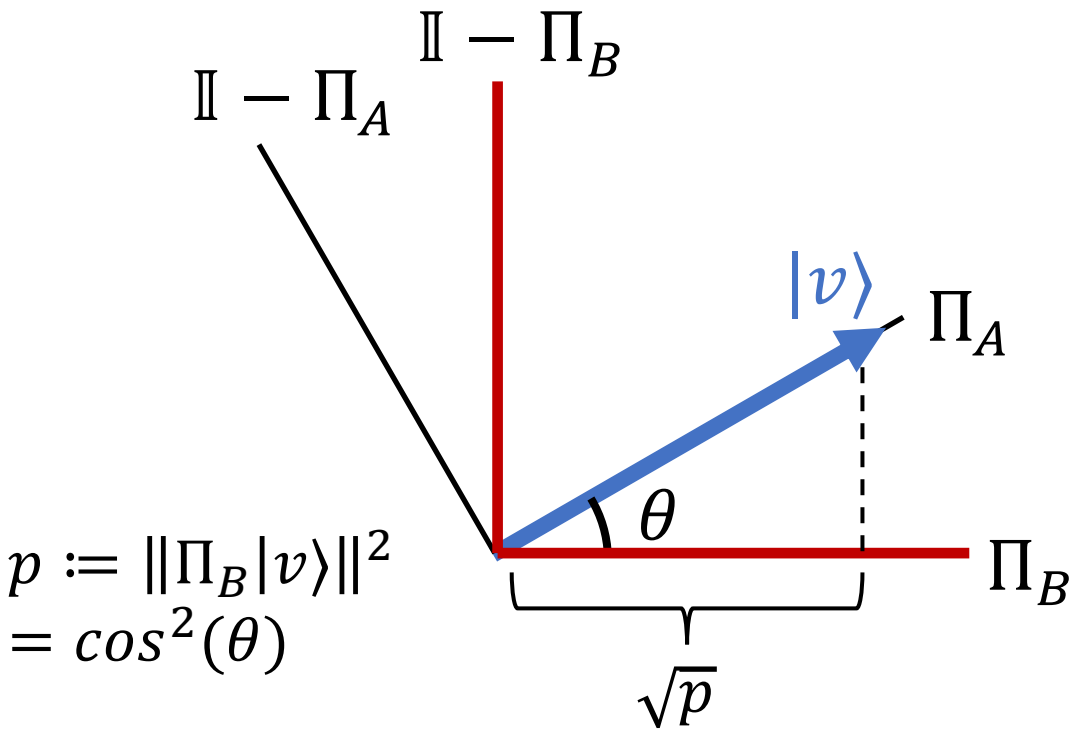


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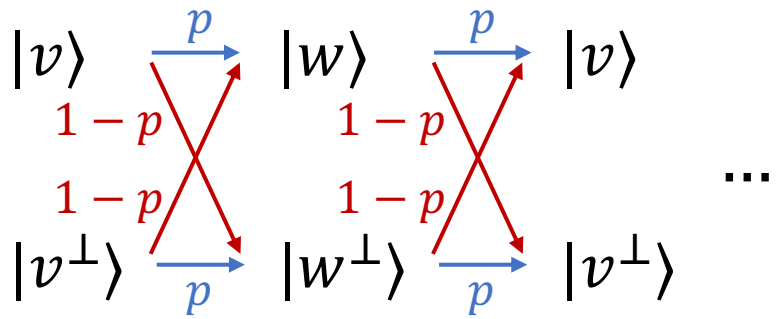

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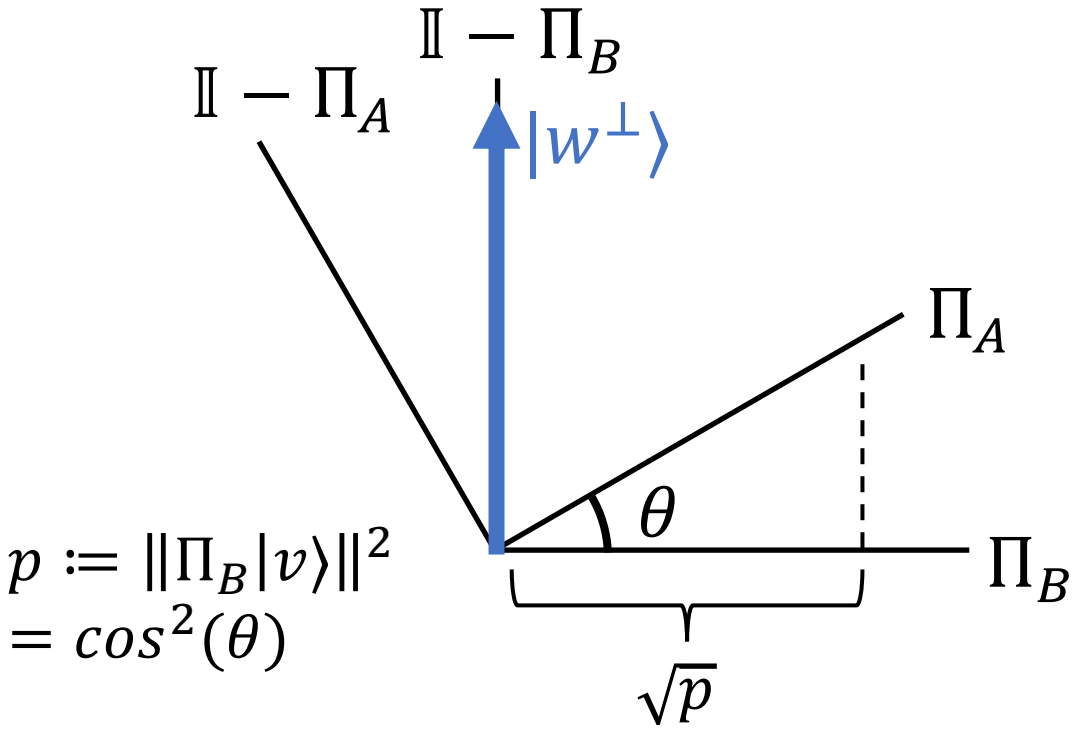


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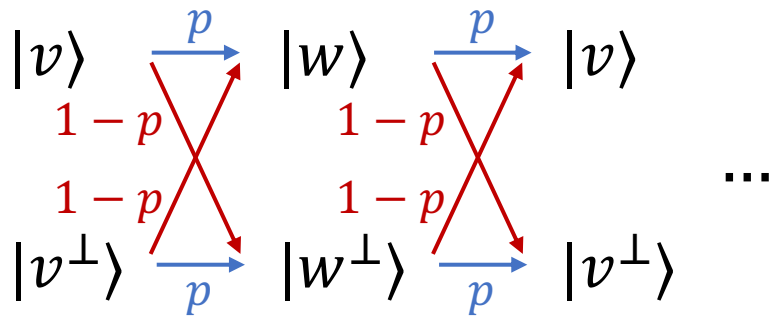

