The cryptographic nature of quantum computation

Fermi Ma

UC Berkeley & Simons Institute

I study **quantum computers**.

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devices that exploit
quantum phenomena

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devices that exploit
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I also study **cryptography**.

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devices that exploit
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build protocols that provably
resist adversarial behavior.

Quantum computers will revolutionize cryptography

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New threats

Shor's algorithm breaks RSA!

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How do we build classical cryptography that resists quantum attack?

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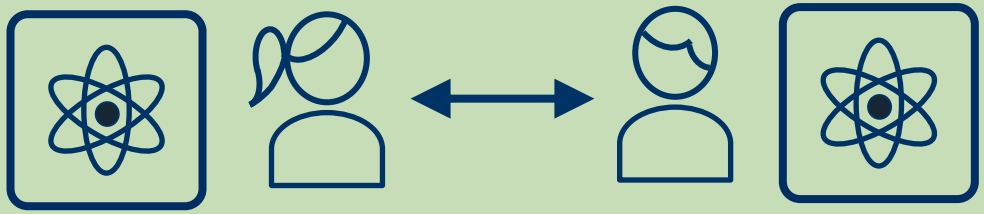

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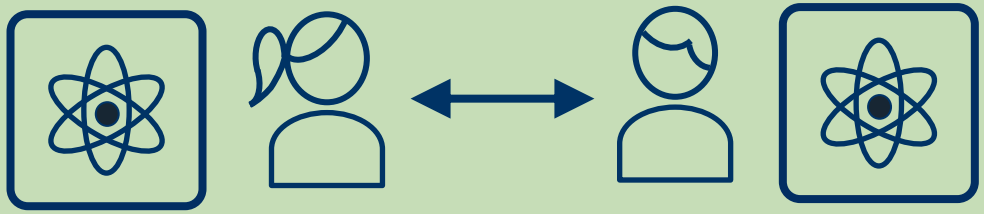

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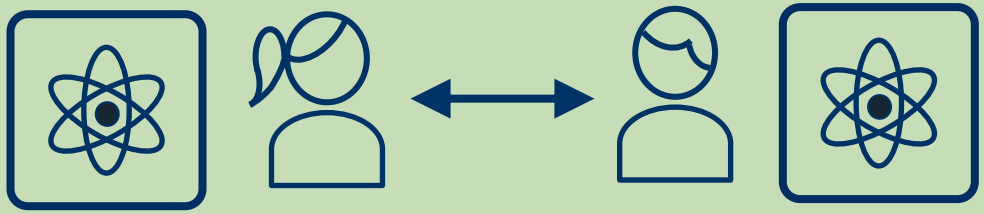

Can we leverage quantum mechanics to build better cryptographic protocols?

quantum cryptography

But quantum cryptography is not just about cryptography…

Recently, quantum cryptography has been showing up in **fundamental physics**.

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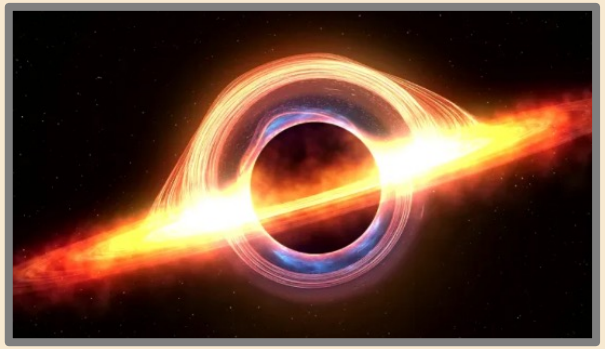

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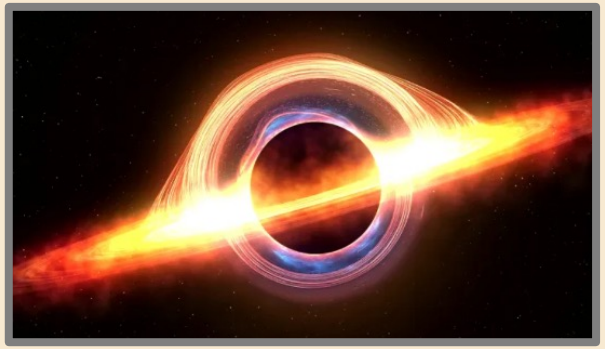

