Commitments to Quantum States

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based on joint work with:

Sam Gunn (UC Berkeley)

Nathan Ju (UC Berkeley)

Mark Zhandry (NTT Research)

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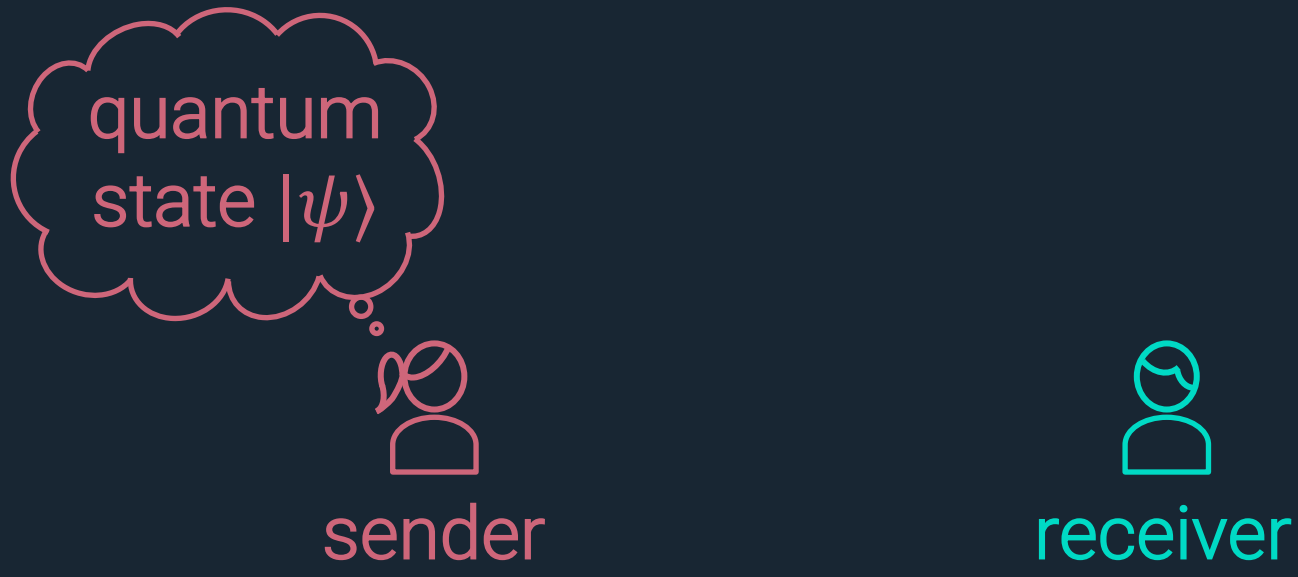

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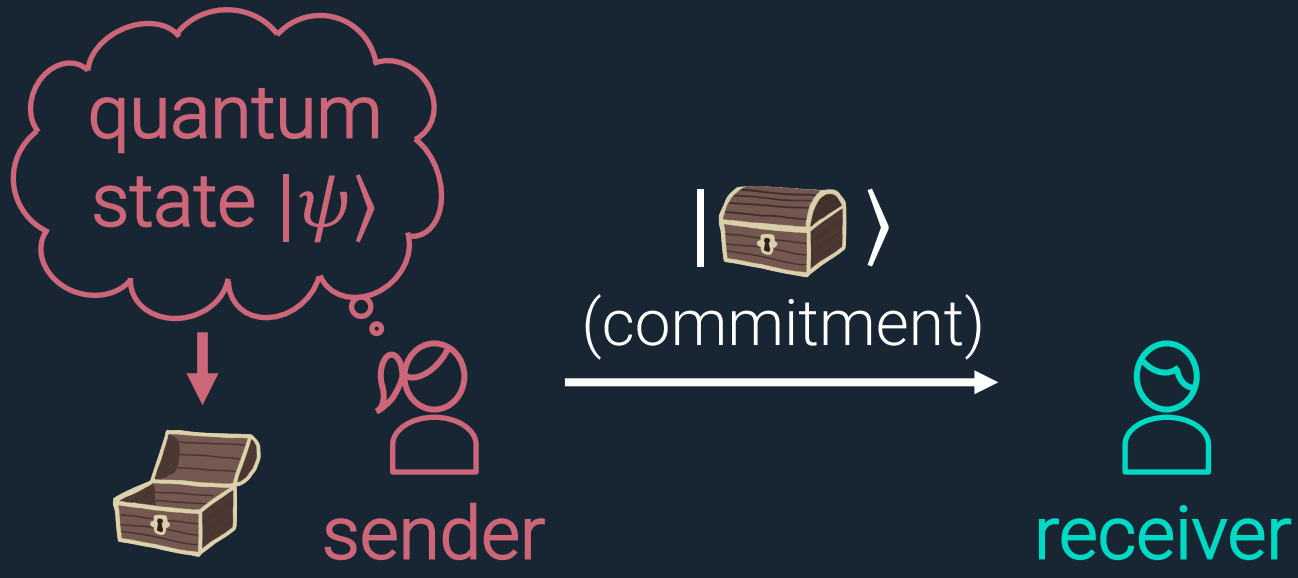

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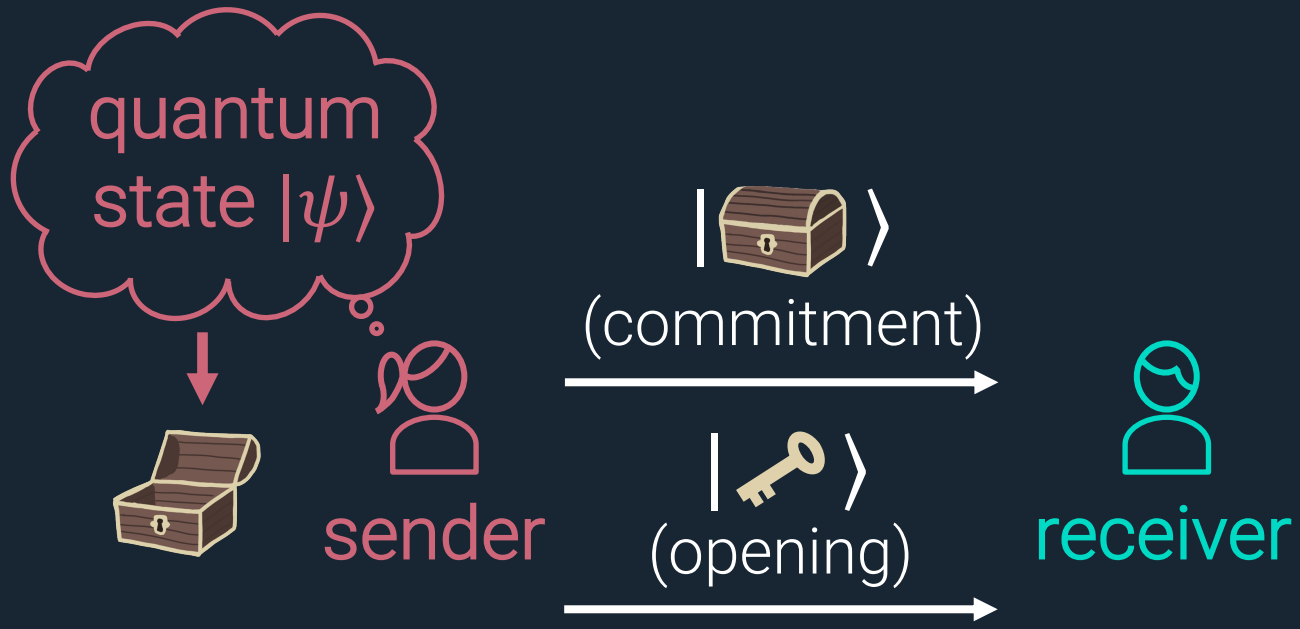

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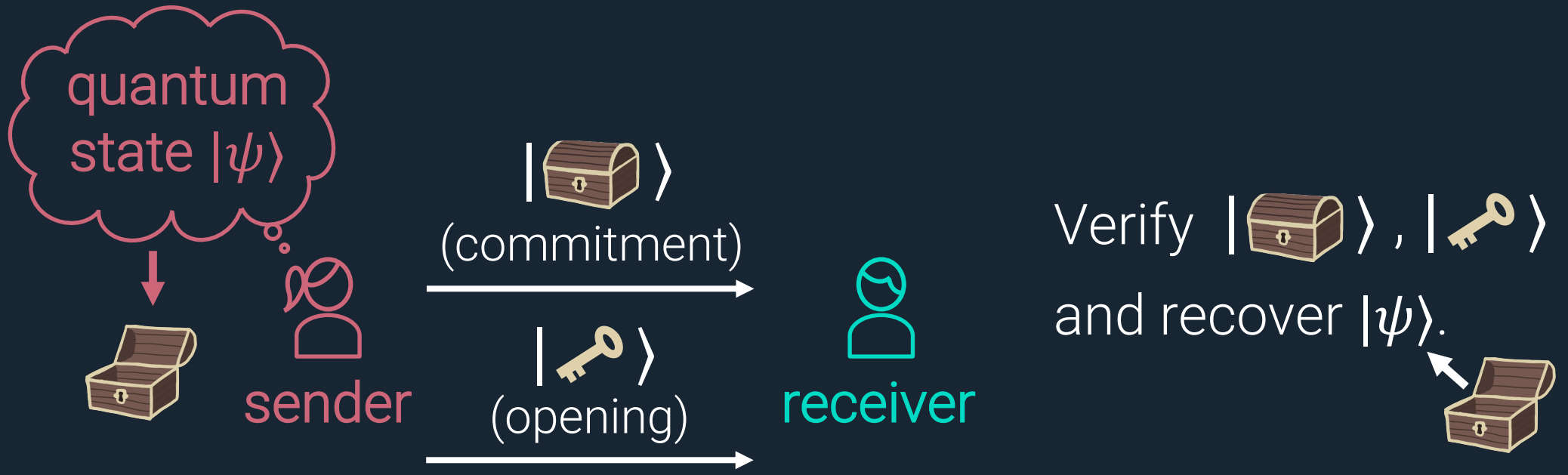

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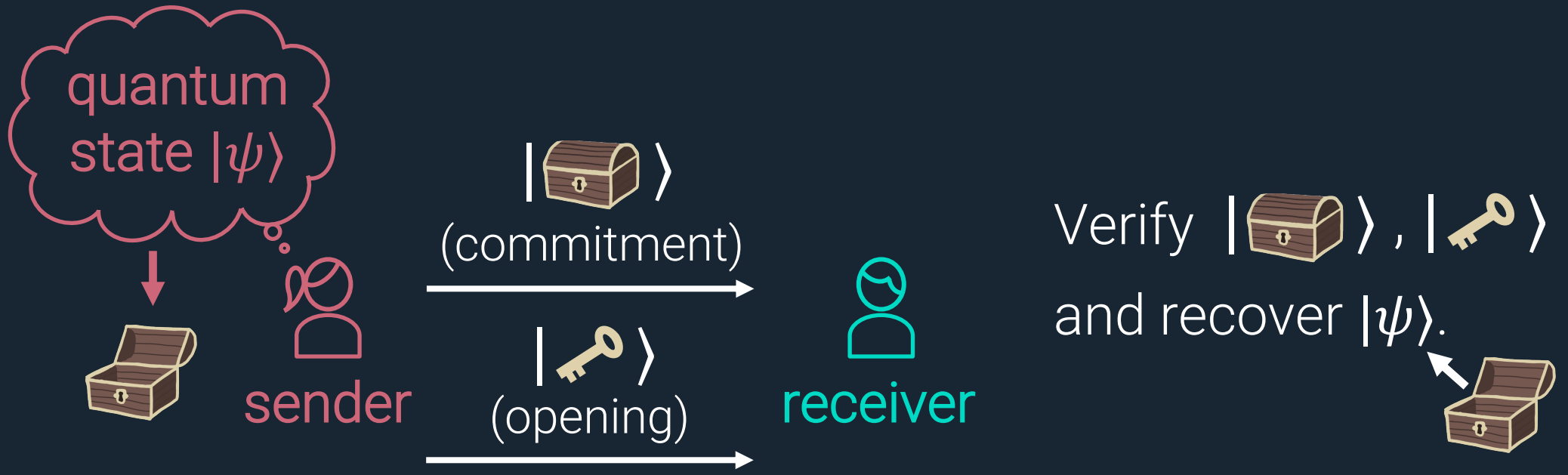

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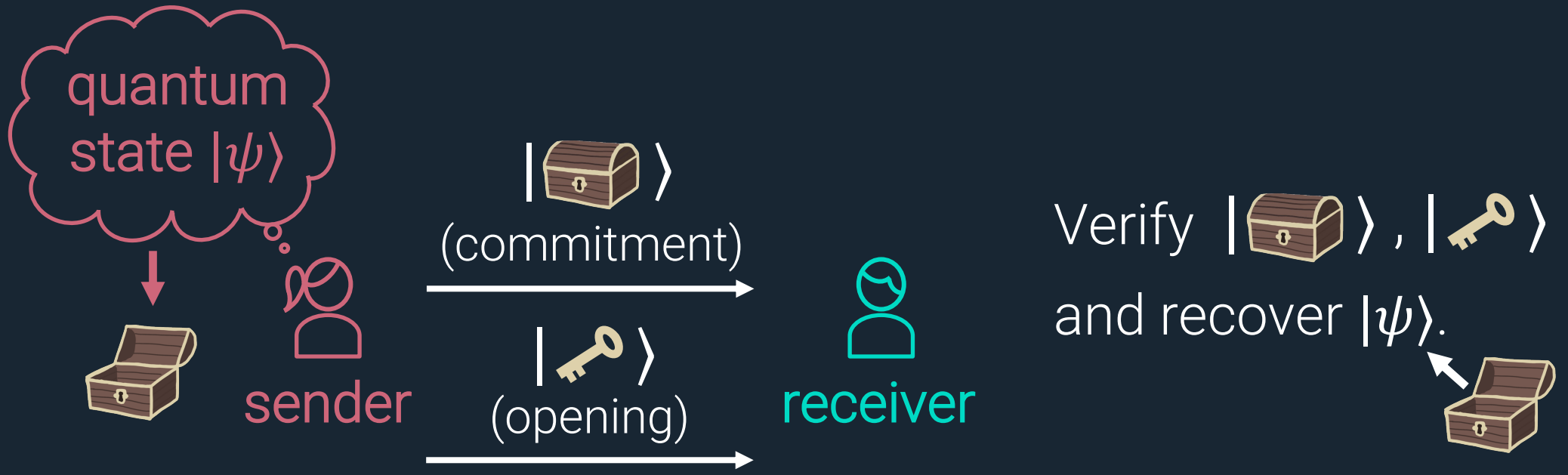