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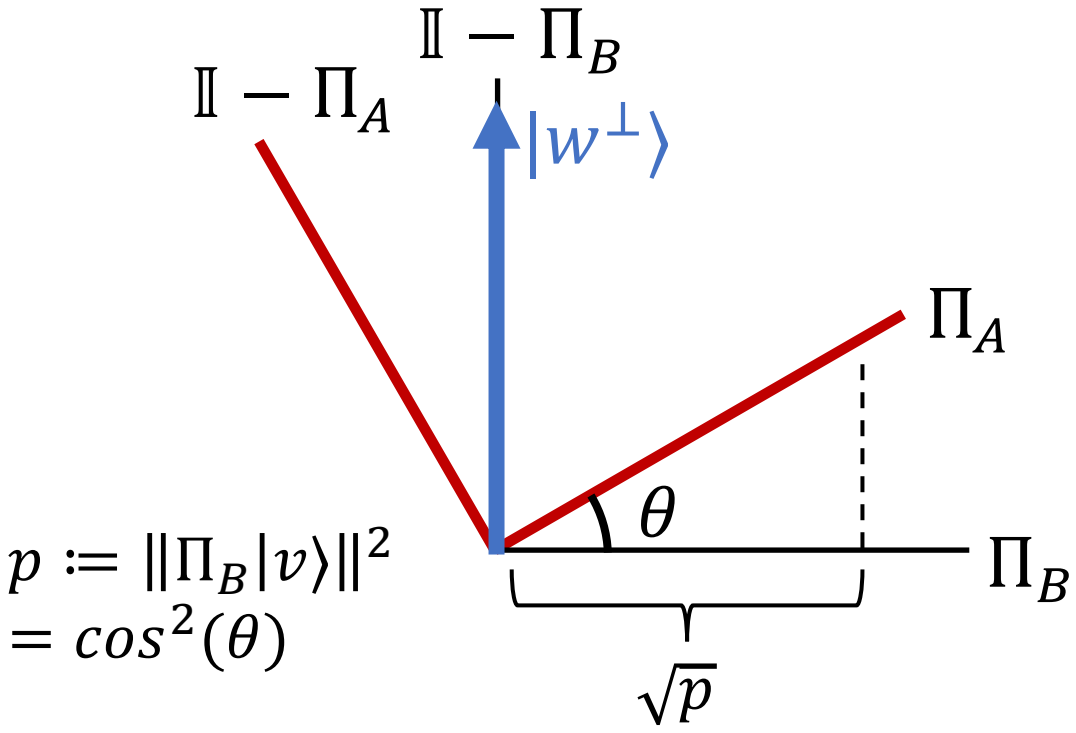


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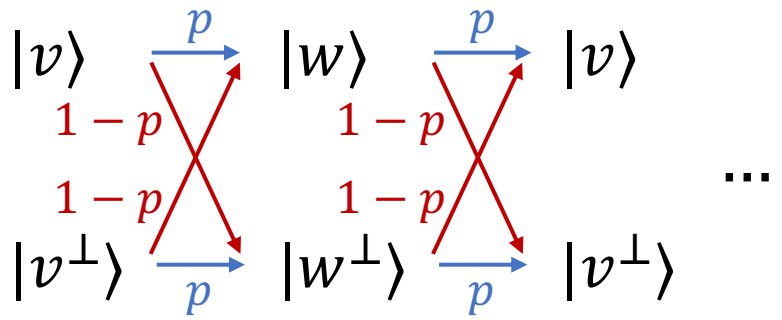

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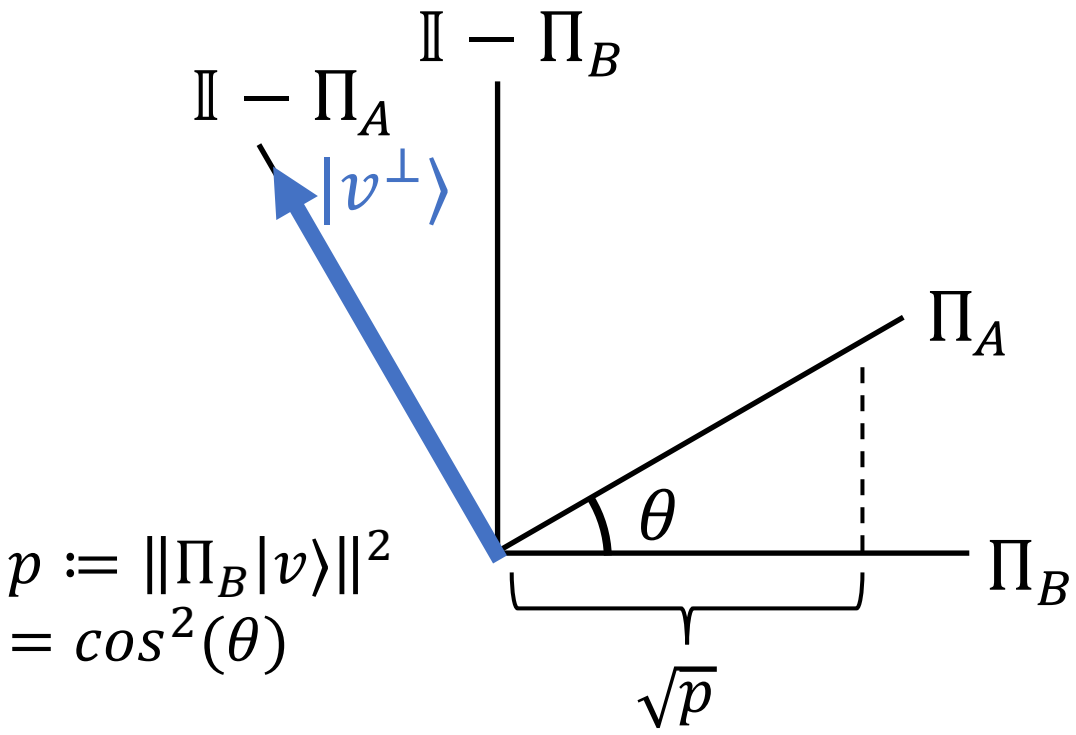