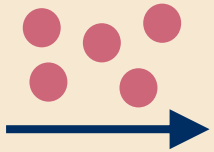

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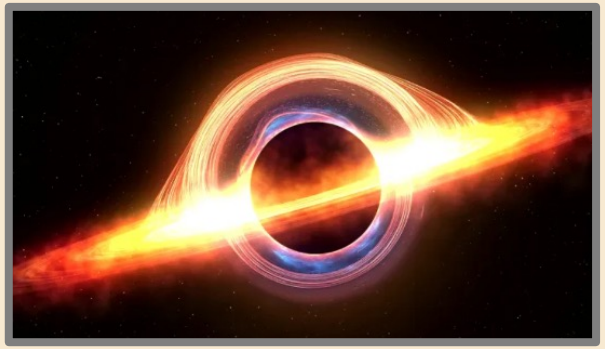

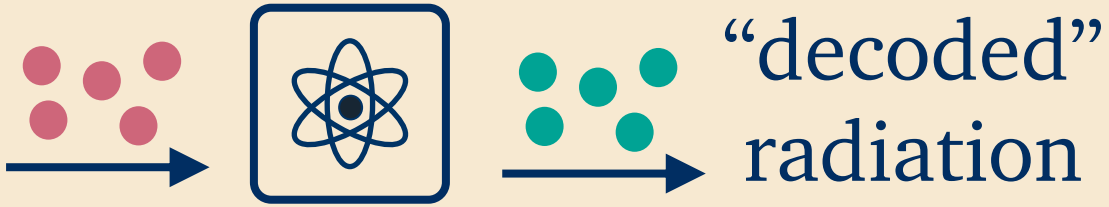

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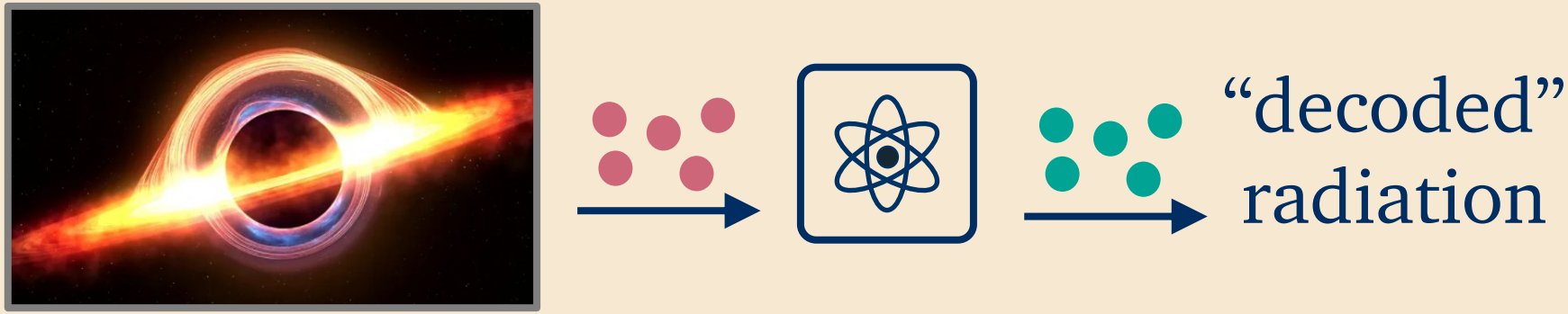

[HH13] counterargument: this is **cryptographically** hard!

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Now common practice: model chaotic systems using cryptographic pseudorandomness

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"We shall assume that the unitary U that describes the formation and the evaporation of the black hole is **pseudorandom**"

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Kim-Preskill (2023)

"We shall assume that the unitary U that describes the formation and the evaporation of the black hole is **pseudorandom**"

Yang-Engelhardt (2023)

"We use the common simplification of modeling black holes and more generally chaotic systems via **(pseudo)random dynamics**"

My research goals

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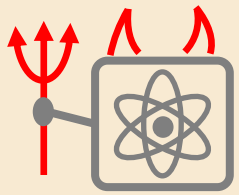
2) Apply these insights to the broader **theory of computation** and **fundamental physics**.