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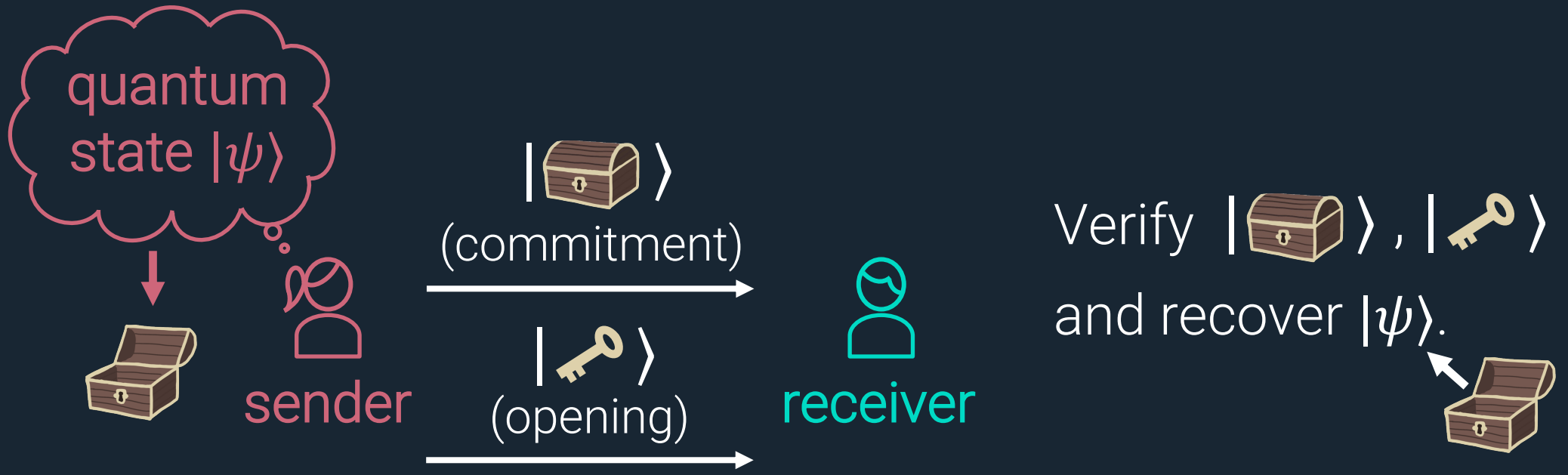

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quantum one-time pad + classical-msg commitments

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quantum state $|\psi\rangle$

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2) $\mathrm{C} = \mathrm{Com}((a,b); r)$

$= (X^a Z^b |\psi\rangle, \mathrm{C})$

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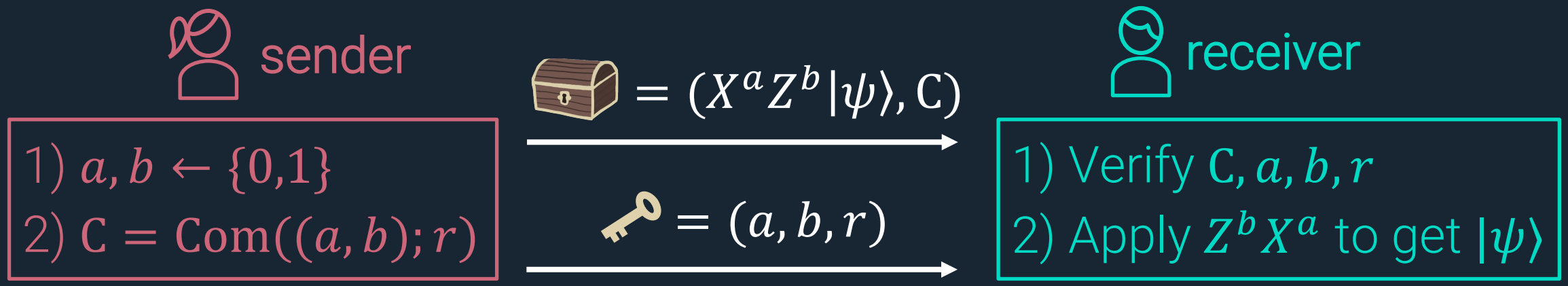

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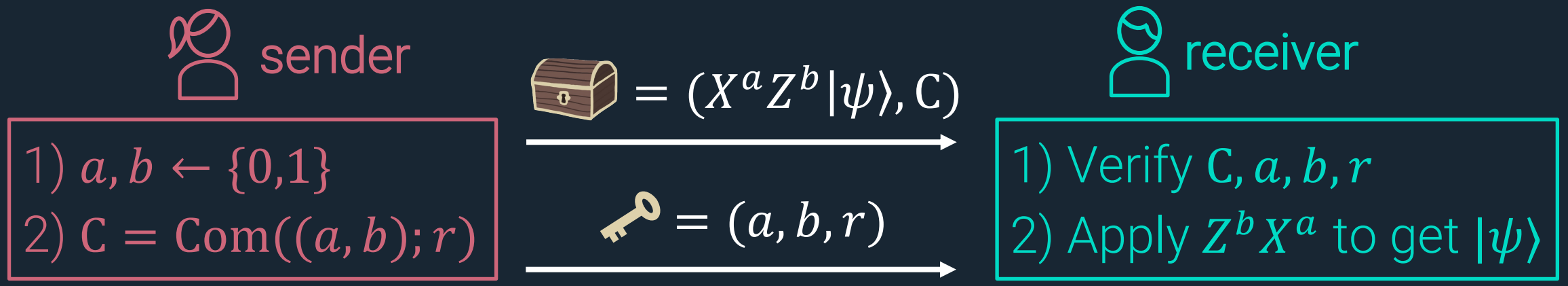

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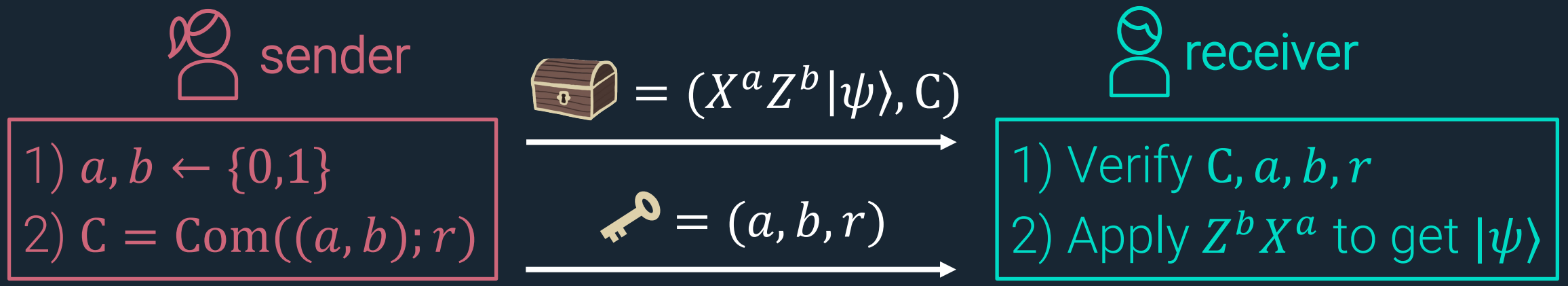

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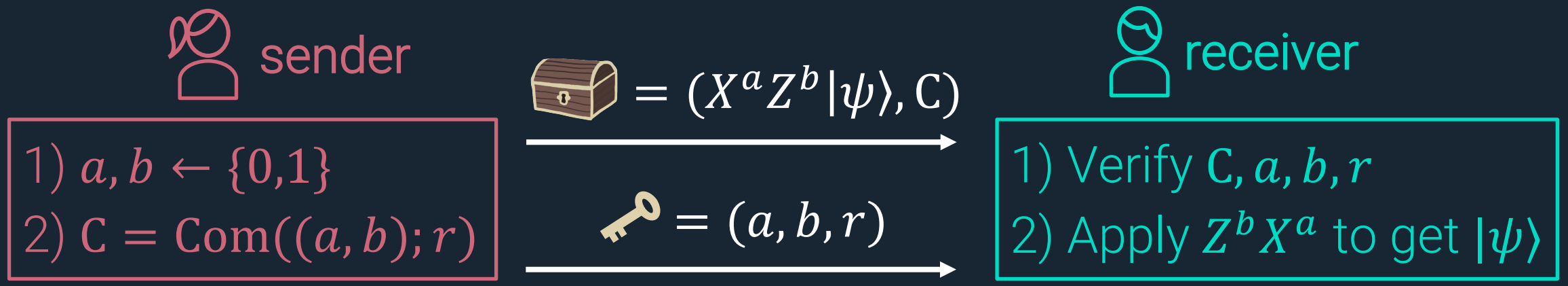

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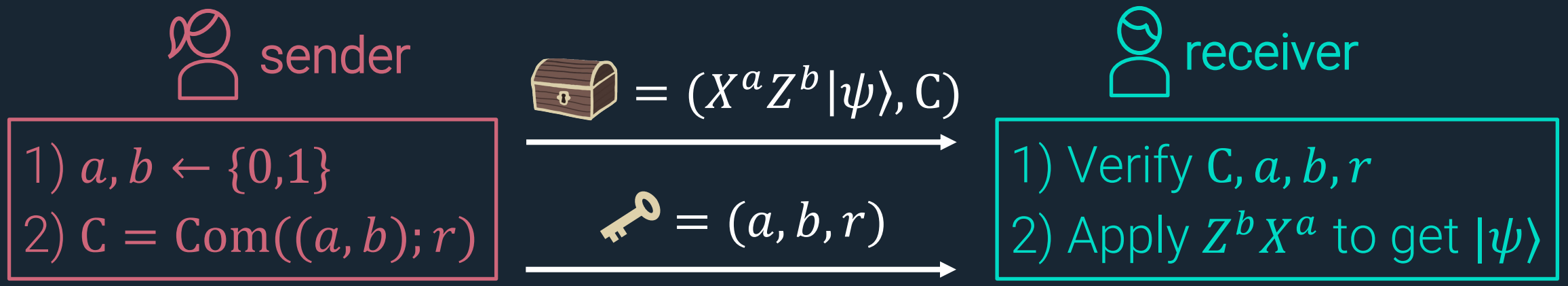

Big problem: totally unclear what this means!

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- …that’s it

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definitions

constructions

applications

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classical comparison: [Kilian92] is 4 messages + assumes CRHFs

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- new tools to rewind quantum protocols/extract quantum states

Plan for today

1) What does it mean to commit to a quantum state?

2) Can we ***succinctly*** commit to a quantum state?

3) Application: quantum succinct arguments

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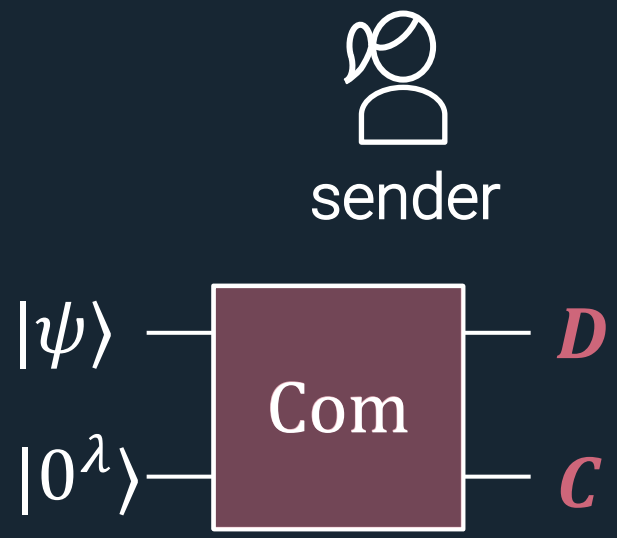

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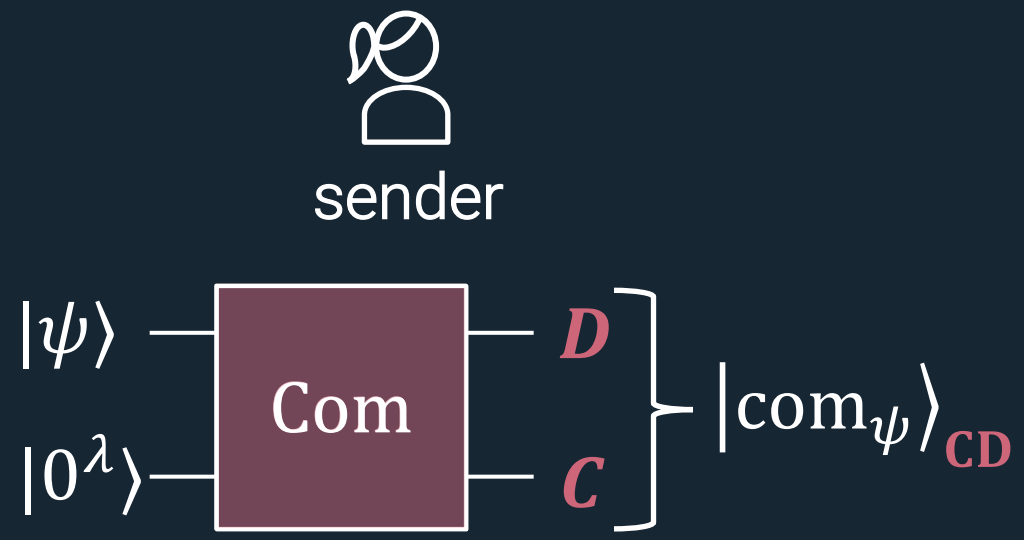

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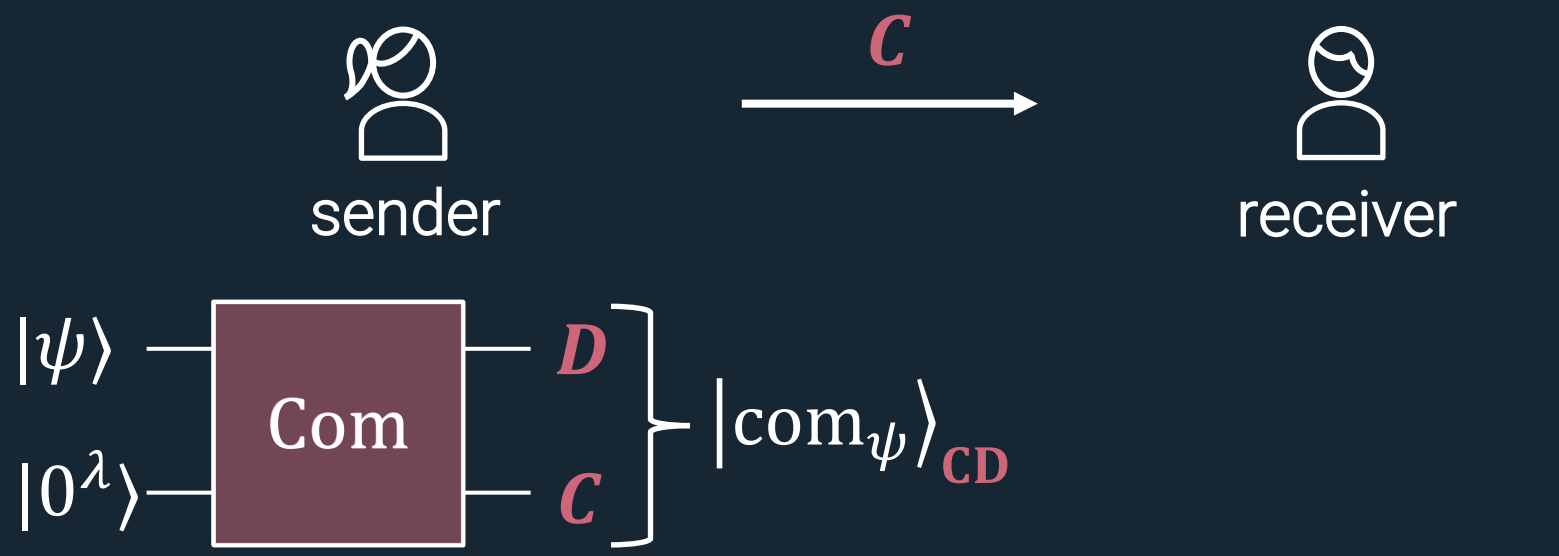