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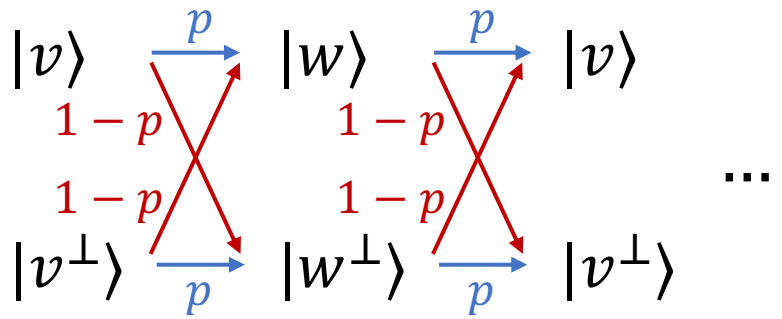

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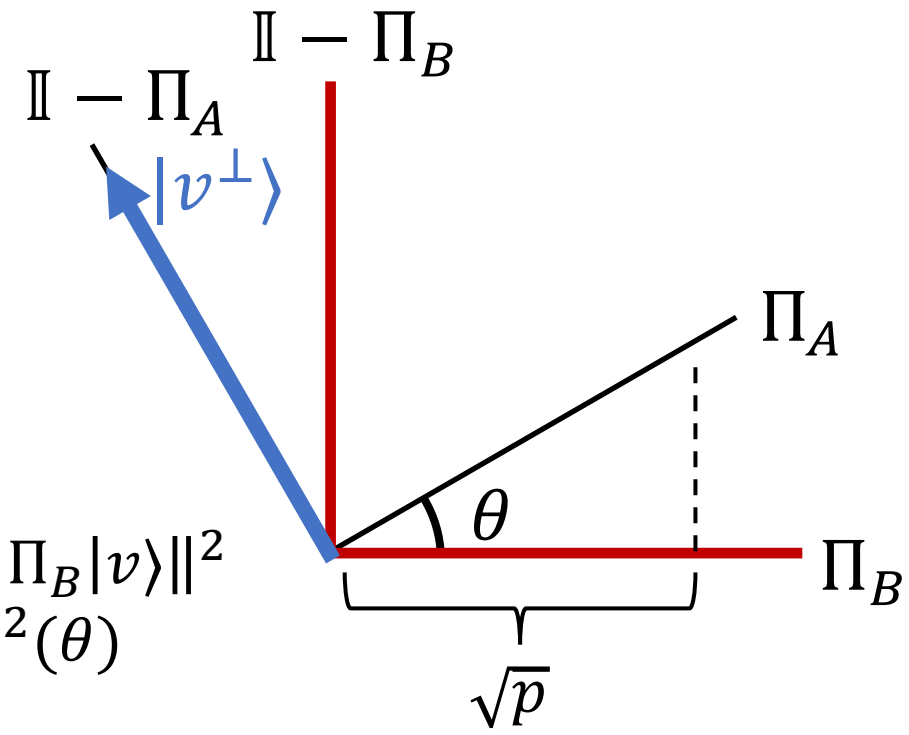


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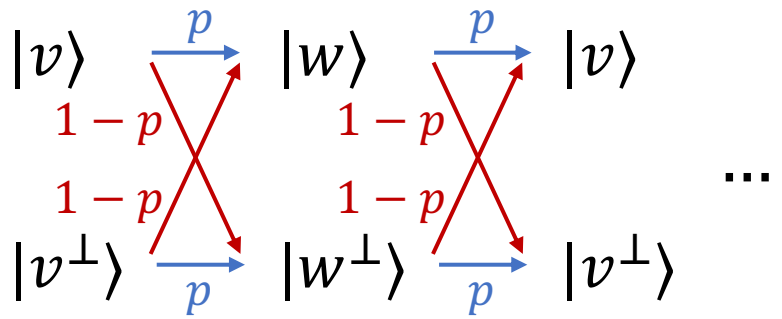

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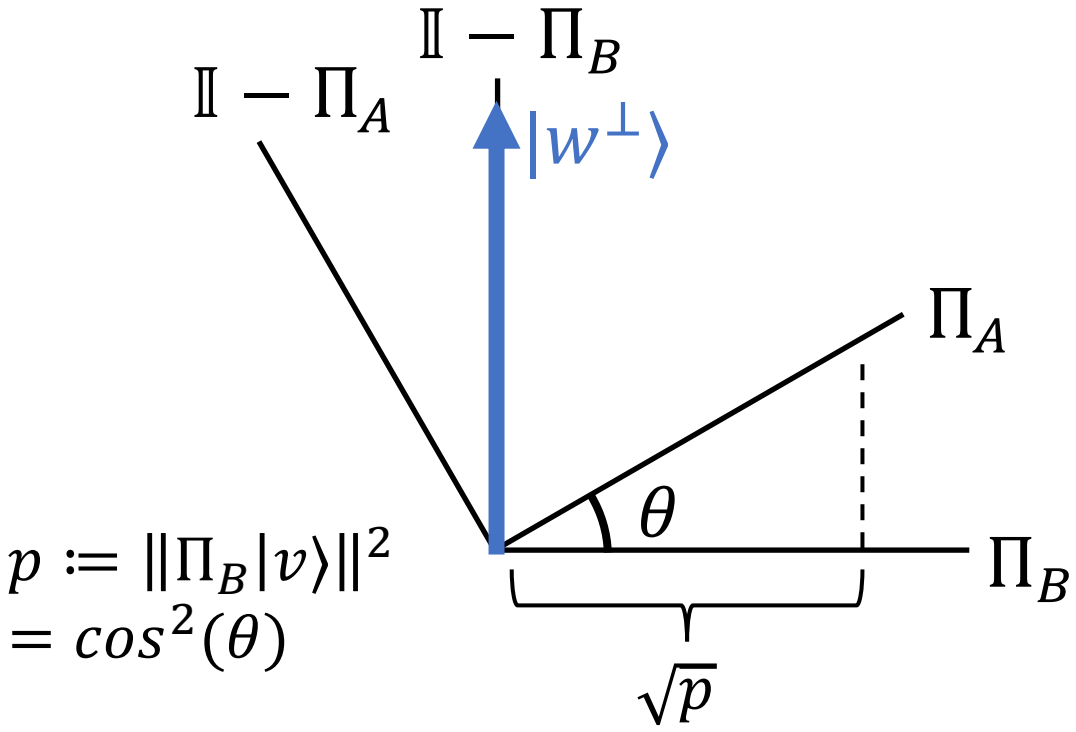


