Quantum Security and Fiat-Shamir for Cryptographic Protocols

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Cryptographic Protocols

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Ex: verify computation, compute on private inputs

- 2) Security against adversarial behavior
- Ex: can't fool verifier, can't learn other party's input





Ex: invert one-way function, factoring, discrete log, lattice problems, etc.



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Any *efficient* attack on the protocol → Break underlying hardness assumption



Many amazing results based on this formula:

- Zero-knowledge proofs [GMR84]
- Secure multi-party computation [Yao86, GMW86]
- Succinct arguments [Kilian92]

But even in settings where secure protocols are known, key challenges remain.

Challenge 1: Quantum Computers Most security proofs consider classical attackers. Does security still hold if a quantum computer is built?



• Use quantum physics to perform computation



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- Likely more powerful than classical computers (e.g., Shor's algorithm enables factoring)



- Use quantum physics to perform computation
- Likely more powerful than classical computers (e.g., Shor's algorithm enables factoring)
- Not far away?

NEWS · 23 OCTOBER 2019

Hello quantum world! Google publishes landmark quantum supremacy claim

[Nature 2019]



[WSJ 2021]

Goal 1:

Understand what happens to crypto if/when a quantum computer is built.

Challenge 2: Removing Interaction

Protocols typically use *interaction*, but interaction is unwieldy in practice.

Through heuristics such as Fiat-Shamir, some protocols can be made *non-interactive*.

Is this secure?

Magical compiler that removes interaction from public-coin interactive protocols.

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Public-Coin Interactive Protocol

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Public-Coin Interactive Protocol

Fruitful approach:

Construct *interactive* protocol for some functionality (e.g., identification, verifiable computation), apply Fiat-Shamir.



Public-Coin Interactive Protocol

Important Caveat:

Not immediately clear if soundness is preserved!



Public-Coin Interactive Protocol

Goal 2:

Understand when the Fiat-Shamir transform preserves soundness.

In sum, this thesis addresses two challenges:

- 1) Our understanding of what constitutes an **efficient algorithm** has changed.
- 2) Our demands for how protocols **will be used** have changed.

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This talk

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Up next:

[CMSZ21]: Post-quantum succinct arguments via rewinding.

How Will Quantum Computers Impact Crypto?

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Immediate consequence: Shor's algorithm breaks commonly used assumptions




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Post-Quantum Cryptography



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Minimum requirement for post-quantum crypto: hard problem should resist quantum attacks

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Post-Quantum Cryptography



Important point:

the *security reduction* must be *quantum-compatible*!

Post-Quantum Cryptography



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the *security reduction* must be *quantum-compatible*!

Reduction must imply:

Any *quantum* attack on the protocol \rightarrow *quantum* attack on the assumption









Reduction 1) Record (a, r, z).













Reduction

break hard problem Problem: unclear how to rewind a quantum adversary since running the adversary may disturb its state!

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Important application where known techniques fail:

Succinct Arguments





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<u>Argument</u> = complete + computationally sound

- Complete: if $(x, w) \in R$, \square accepts.
- Sound: if $x \notin L(R)$, malicious poly-time $\overset{\frown}{\bigcup}$ can't fool $\overset{\frown}{\Box}$



[Kilian92]: Succinct arguments for NP exist assuming collisionresistant hash functions (CRHFs).



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Many applications: Succinct non-interactive arguments (SNARGs) [Micali94], Universal arguments [BG01], non-black-box zero knowledge [Barak01], ... Kilian's security proof fundamentally relies on rewinding, and does not extend to quantum attackers.

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[Kilian92]: succinct arguments from CRHFs



post-quantum succinct arguments from *postquantum* CRHFs

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Kilian's protocol is post-quantum secure when instantiated with a post-quantum hash function*.

*collapsing hash function [U16]

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Extract **unbounded** number of accepting transcripts from quantum adversary.

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Technique

Extract **unbounded** number of accepting transcripts from quantum adversary.

Prior work [U12,U16]: extract constant number of transcripts.

Kilian's protocol





 $\boldsymbol{\chi}$

Encode w as PCP







sends short commitment to PCP π .

$com = Merkle_h(\pi) \quad x, w$ CRHF h com r r $PCP \pi$ Kilian's protocol <math display="block">r





 $\{P\}$ sends $\pi[Q_r]$ + short opening proofs

 Q_r = indices PCP verifier checks on random coins r





Proving Security, Classically





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Assume \bigcup fools \Box into accepting on $x \notin L$.
Proving Security, Classically



Reduction's goal: record *many* accepting transcripts (r_i, z_i)

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Reduction's goal: record *many* accepting transcripts (r_i, z_i) Eventually finds impossible π OR collision. Pr[PCP verifier accepts π] > PCP soundness error

Proving Security, Classically



$$\begin{array}{c} r \\ \hline s \\ \hline z \\ \hline \end{array} \begin{array}{c} r \\ \hline z \\ \hline \end{array} \end{array} \begin{array}{c} p \coloneqq Pr \\ r \leftarrow R, \\ z \leftarrow s \\ \hline s \\ (r) \end{array} \left[\begin{array}{c} \hline s \\ \hline s \\ \hline s \\ \hline s \\ (r,z) \end{array} \right]$$

Goal: given $\{x_i\}$ with large enough success probability p, obtain k accepting transcripts (r_i, z_i) .