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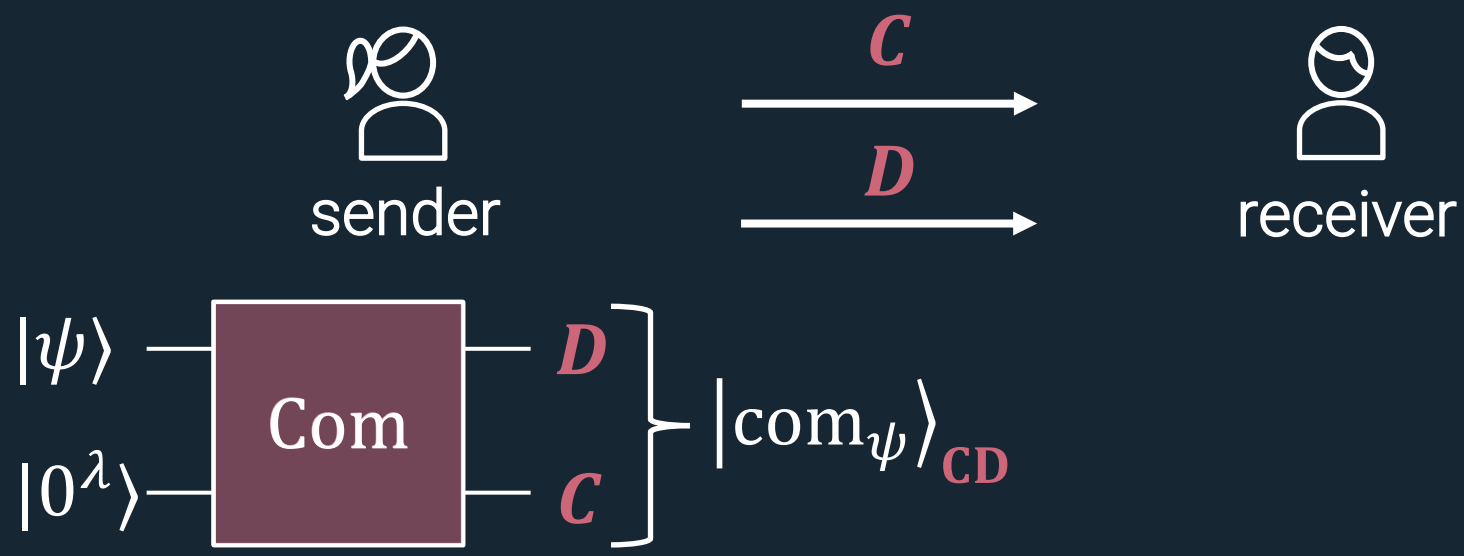

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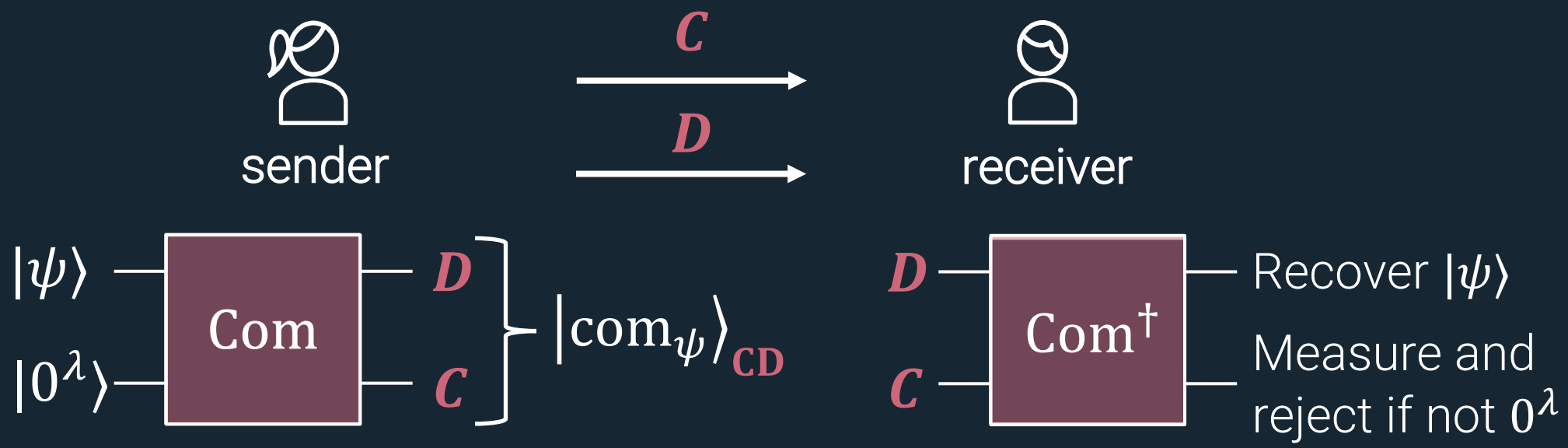

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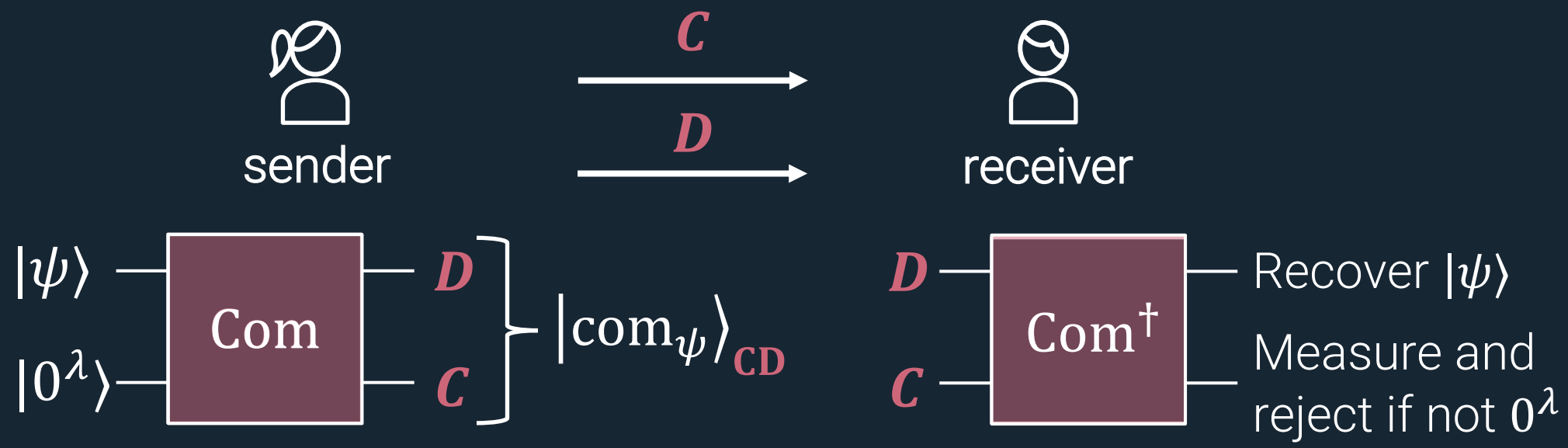

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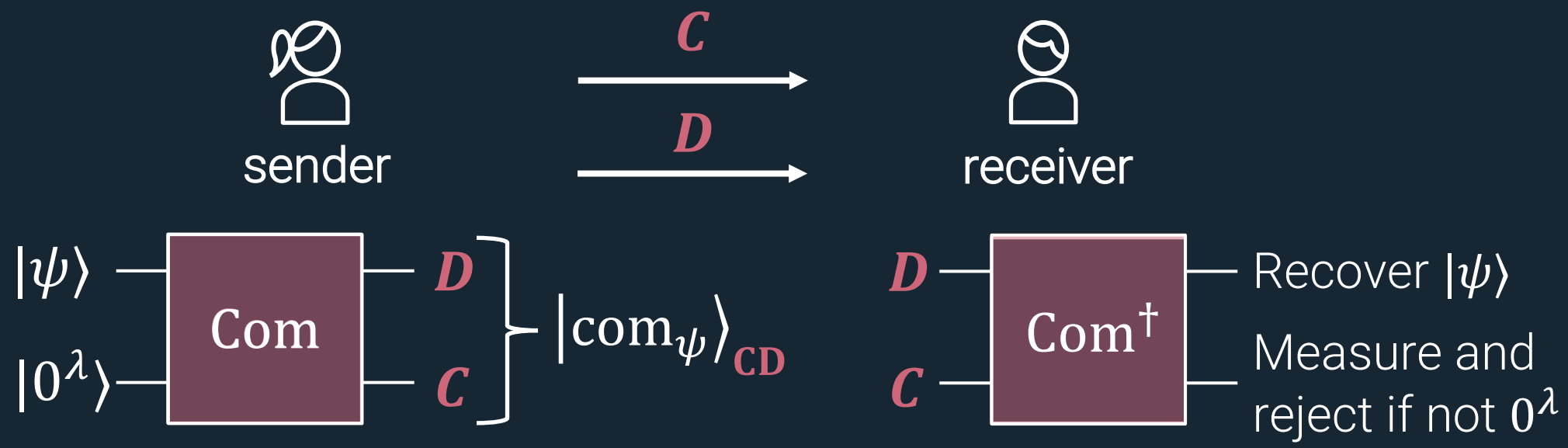

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(any QSC can be rewritten in this form)

What does it mean to commit to a quantum state?

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A priori, not obvious that a cryptographic definition is possible!

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As a result, our new definition looks quite different…

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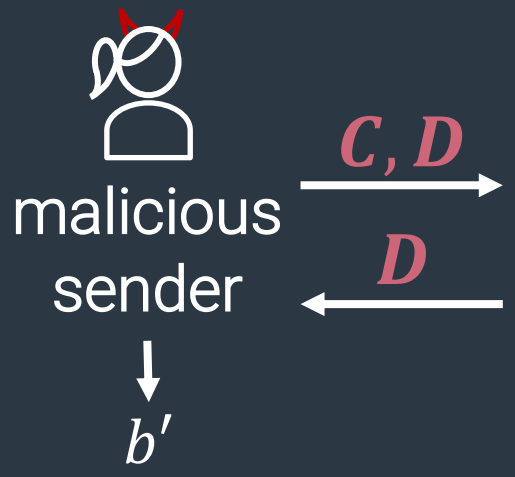

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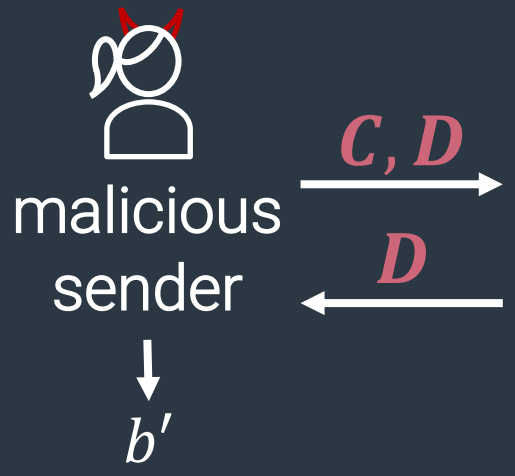

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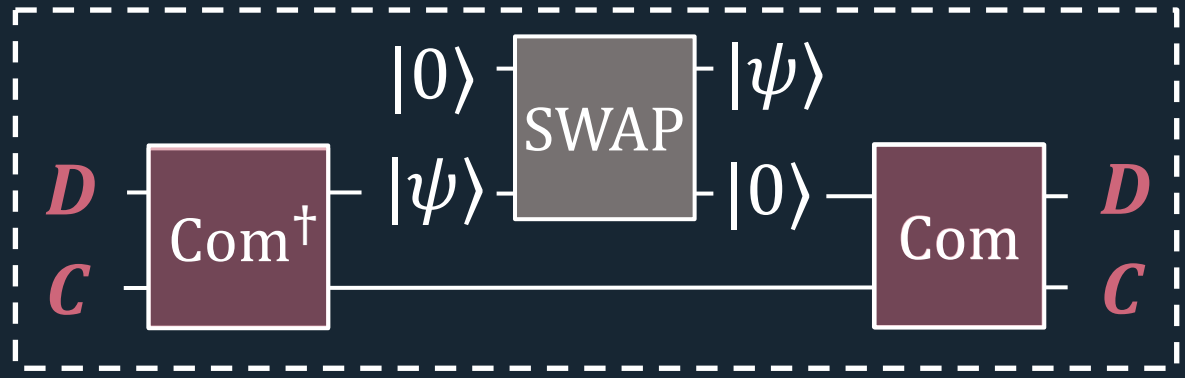
"swap $|\psi\rangle$ with junk" =



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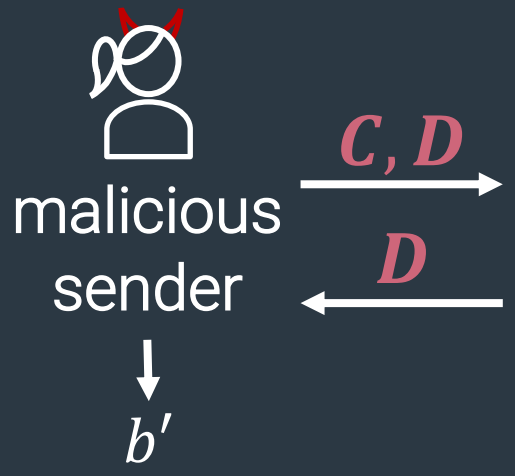

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Informally: $|\mathrm{com}_{\psi}\rangle_{\boldsymbol{CD}} \approx |\mathrm{com}_{0}\rangle_{\boldsymbol{CD}}$

(given only the $\boldsymbol{D}$ part)

Why this definition?