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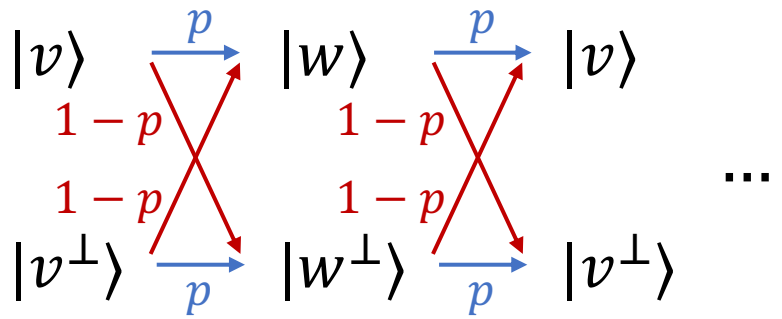

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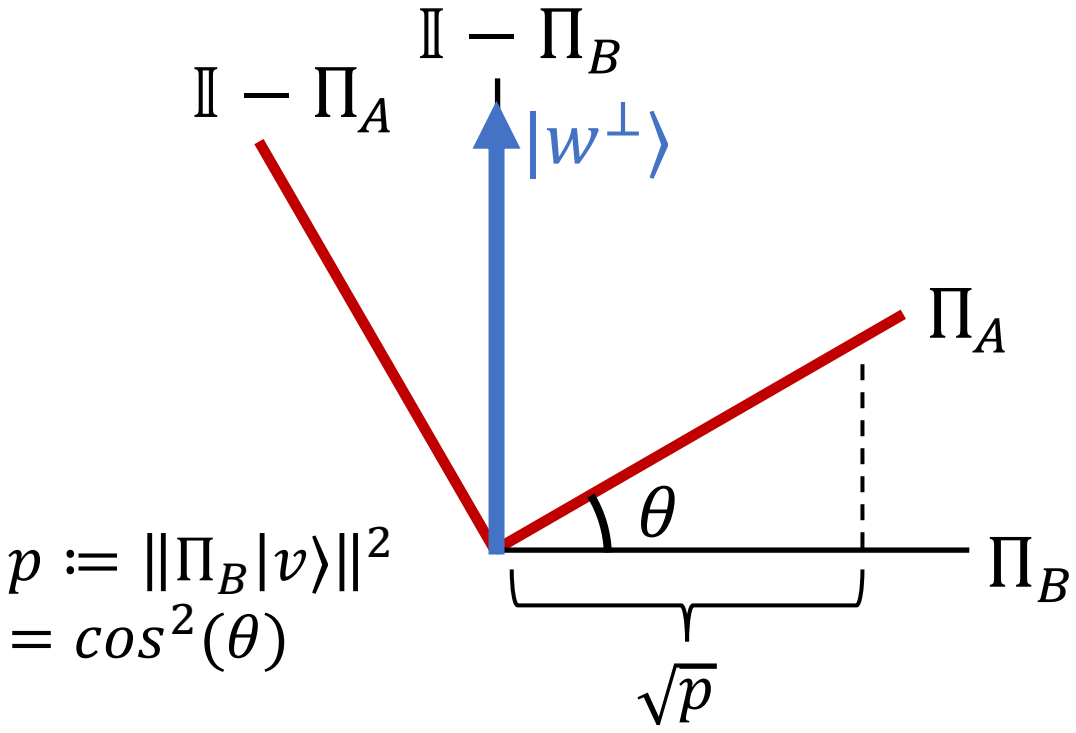


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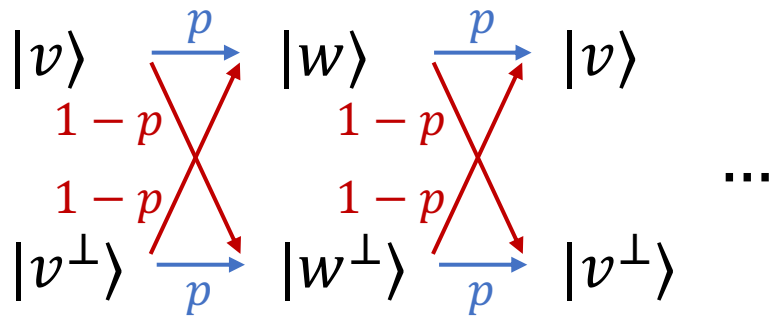

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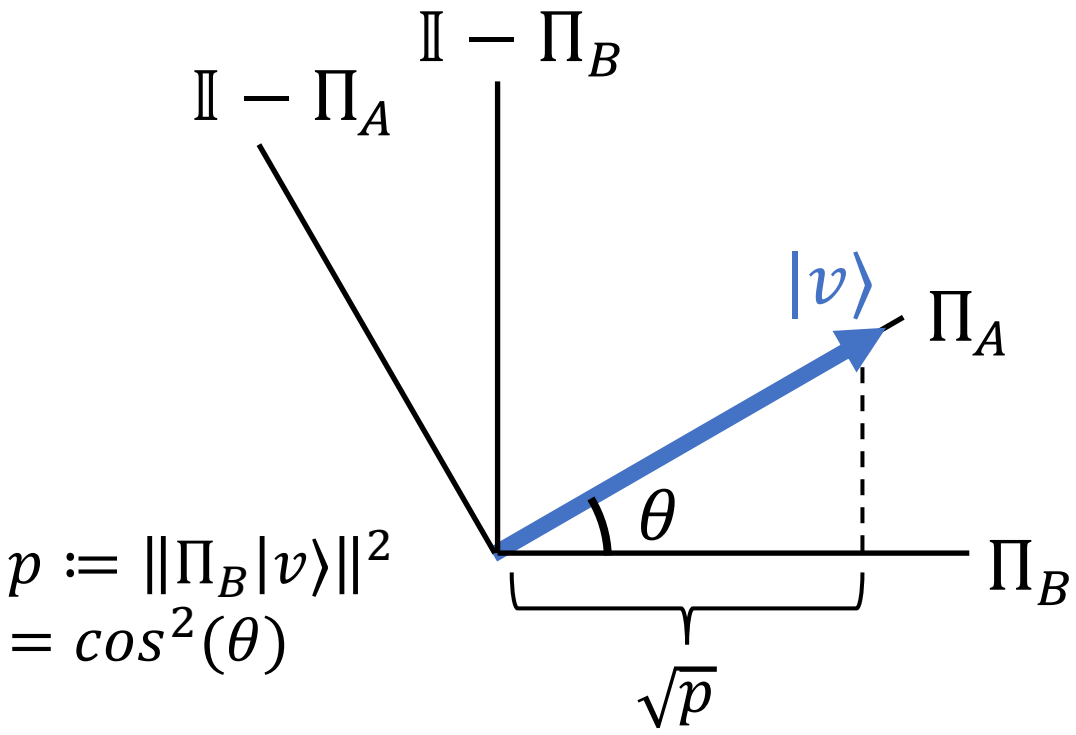