Highlights of my research



quantum
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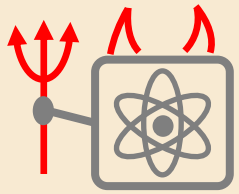

quantum security

- [CMSZ21, LMS22]: proved that textbook crypto protocols are quantum secure via “quantum rewinding”

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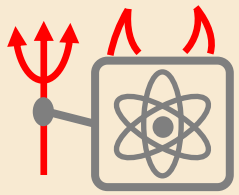

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- [CMSZ21, LMS22]: proved that textbook crypto protocols are quantum secure via “quantum rewinding”

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- These techniques have found widespread use

  [BBK22,BKLMMVVY23,BQSY24,CDGS24,MNZ24,…]

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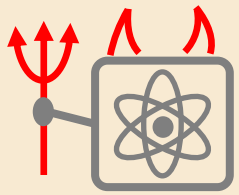

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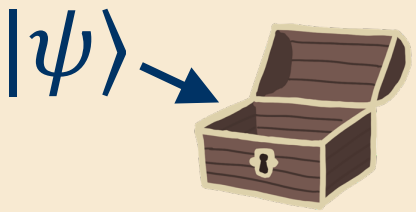

quantum primitives

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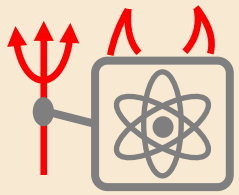

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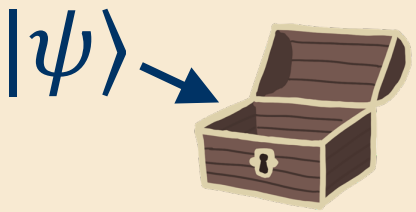

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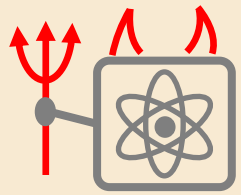

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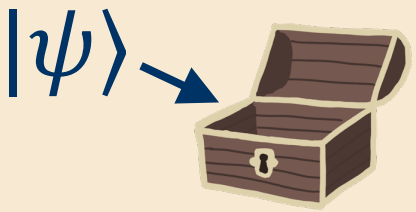

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- [HH13,A16,B23]+[M23]: decoding black hole radiation is equivalent to breaking a “maximally entangled” commitment

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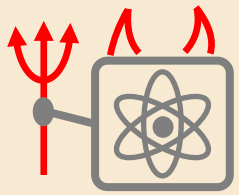

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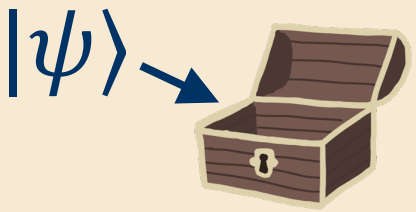

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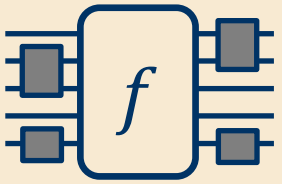

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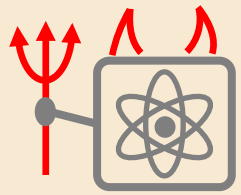

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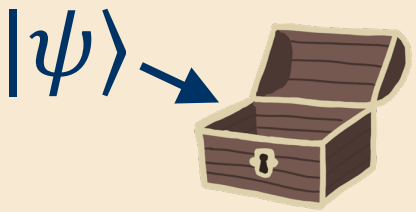

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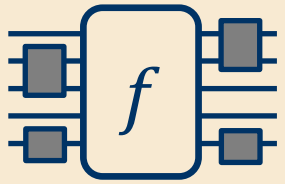

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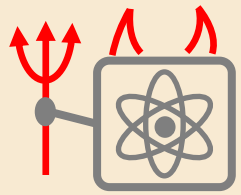

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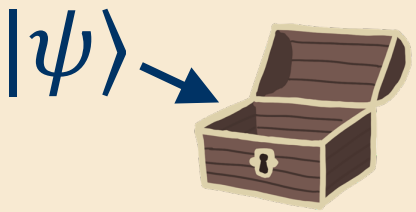

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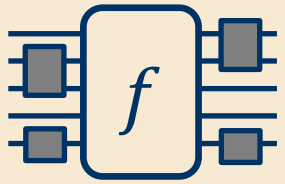