Swapping-Distinguishing Equivalence [AAS20]

These are **equivalent** for orthogonal $|x\rangle, |y\rangle$:

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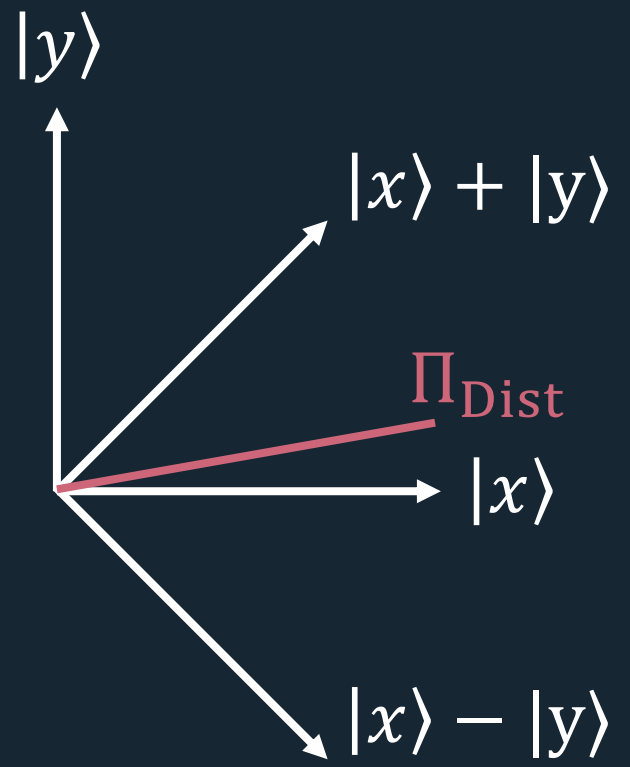

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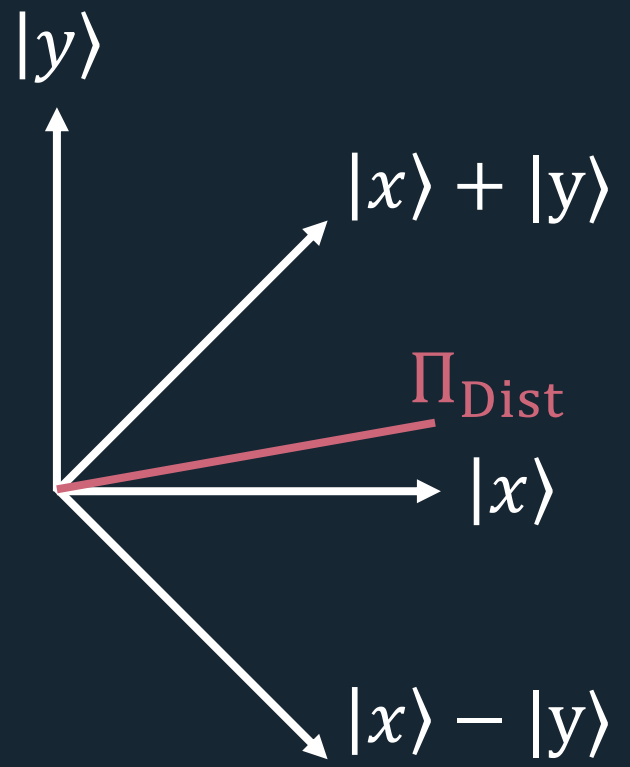

Proof Sketch:

Π_{Dist} distinguishes $|x\rangle + |y\rangle$ vs $|x\rangle - |y\rangle$ iff $2\Pi_{\mathrm{Dist}} - \mathrm{Id}$ (reflection about Π_{Dist}) swaps $|x\rangle \leftrightarrow |y\rangle$.

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This is (essentially) [Unruh16]'s collapse-binding definition, extended to quantum bit commitments.

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Swapping-distinguishing makes it possible to ***test*** if sender changes ψ.

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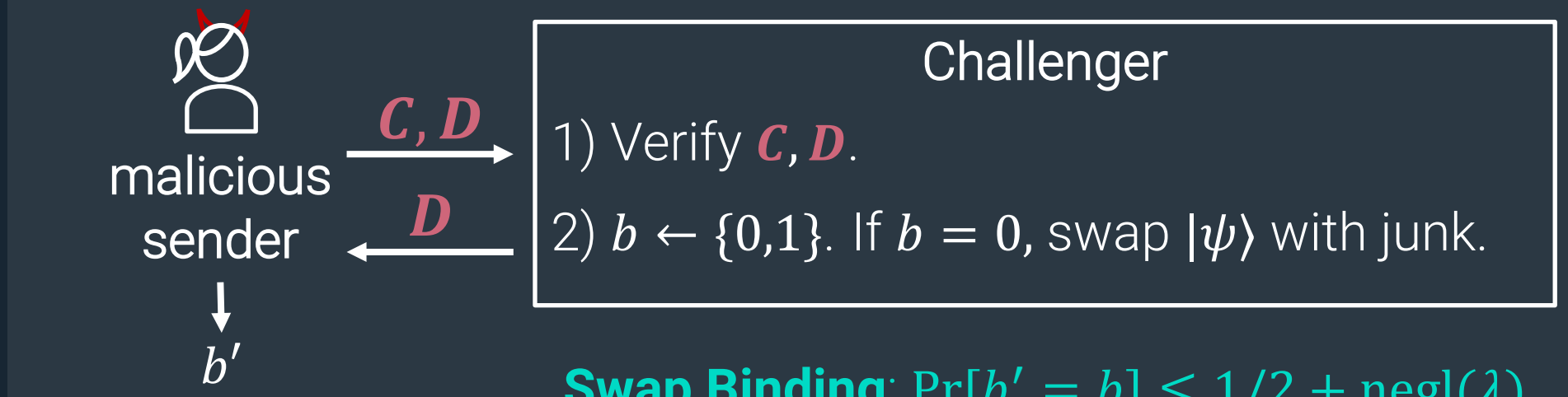
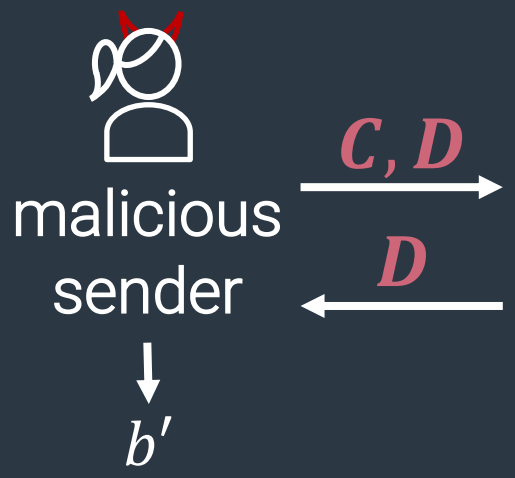
This paper: swap-binding captures security against *arbitrary malicious senders*.



Additional Properties



Challenger

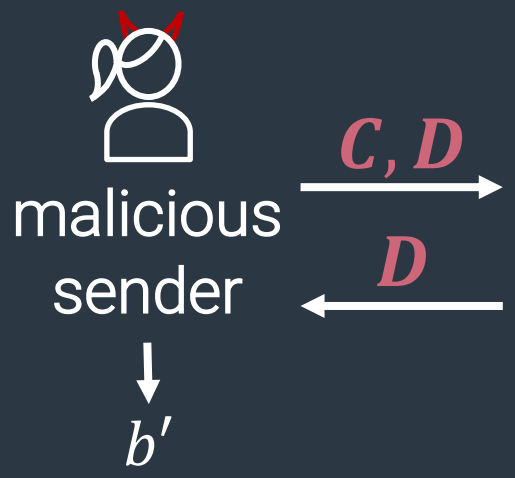
1) Verify C, D.

2) $b \leftarrow \{0,1\}$. If $b = 0$, swap $|\psi\rangle$ with junk.

Swap Binding: $\Pr[b' = b] \leq 1/2 + \mathrm{negl}(\lambda)$

Additional Properties

- handles entangled messages
- composable
- statistical or computational
- formalizes folklore (QSCs exist iff quantum bit commitments exist)



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Additional Properties

Bonus: swap-binding is dual to hiding!

- swap-binding means D hides ψ
- hiding means C hides ψ (same game but adversary gets C)

Hiding-Binding Duality

- Primal scheme: $\boldsymbol{C}$ is the commitment and $\boldsymbol{D}$ is the opening.
- Dual scheme: $\boldsymbol{D}$ is the commitment and $\boldsymbol{C}$ is the opening.

Hiding-Binding Duality

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Immediate consequence of hiding/binding definitions:

For $\mathrm{X} \in \{\mathbf{statistically}, \mathbf{computationally}\}$:

primal is X-binding $\Leftrightarrow$ dual is X-hiding

primal is X-hiding $\Leftrightarrow$ dual is X-binding

Plan for today

1) What does it mean to commit to a quantum state?

2) Can we ***succinctly*** commit to a quantum state?

3) Application: quantum succinct arguments

Succinct commitment to an n-qubit state

Succinct QSC:

- |commitment| < n qubits
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$$\sum_k |k\rangle_C \otimes X^{\mathrm{G}_0(k)} Z^{\mathrm{G}_1(k)} |\psi\rangle_D$$

Succinct QSC: $\boldsymbol{C}$ (n/2 qubits) $\boldsymbol{D}$ (n qubits)

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Note: classical succinct commitments from PRGs is *not* known!

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In fact, one-way functions might not be necessary! In the paper:

- This kind of encryption can be built from any pseudorandom unitary and (by [K21]) is potentially *weaker* than OWFs.

Useful property: succinct QSCs compose easily.

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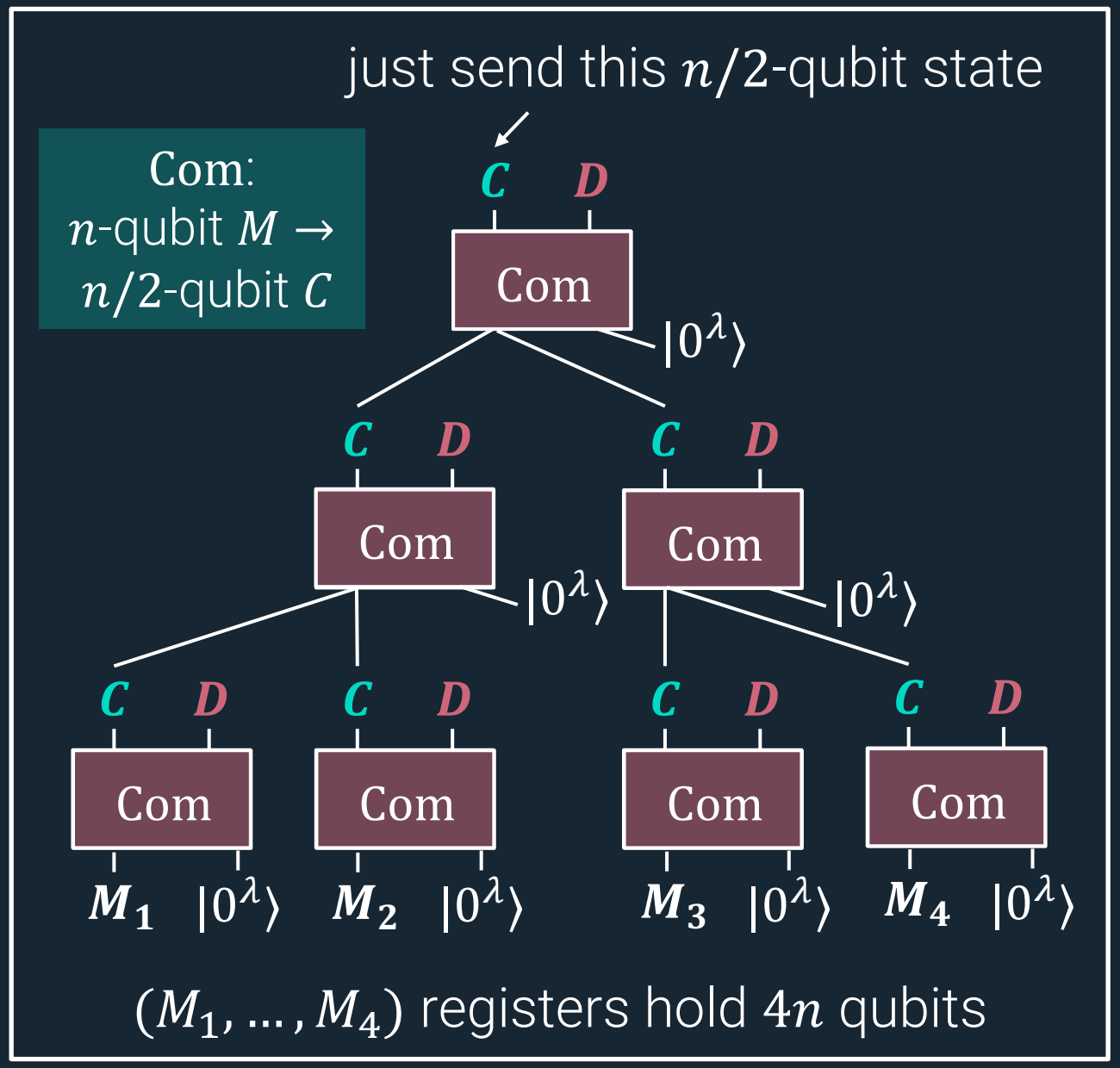
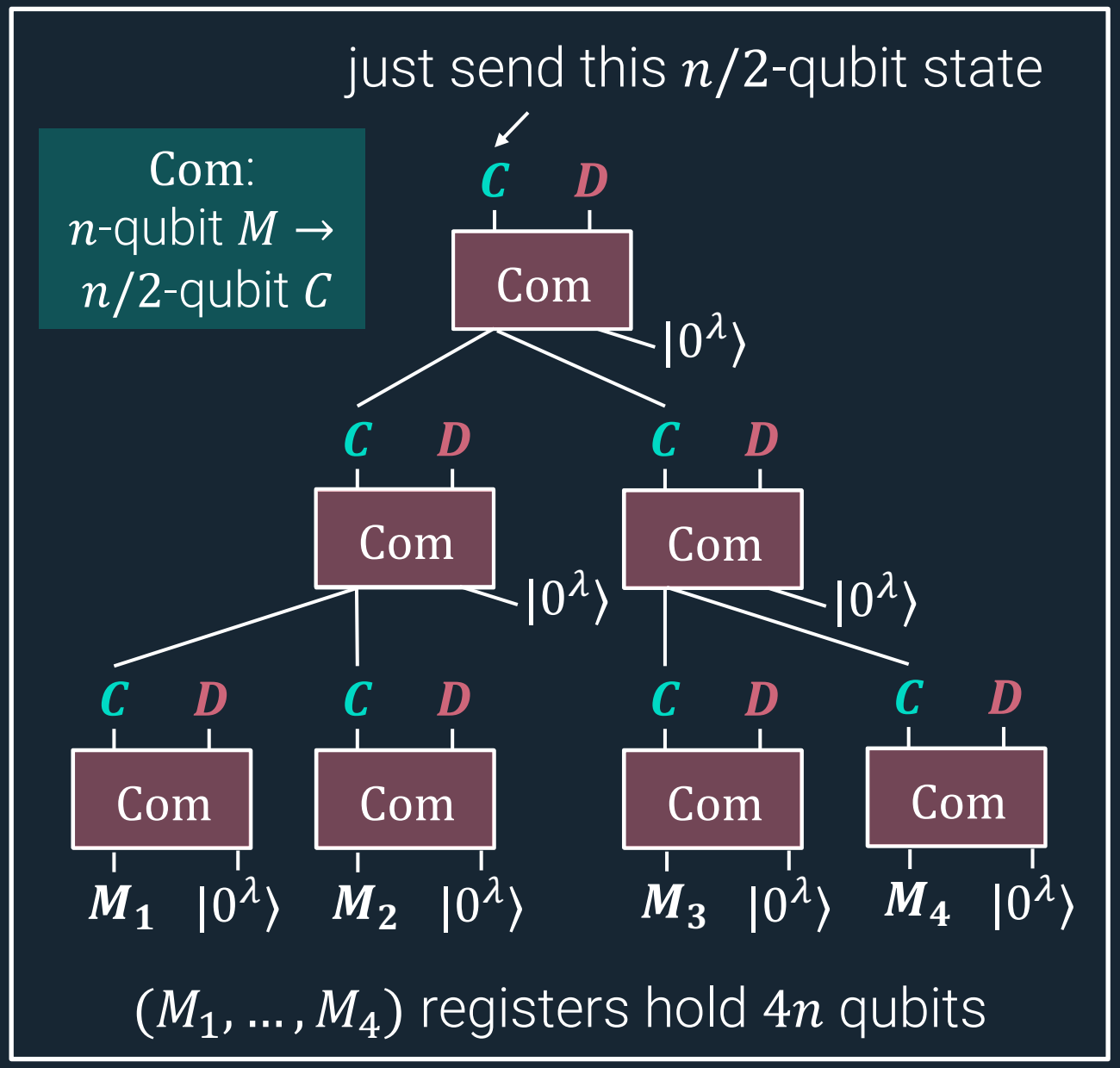
- Domain extension: 1-qubit compression → any compression (see paper)
- Merkle tree: succinct commitments with local openings (we'll see this next)