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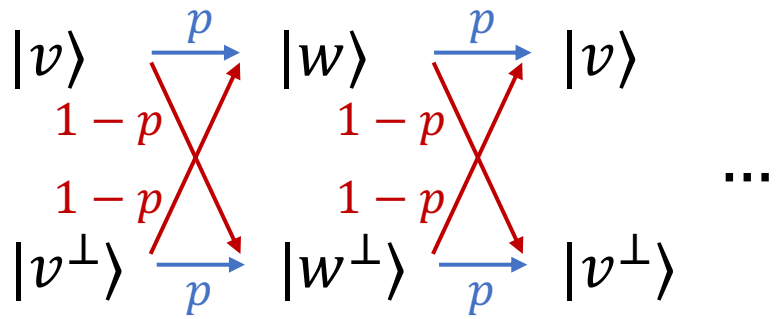

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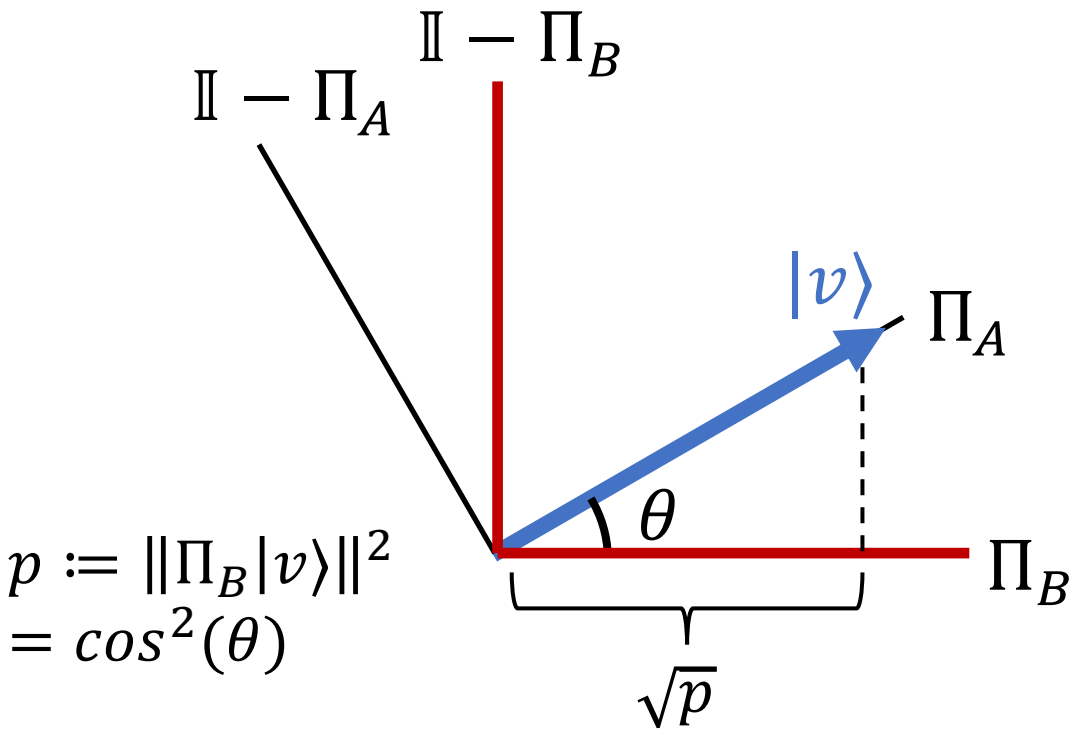


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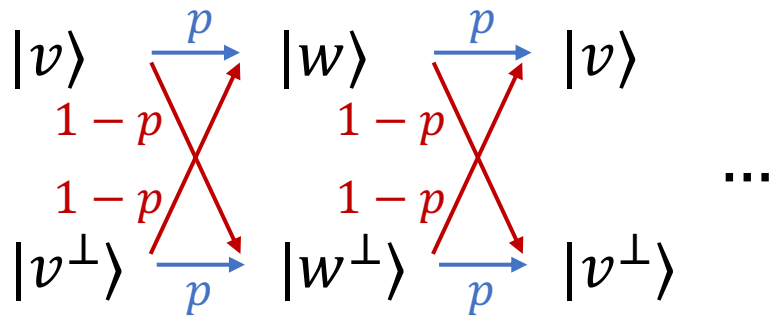

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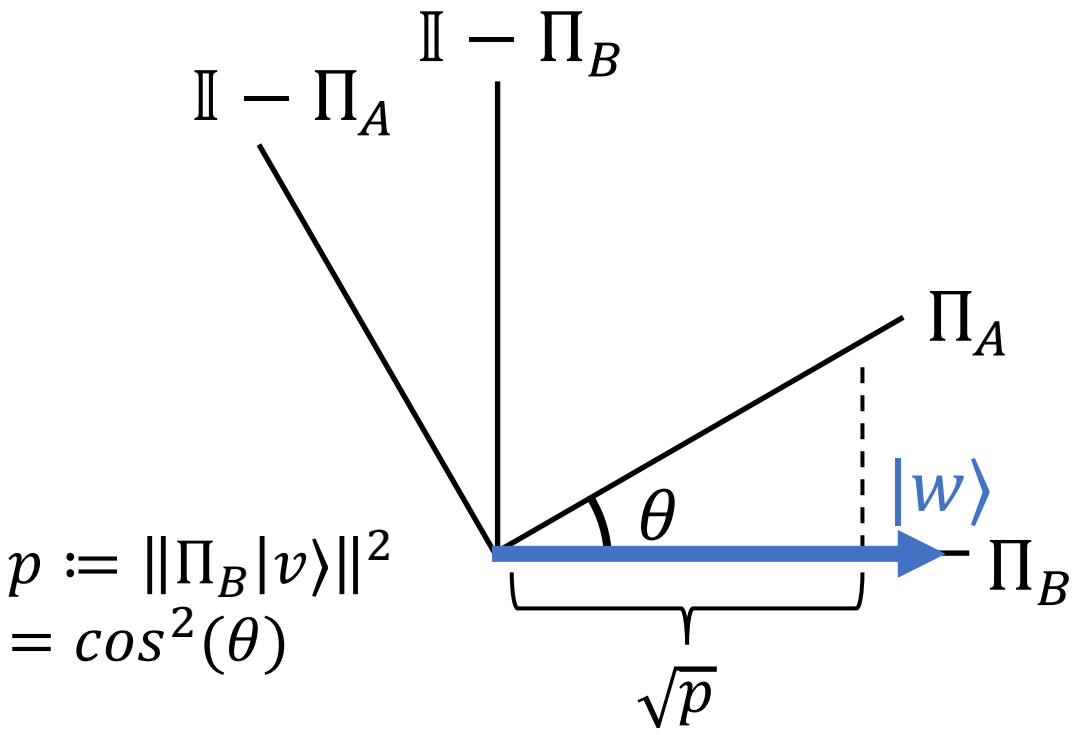