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Goal: given $\{x_i\}$ with large enough success probability p, obtain k accepting transcripts (r_i, z_i) .

If S is classical, perform independent trials:



obtain k accepting transcripts in k/ptrials (expected)



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What happens when $|S\rangle$ is quantum?

$$\begin{array}{c} r \\ \hline z \\ \hline z \end{array} \end{array} \qquad p \coloneqq \Pr_{\substack{r \leftarrow R, \\ z \leftarrow [S] \\ (r)}} \left[\begin{array}{c} \hline z \\ \hline z \\ \hline z \\ \hline \end{array} \right]$$

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Independent trials violate no-cloning!



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Goal: given $(s_i)^{*}$ with large enough success probability p, obtain k accepting transcripts (r_i, z_i) .

Independent trials violate no-cloning!



Goal: given $\{ j \} \}$ with large enough success probability p, obtain k accepting transcripts (r_i, z_i) .

Independent trials violate no-cloning!



Goal: given $\{j_{k}\}$ with large enough success probability p, obtain k accepting transcripts (r_i, z_i) .

Independent trials violate no-cloning!







 $p_i \coloneqq \Pr[\text{Trial } i \text{ successful}]$ Unruh's Lemma [U12,DFMS19]: $p_i \ge p_1^{2d-1}$ i=1In words: a constant number of sequential trials all succeed with non-negligible probability.

If we could improve this analysis, we'd get better quantum rewinding!

 $p_i \coloneqq \Pr[\text{Trial } i \text{ successful}]$

Unruh's Lemma [U12,DFMS19]:

$$\prod_{i=1}^d p_i \ge p_1^{2d-1}$$

In words: a constant number of sequential trials all succeed with non-negligible probability.

Bad news: We show p_i can decay exponentially fast. This means "sequential trials" rewinding is stuck at obtaining a constant number of transcripts.

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- We don't run the next trial on the leftover state from the previous trial.
- Instead, run a procedure to *repair* the success probability of the leftover state before the next trial.



• Quantum state: $\sum_{x} \alpha_{x} |x\rangle$ superposition over classical x



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• Can do "partial" measurements that don't fully collapse the state

SUCCESS prob p^* |S> initial state

Before any trials, (somehow) measure the adversary's success probability



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Next, perform trial on $|S_0\rangle$ with **known** success probability p.



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How do we implement these procedures?

Fact: exists special basis $\{|T_p\rangle\}_{p\in[0,1]}$ where each $|T_p\rangle$ is adversary with success prob p.

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$$|S\rangle = \sum_{p} \alpha_{p} |T_{p}\rangle$$
 w/ success prob p^{*}
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[MW04,Z20]: Can *approximately* measure in this basis: $|S\rangle = \sum_{p} \alpha_{p} |T_{p}\rangle$ collapses to $\approx |T_{p}\rangle$ w/ prob $|\alpha_{p}|^{2}$.

Measure success prob



[MW04,Z20]





[MW04,Z20]

New techniques needed!

[CMSZ21] State Repair

After we measure success prob

$$p, |S_0\rangle$$
 lies in subspace
 $V \coloneqq span(|T_q\rangle)_{q \ge p}$
i.e., $V =$ subspace of attacker
states with success prob $\ge p$.














If $|S_1\rangle$ not far from V, must have non-trivial component in V.

We show: can *amplify* this component to output $|S_1^*\rangle \in V$







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[CMSZ21] State Repair

Looks like *amplitude amplification*, but requires care since we can only *approximately* project onto *V*.







[CMSZ21] Summary: our quantum rewinding approach extends many classical reductions to the quantum setting.



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Consequences: post-quantum succinct arguments + optimal post-quantum security guarantees for other protocols

So far, we've considered quantum adversaries.

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But in the long-term, even honest parties may possess quantum computers.

[BB84] Quantum Key Distribution

- Quantum parties + quantum channel agree on key
- Security is *information-theoretic* (i.e., no assumptions)



After [BB84], significant efforts (e.g. [BCJL93]) to build more information-theoretic quantum crypto.

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Use quantum to build crypto without assumptions!

Bad news [M97,LC97]: Information-theoretic quantum bit commitments are impossible.

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Does this mean for most crypto tasks, quantum information doesn't help?

Not necessarily! May be possible to use quantum information to build crypto from *weaker assumptions*.



Goal: Learn joint function $f(x_A, x_B, x_C)$ on private inputs **Security:** reveal nothing else about x_A, x_B, x_C



[Yao86,GMW87]: Classical MPC

- Equivalent to "oblivious transfer"
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Recap

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Upcoming work [LMS21]:

- obtain state-preserving version of [CMSZ21] rewinding
- one application: the [GMW86] graph non-isomorphism protocol is zero-knowledge against quantum verifiers

Recap

[CMSZ21]: Obtain quantum analogue of classical rewinding and prove Kilian's protocol secure against quantum.

[BCKM21a]: Construct secure multiparty computation from oneway functions + quantum information.

Rest of thesis: new results on Fiat-Shamir

[BBHMR19]: Barriers to provably-secure succinct non-interactive arguments from Fiat-Shamir.

[CLMQ21]: Investigate whether Fiat-Shamir hash function must be "cryptographic"

Thank You!

Slide Artwork by Eysa Lee