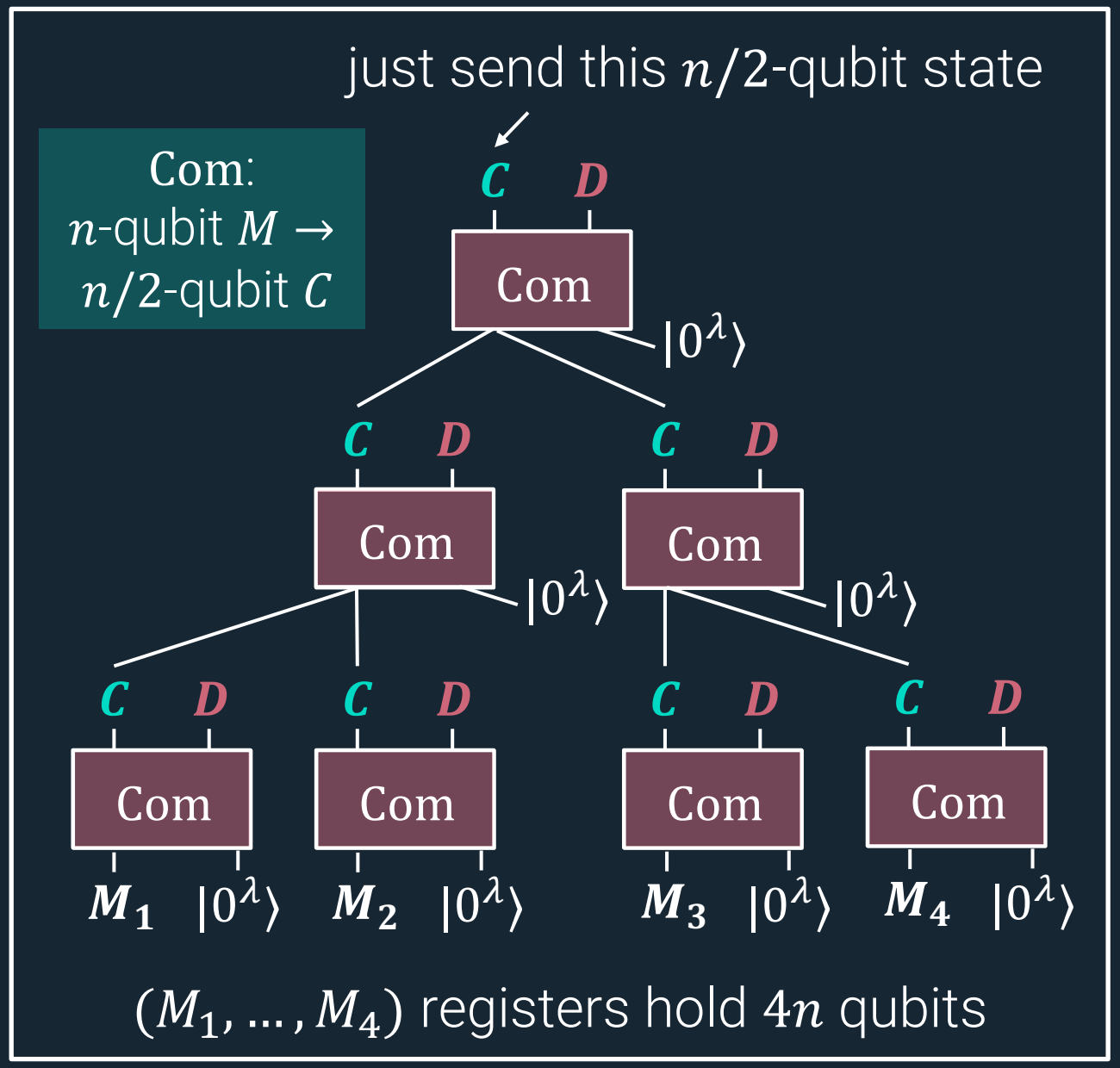


Merkle tree commitments



Merkle tree commitments

- since swap binding composes, this is swap-binding on every local opening.



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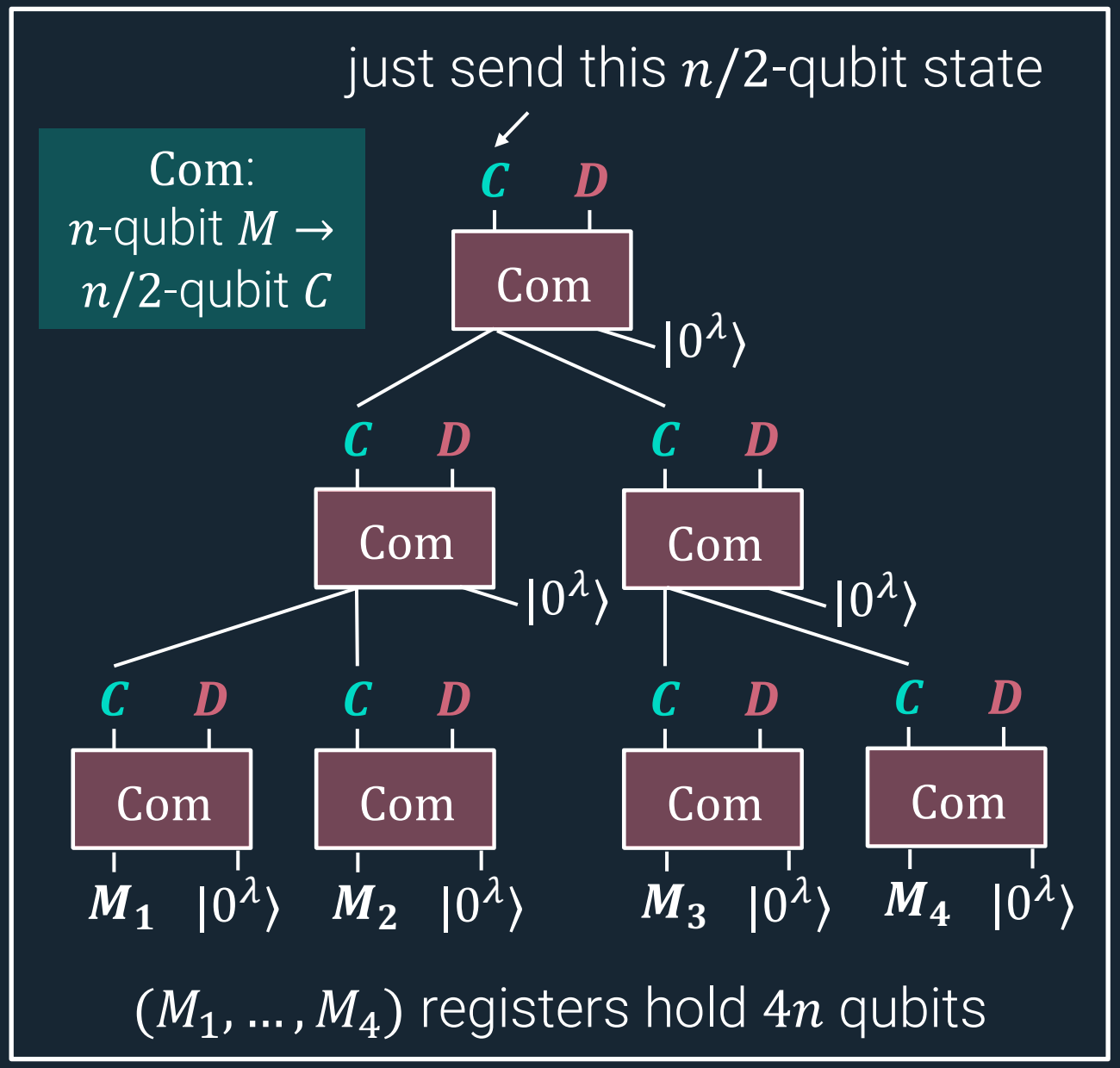

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Note: this is similar to a heuristic proposal of [Chen-Movassagh22]

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Classically, tree commitments are **the key cryptographic component** of succinct arguments for NP [Kilian92].

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Succinct Arguments [Kilian92,Micali94]

Claim: $x \in$ 3SAT

- "Succinct" = very short communication + verification
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Kilian's protocol: tree commit to a locally checkable proof π (aka PCP).

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quantum tree commitments + quantum PCP → quantum succinct arguments

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quantum PCP means quantum proof $|\pi\rangle$ that can be verified by checking a few qubits.

(includes classical PCPs!)

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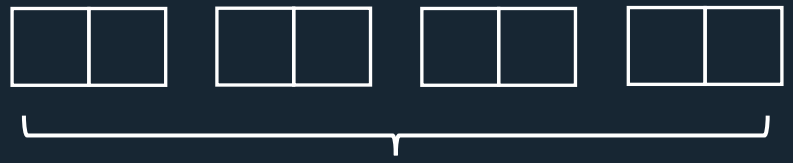

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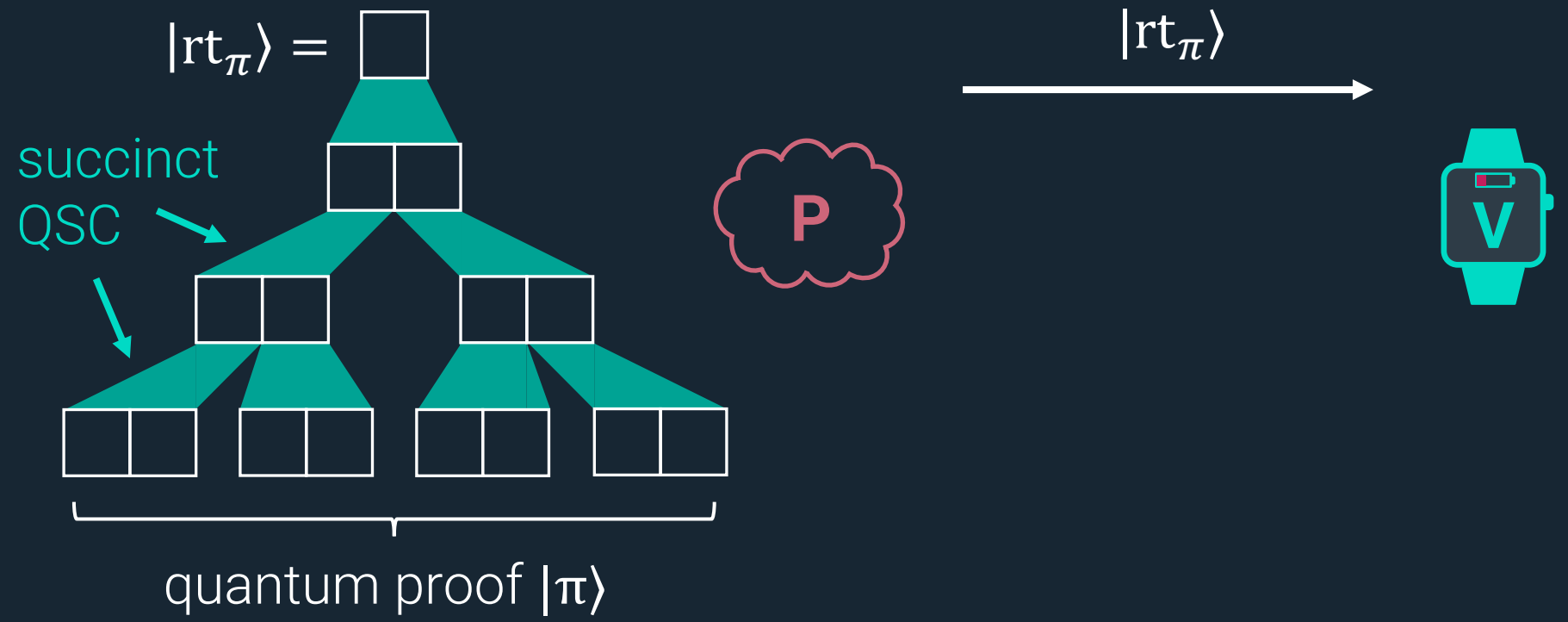

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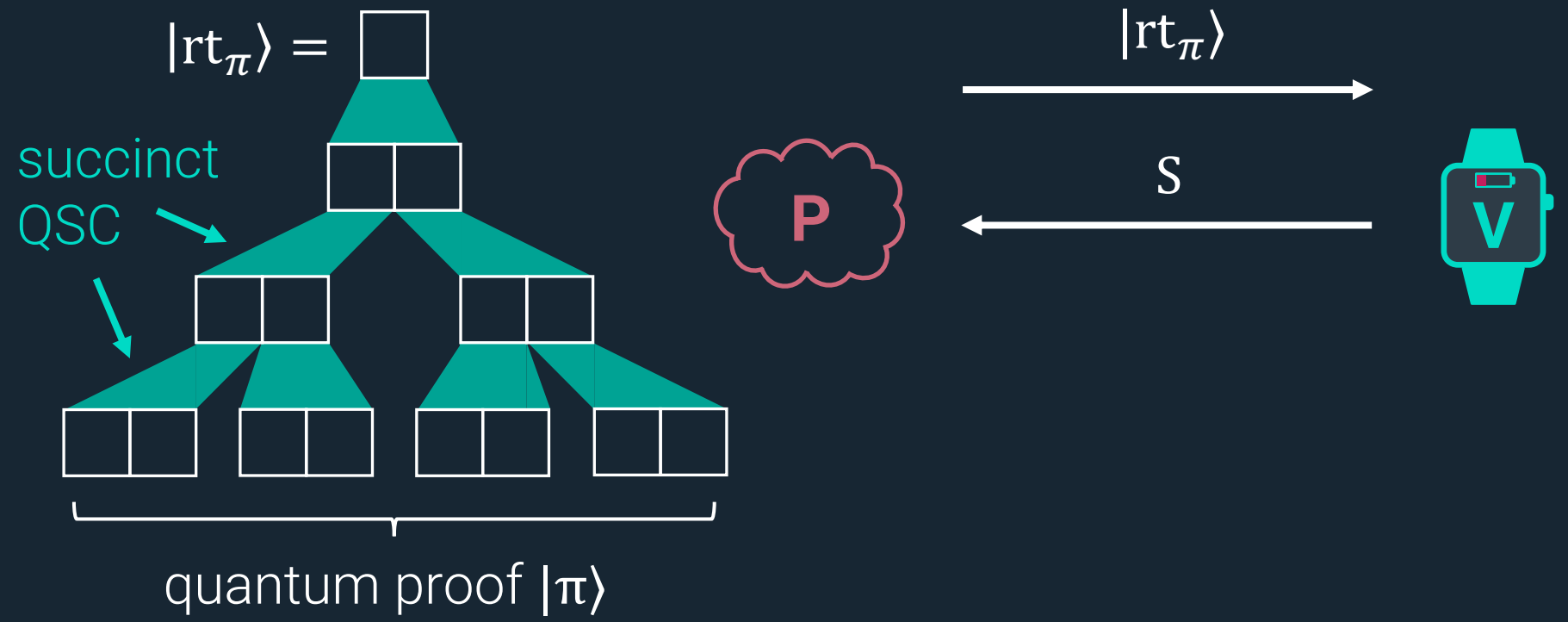