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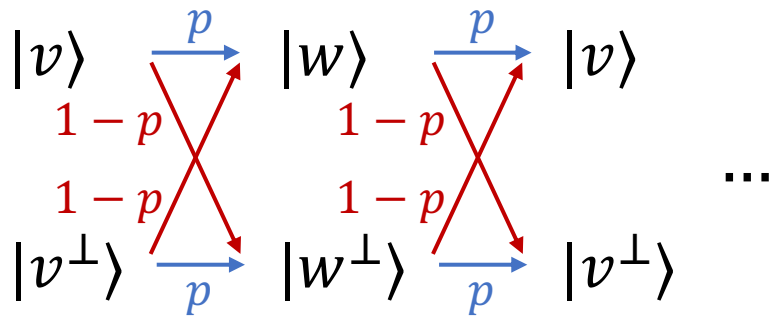

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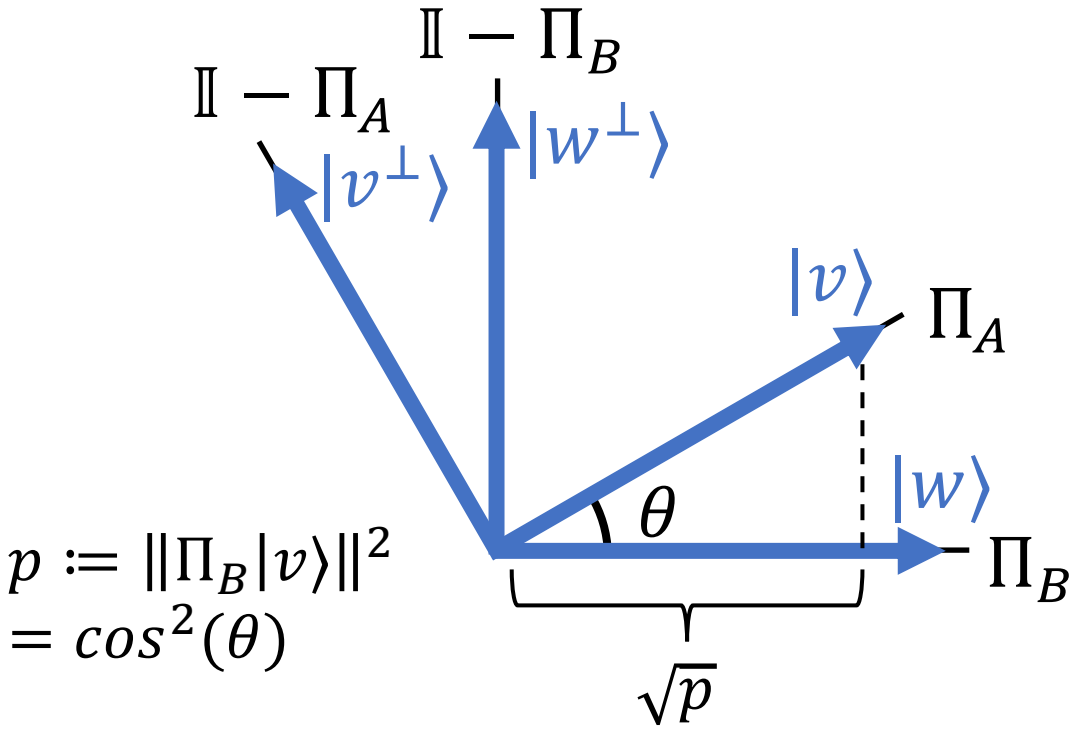


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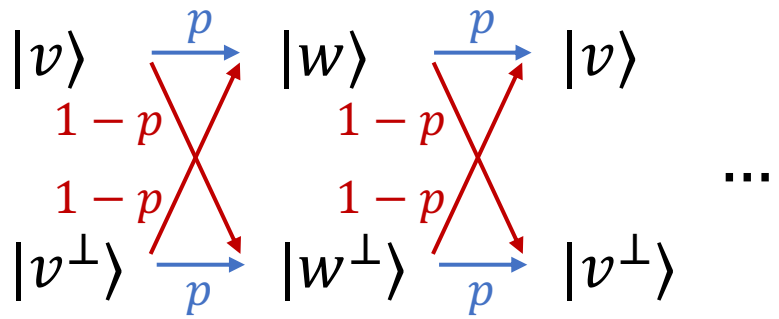


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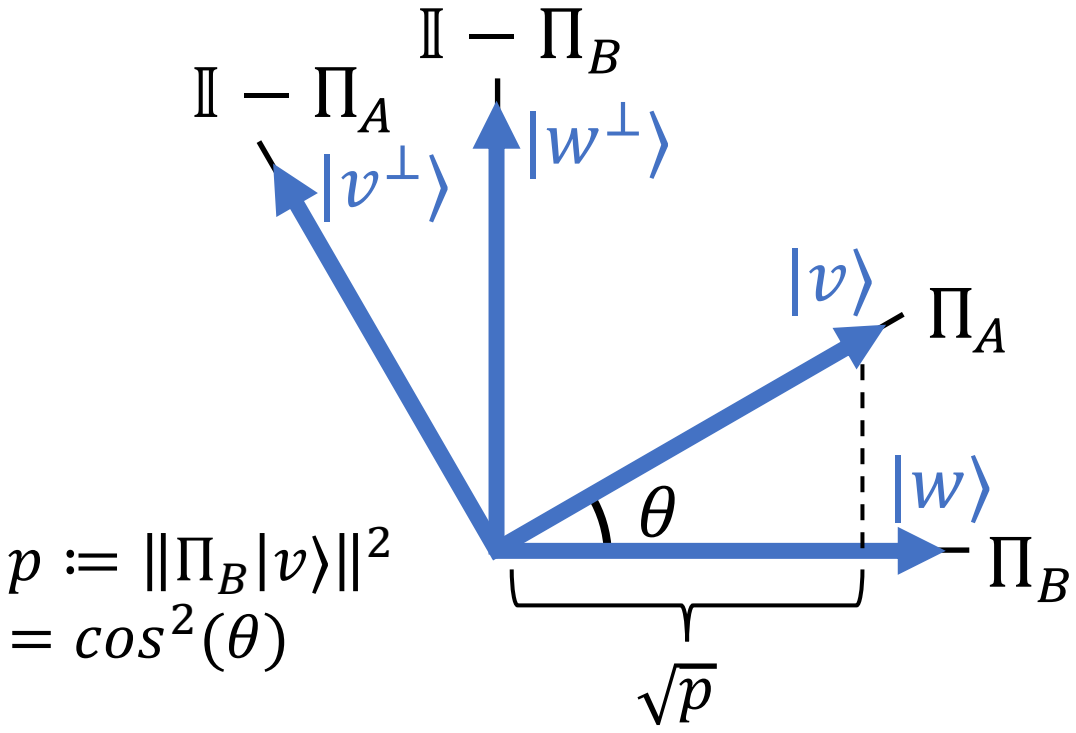