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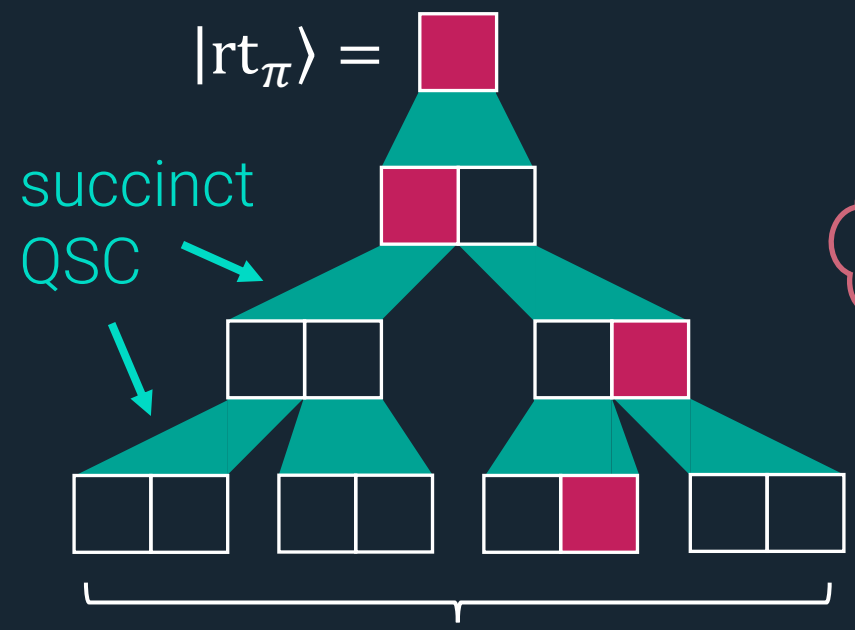

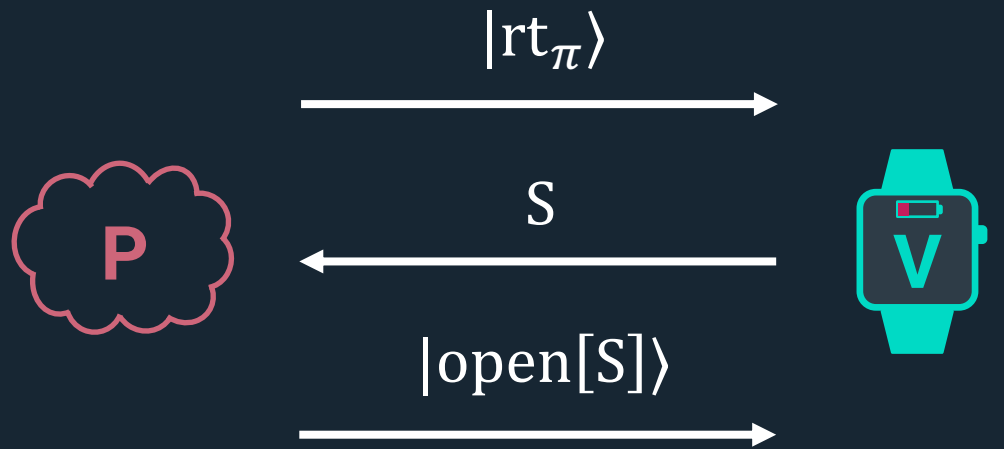

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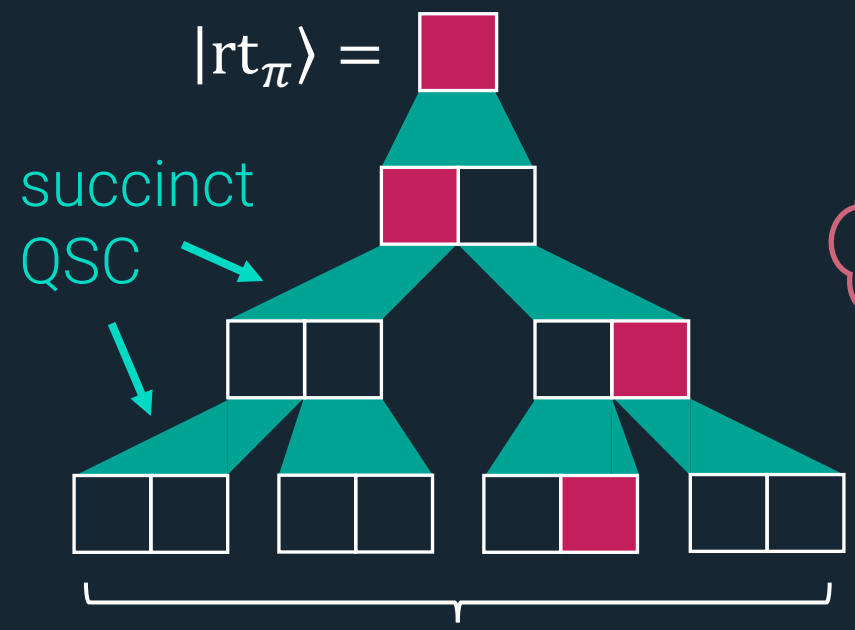

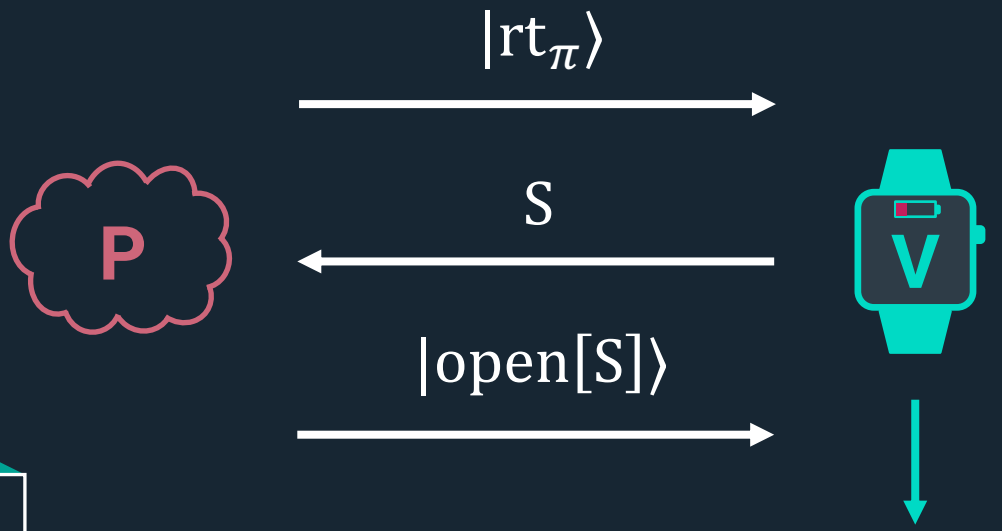

1) Check if $|\mathbf{open}[S]\rangle$ is valid.

2) Apply $\mathbf{Com}^\dagger$ to reveal $|\pi[S]\rangle$ and run PCP verifier.

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Corollary: 3-msg succinct arguments for QMA from less-than-OWF assumptions + quantum PCP conjecture.

We'll prove soundness by showing how to extract a *quantum PCP* from any (successful) malicious prover.

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First: recall how this works for Kilian's *classical* protocol.

Soundness of Kilian's *classical* protocol

Classical adversary: record $\widetilde{\mathbf{P}}$'s answers to many random $\mathbf{s}$.

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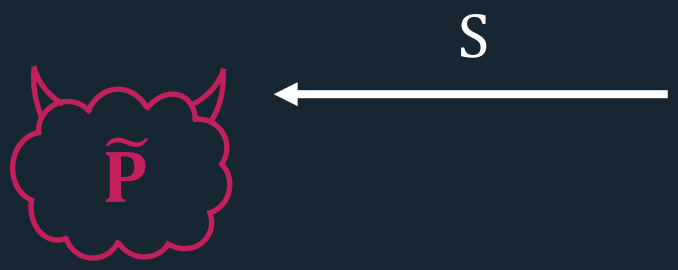

Classical reduction

1) query $\widetilde{\mathbf{P}}$ on many random $\mathbf{S}$, record $\pi[\mathbf{S}]$.

extracted proof

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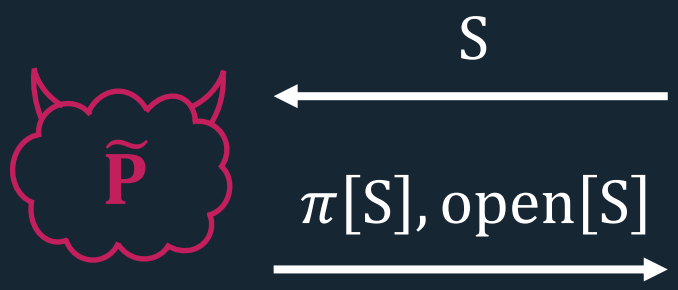

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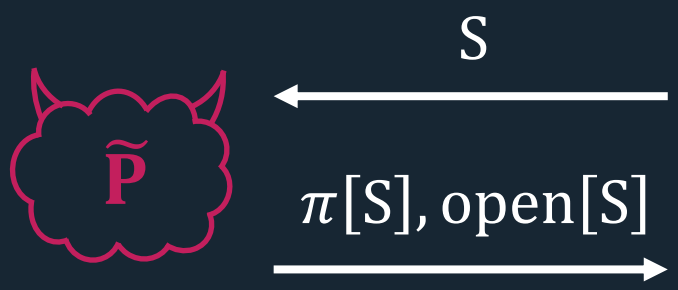

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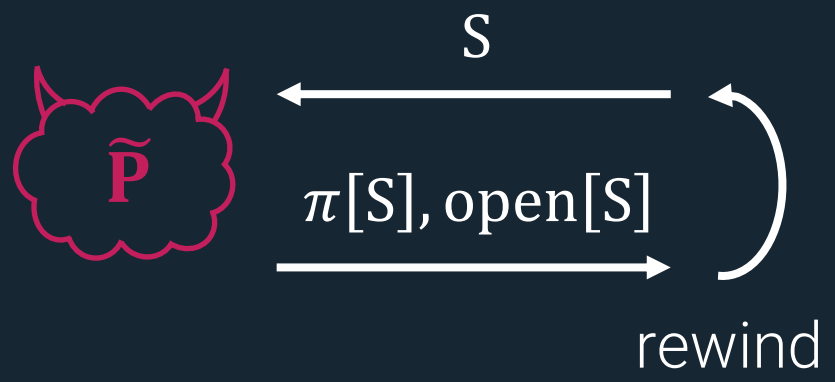

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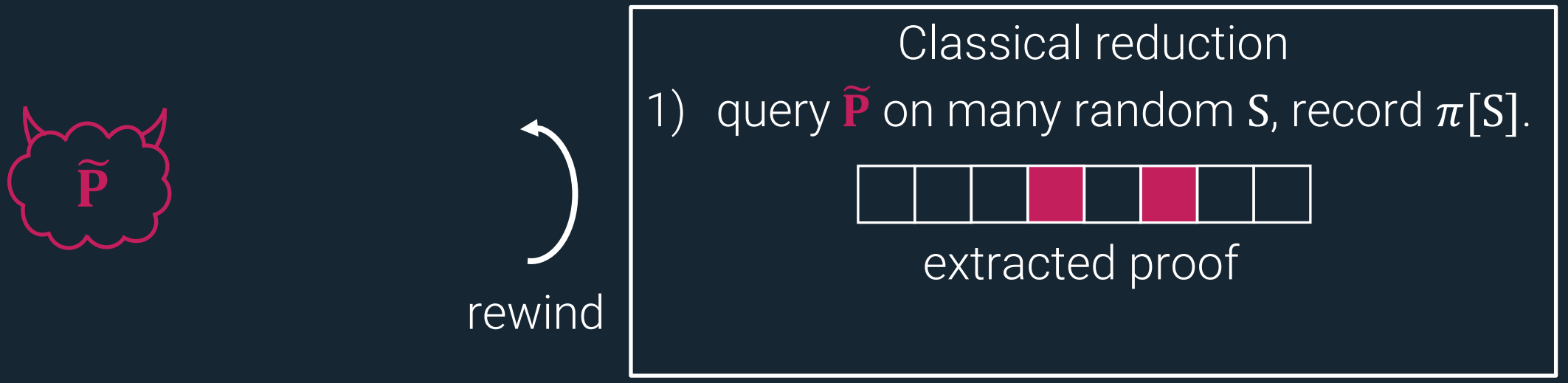

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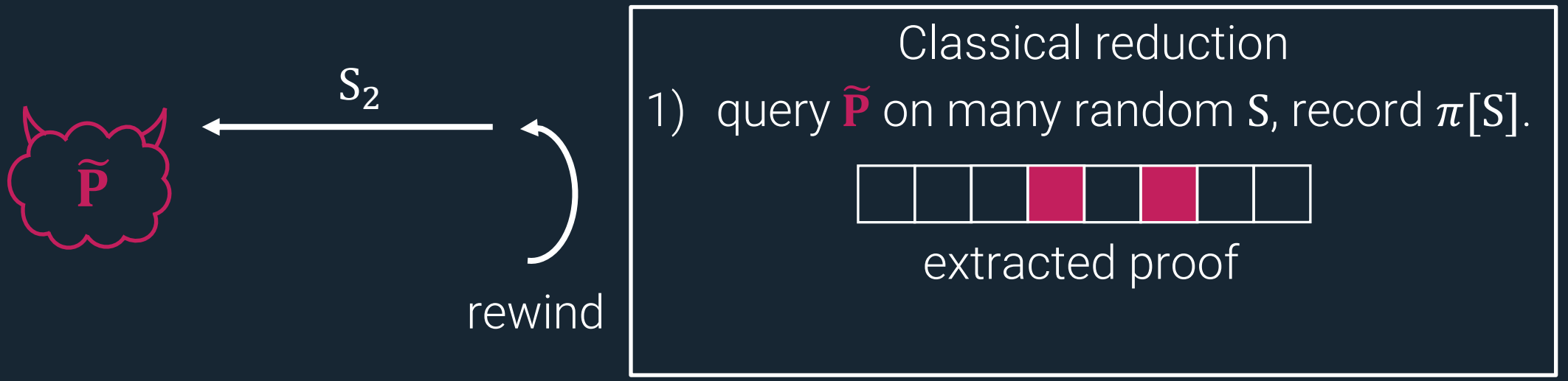