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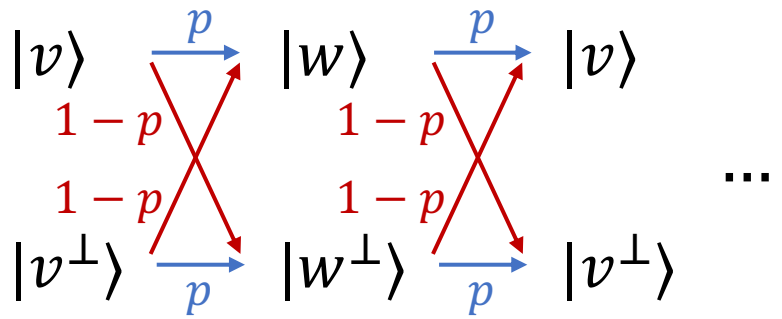


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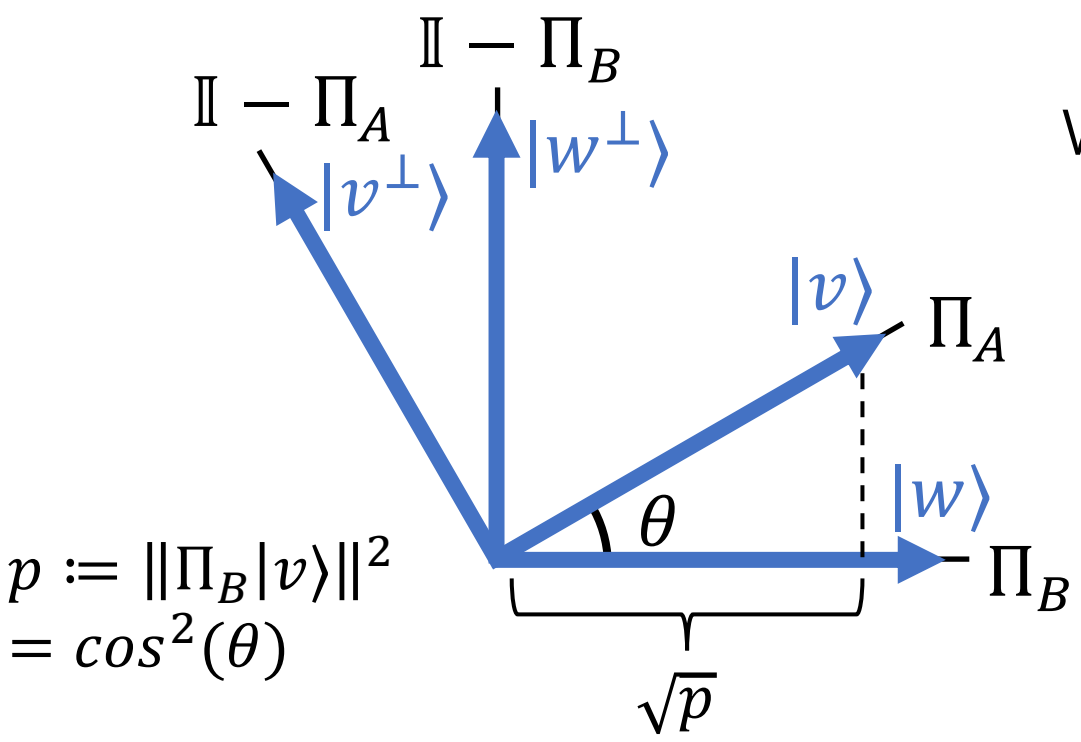


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These are the guarantees we want, but $\Pi_0, \Pi_G$ don't live in 2D!

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# Extremely Useful Tool

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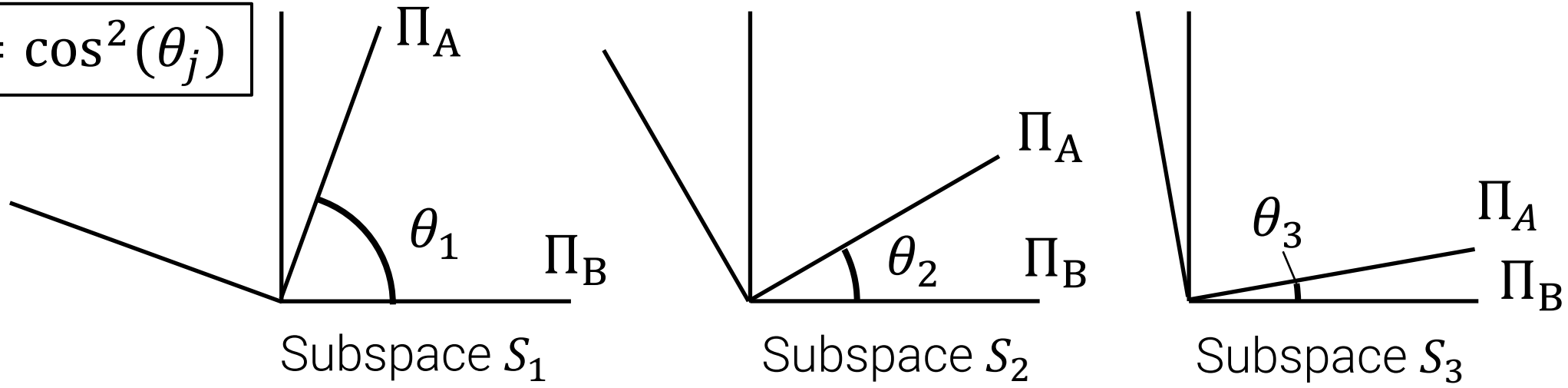

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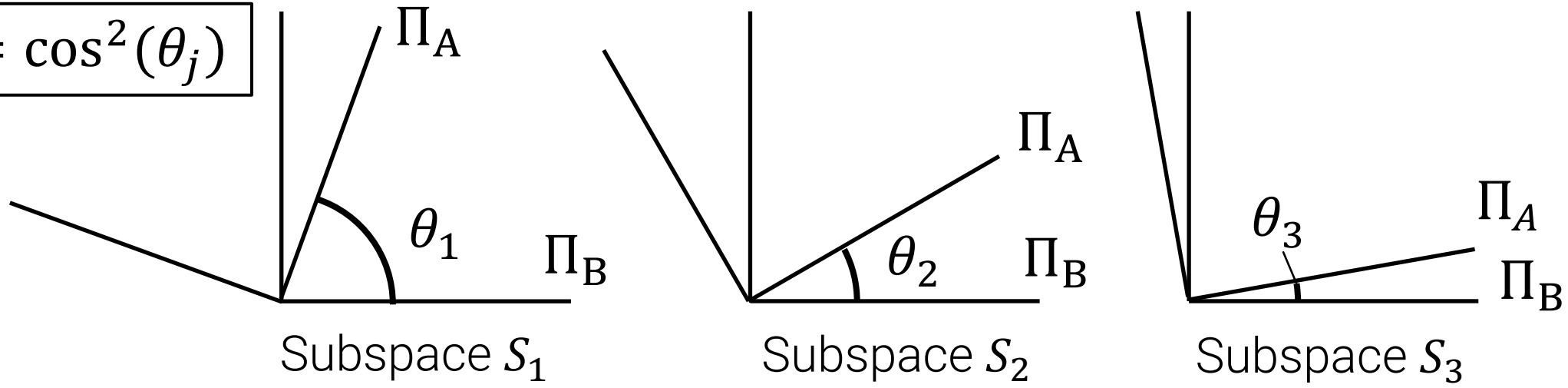


To analyze our simulator, it will be helpful to understand the Jordan subspace decomposition for $\Pi_0, \Pi_G$.

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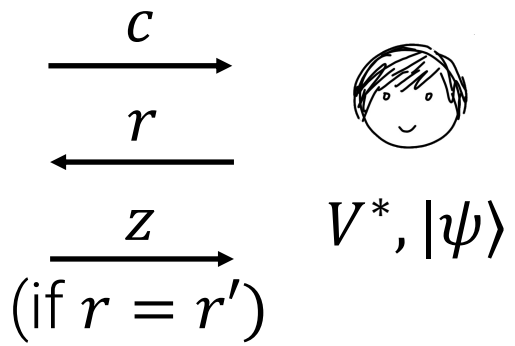
2) If $r = r'$, output $(c, r, z)$. Otherwise $\perp$.



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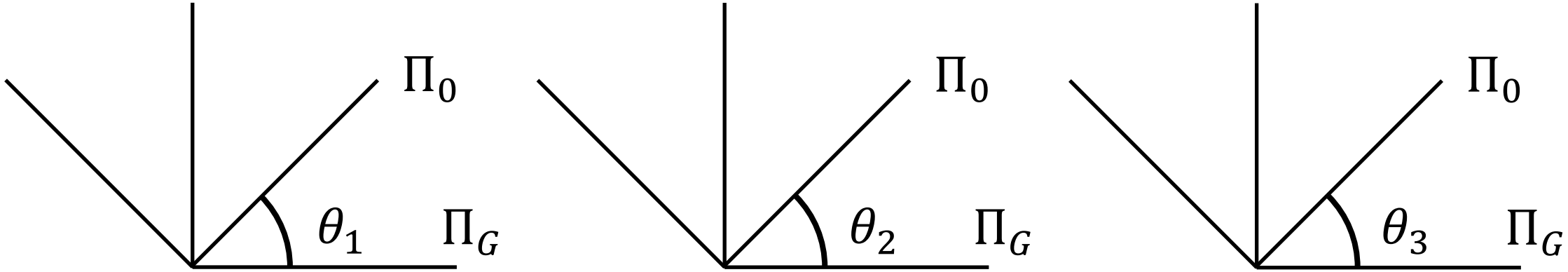


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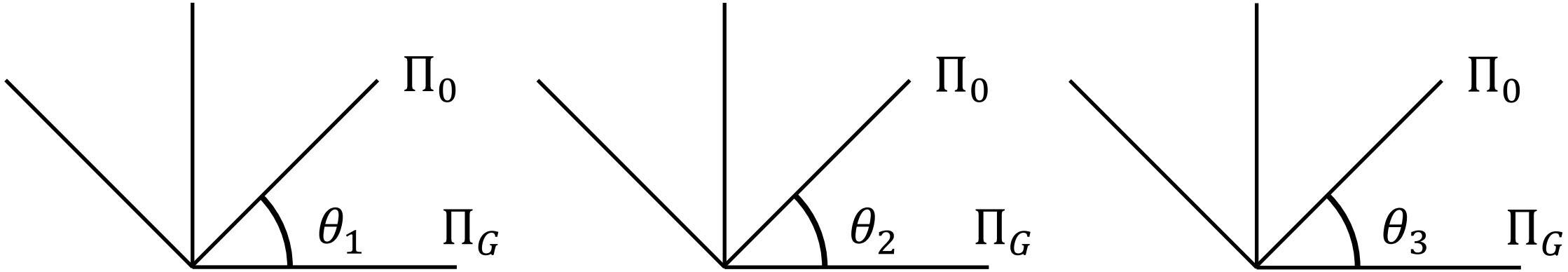


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We can now extend the 2-D analysis to our simulator!

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These claims extend to high-dim if all $(\Pi_A, \Pi_B)$-Jordan subspaces have roughly equal $p_j$.

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Intuition for Claim 1: the 2-D runtime analysis extends to higher dimensions because the $\Pi_A, \Pi_B$ measurements act independently on each Jordan subspace.

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Intuition for Claim 2:

- Consider $|v\rangle = \sum_j \alpha_j |v_j\rangle$. In each $S_j$, the state after $(\Pi_B, \mathbb{I} - \Pi_B)$ accepts is $\propto \Pi_B|v_j\rangle$ by our analysis of the 2-D case.

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- Alternating measurement results only depend on $p_j$, but since all $p_j \approx p$, the measurement outcomes give no signal about $j$.
- So the final state is $\propto \sum_j \alpha_j \Pi_B |v_j\rangle = \Pi_B|v\rangle$.

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- By Claim 1, the simulator is efficient.
- By Claim 2, when $M_G \to 1$, the state is $\propto \Pi_G|\psi\rangle|0\rangle$ as desired.

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Zero knowledge:

- Key tool: obtain a quantum analogue of the classical "repeated-guessing" simulator using alternating projectors.
- Analyze alternating projectors via Jordan's lemma

Thank You!

Questions?