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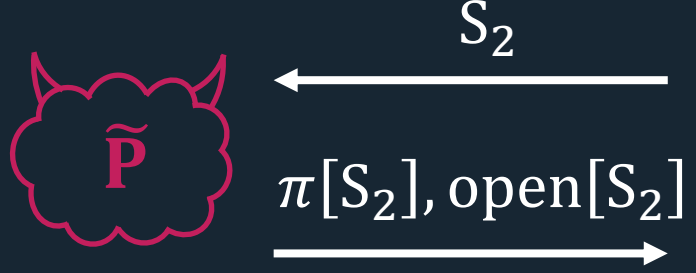

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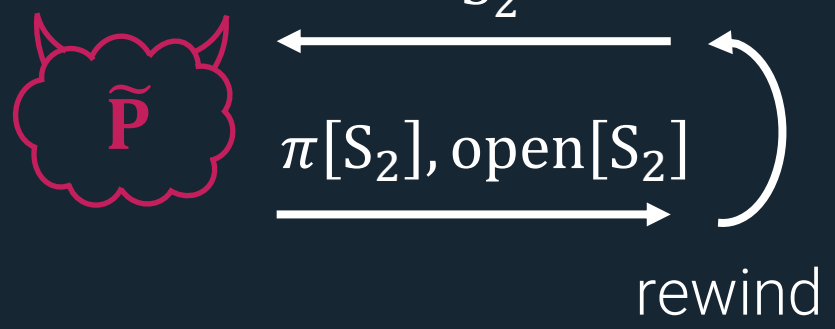

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rewind

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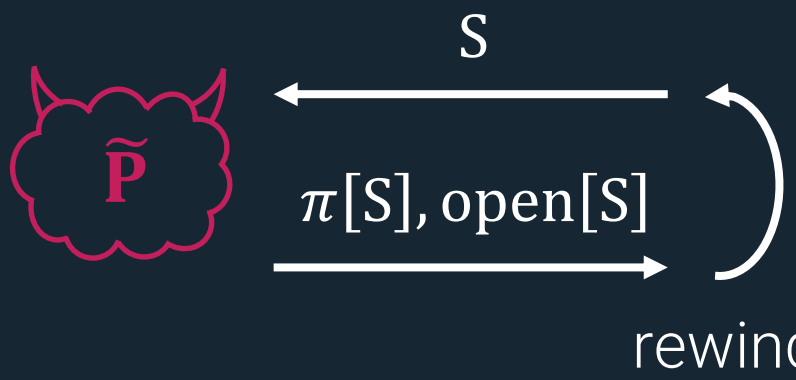

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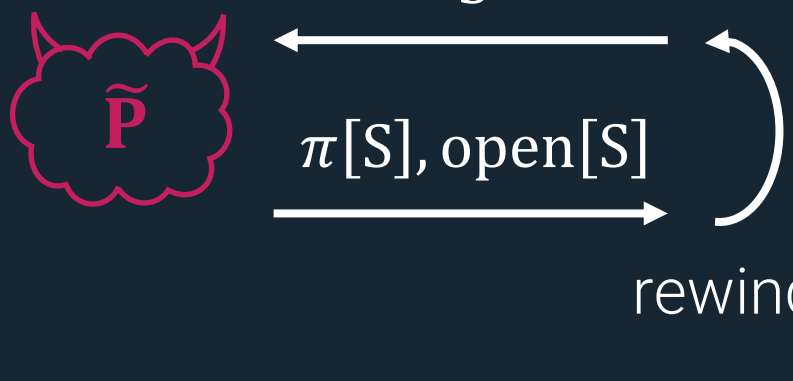

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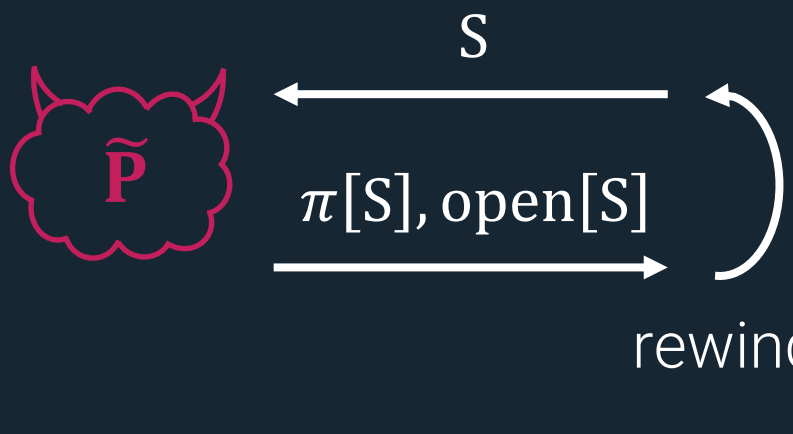
Quantum adversary: recording $\widetilde{\mathbf{P}}$'s answers is not straightforward because measurement can disturb $\widetilde{\mathbf{P}}$'s state (rendering it useless for future queries).

Soundness of Kilian's *classical* protocol

Classical adversary: record $\widetilde{\mathbf{P}}$'s answers to many random $\mathbf{S}$.



Classical reduction

1) query $\widetilde{\mathbf{P}}$ on many random $\mathbf{S}$, record $\pi[\mathbf{S}]$.

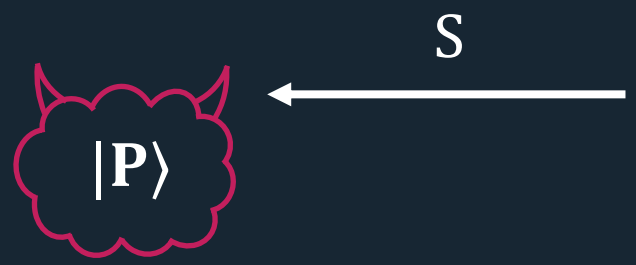
extracted proof

2) get convincing PCP *or* CRHF collision.

Quantum adversary: recording $\widetilde{\mathbf{P}}$'s answers is not straightforward because measurement can disturb $\widetilde{\mathbf{P}}$'s state (rendering it useless for future queries).

Recent work [CMSZ22]: there is a way for "disturbed" $\widetilde{\mathbf{P}}$ to answer later queries

Quantum soundness of Kilian's *classical* protocol [CMSZ22]



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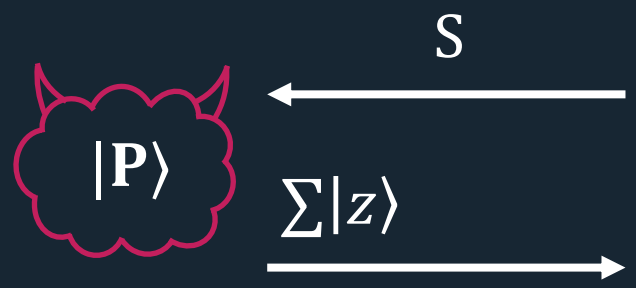

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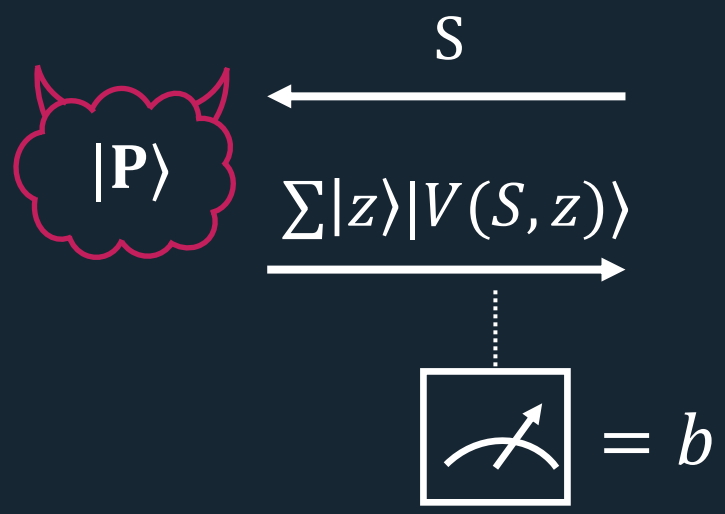

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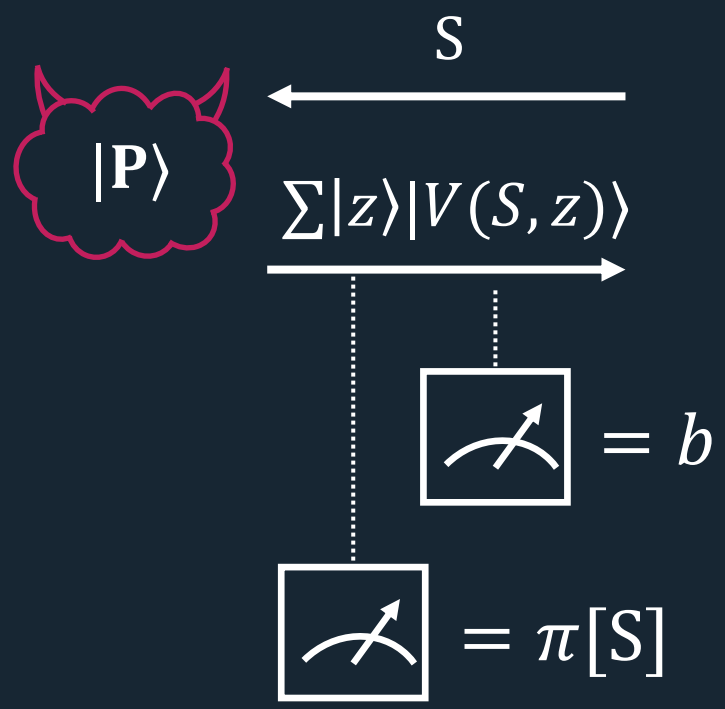

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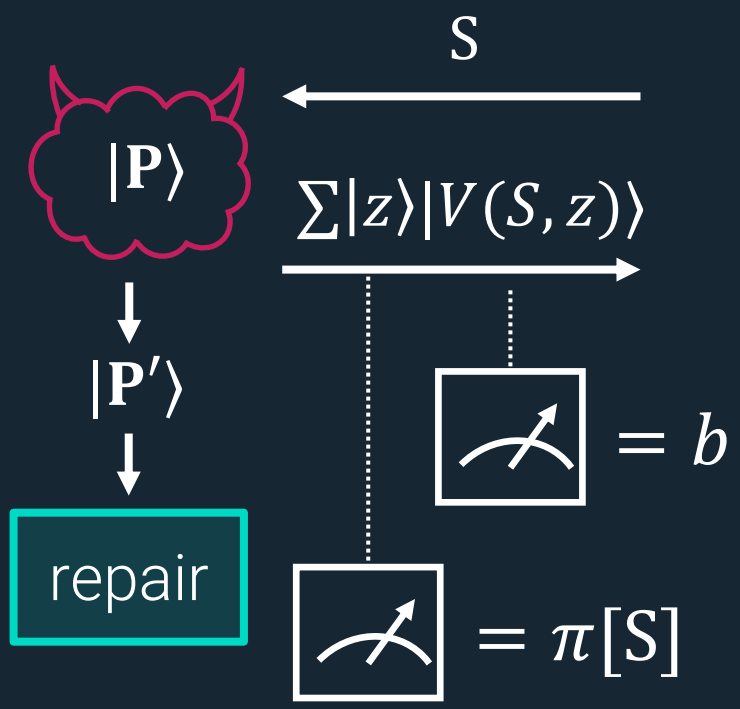

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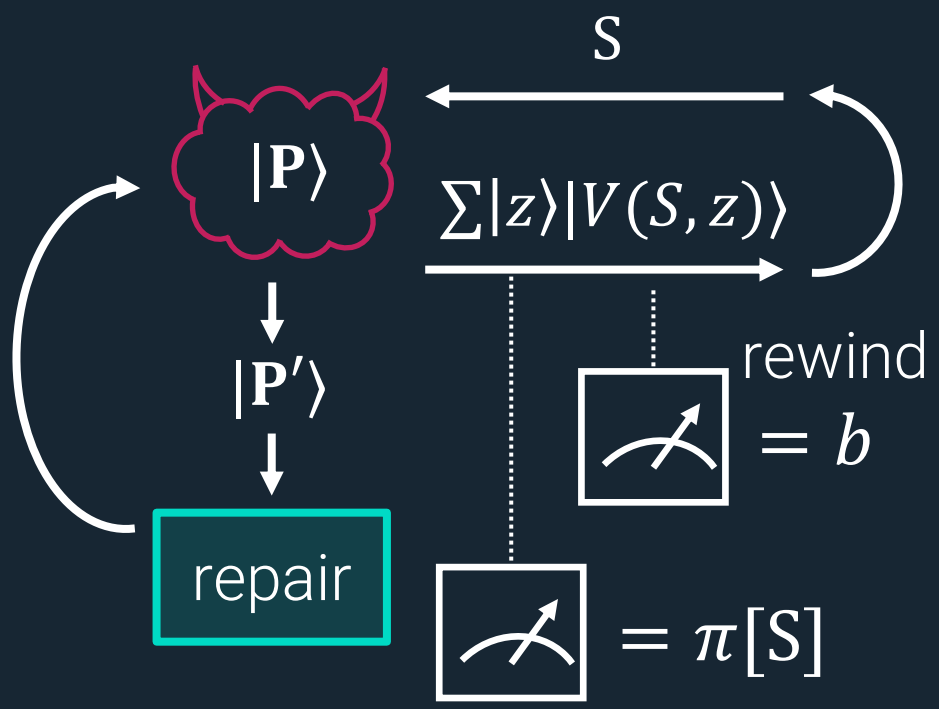

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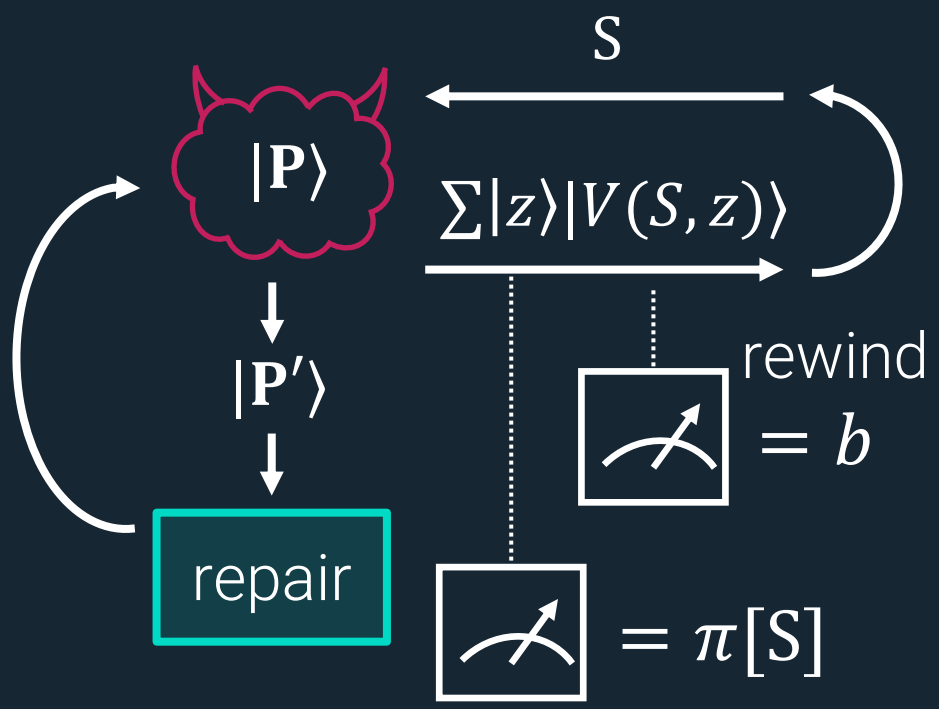

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Main idea [Unruh16, CMSZ22]

- Collapse-binding → step 2 causes no detectable disturbance
- If step 2 doesn't disturb state, "repair" will restore prover's success prob.

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But our definition suggests what to do instead!

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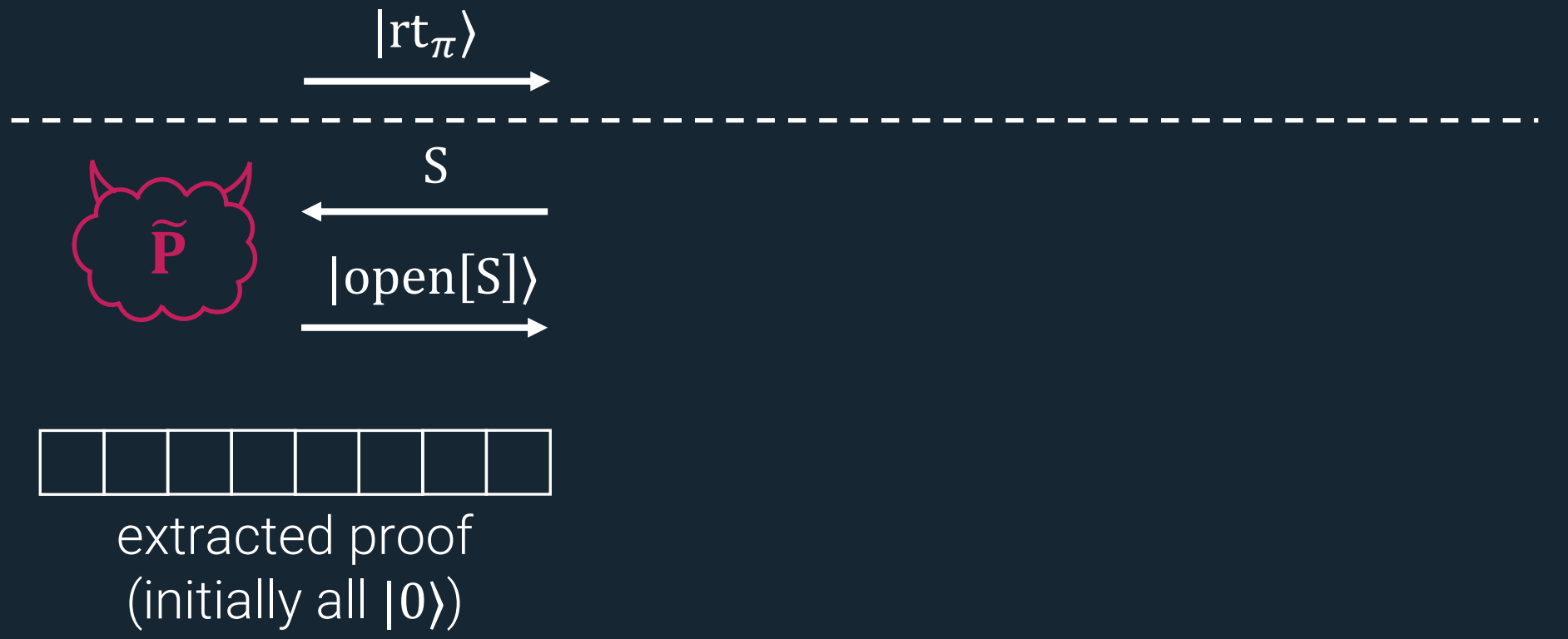

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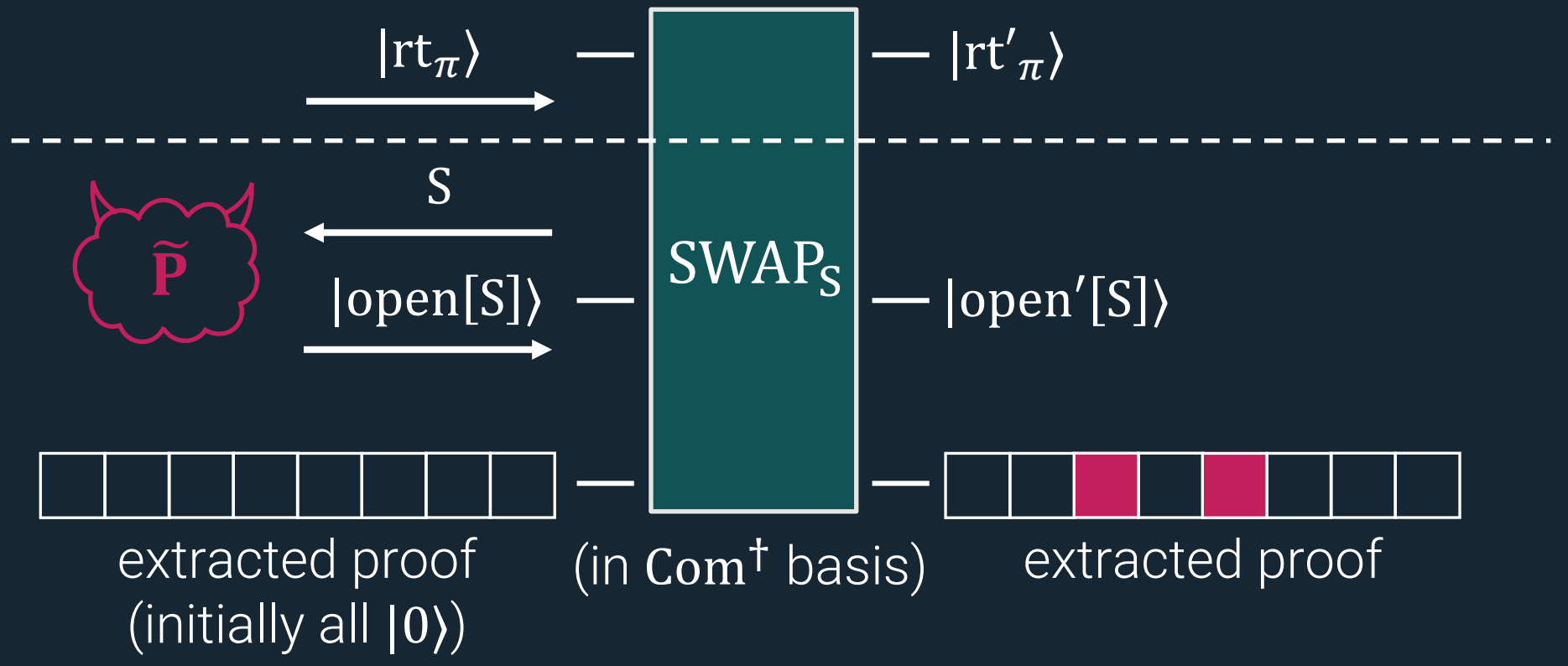

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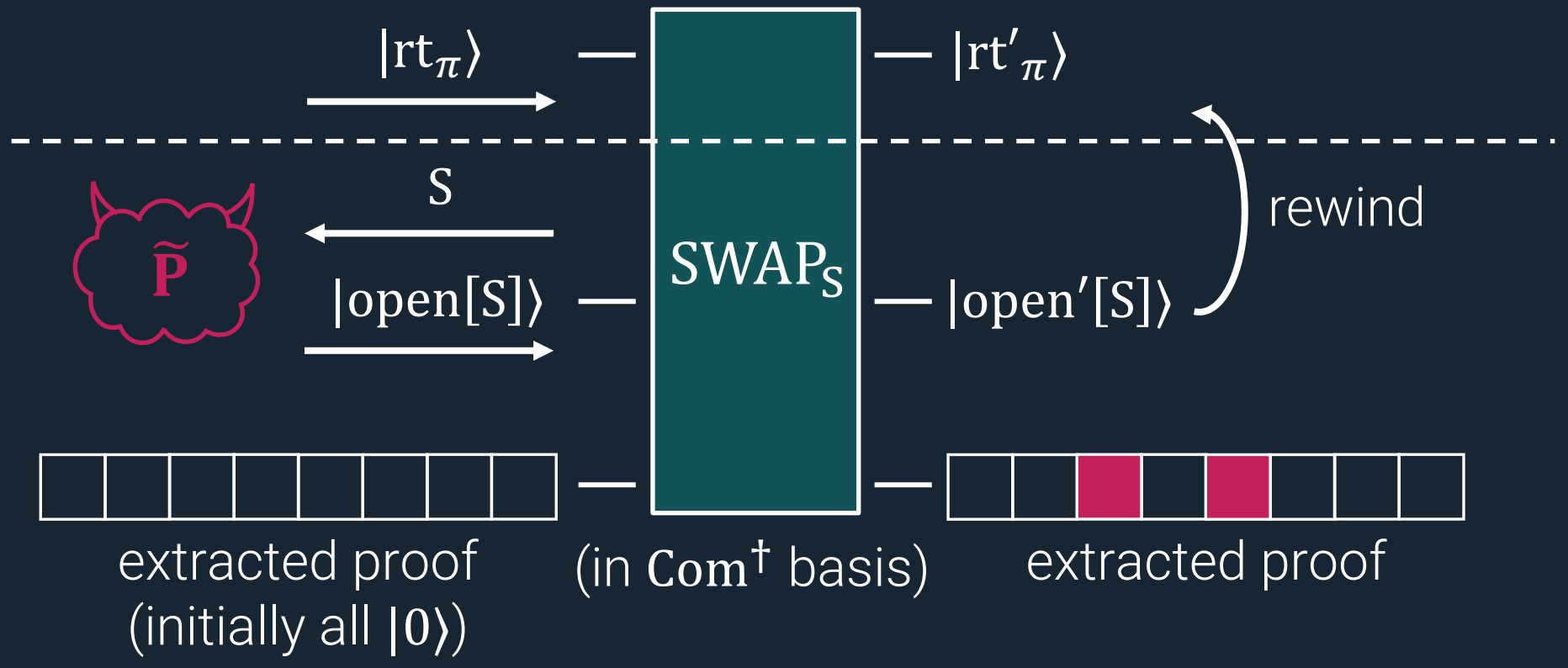

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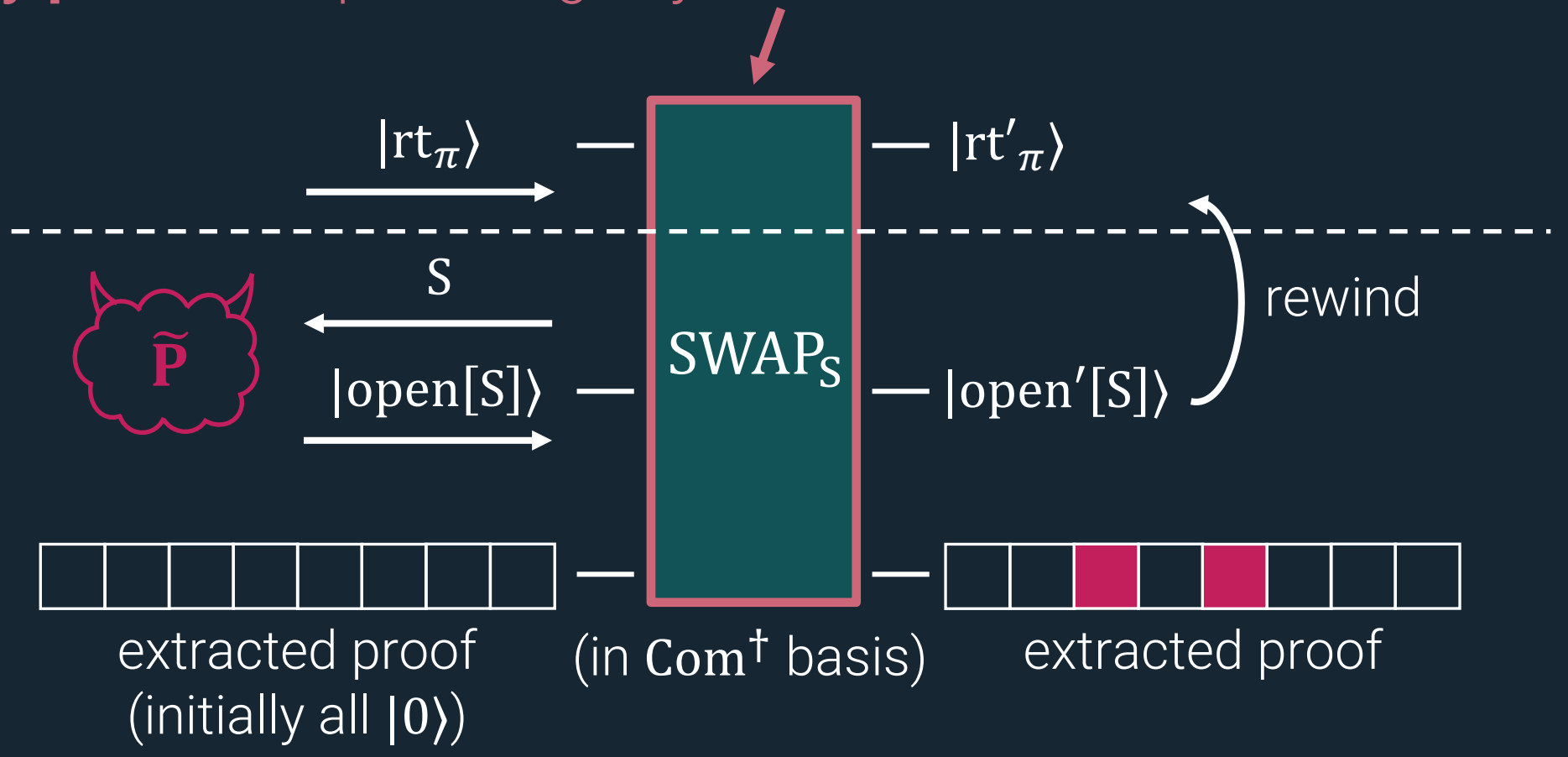
Key point: swap-binding says this is undetectable!



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Note: This is an issue even if we're only extracting classical information from quantum commitments.

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Our solution (see paper)

- Main technical step: Prove that swap-binding remains hard even if the adversary is given an oracle to verify commitments + act on the message
- Show that this oracle suffices to implement CMSZ-style rewinding.

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- do pseudorandom states imply succinct QSCs?
- classical communication QSCs?
- simpler techniques for rewinding quantum protocols?

Thank You!

Questions?