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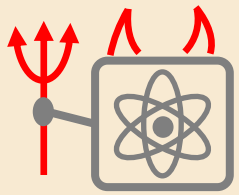

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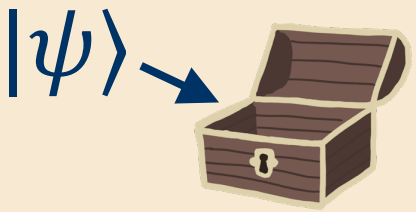

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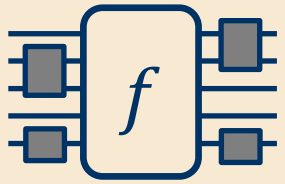

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(STOC 2024)

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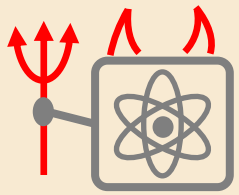

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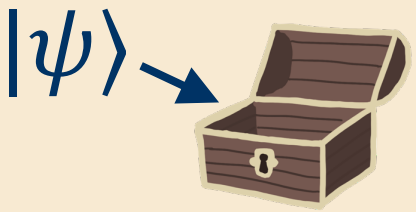

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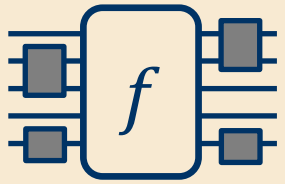

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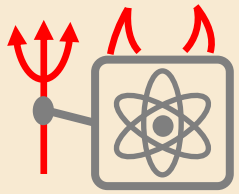

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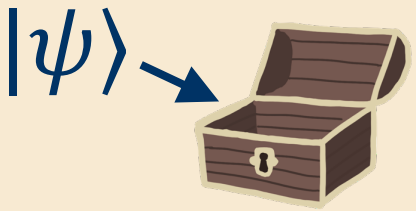

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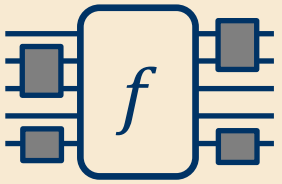

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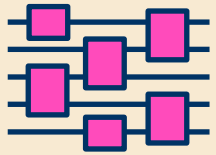

pseudorandom unitaries

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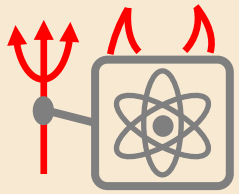

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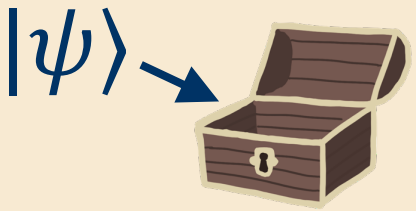

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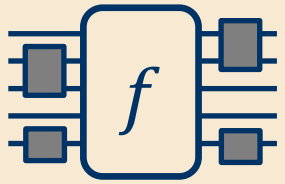

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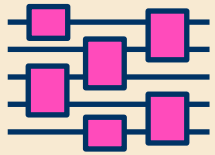

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[MH24]: first proof that pseudorandom unitaries exist (under crypto assumptions).

(STOC 2025, QIP 2025 plenary talk)

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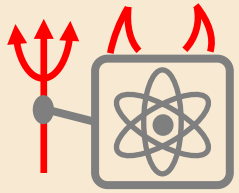

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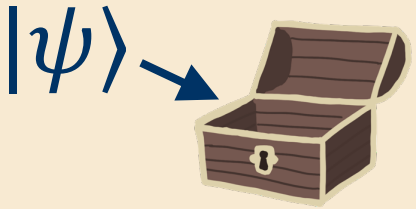

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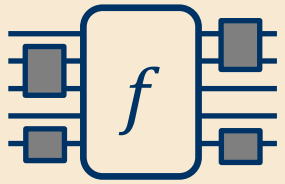

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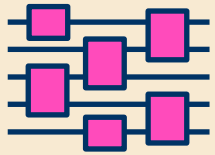

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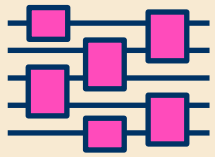

≈ Haar

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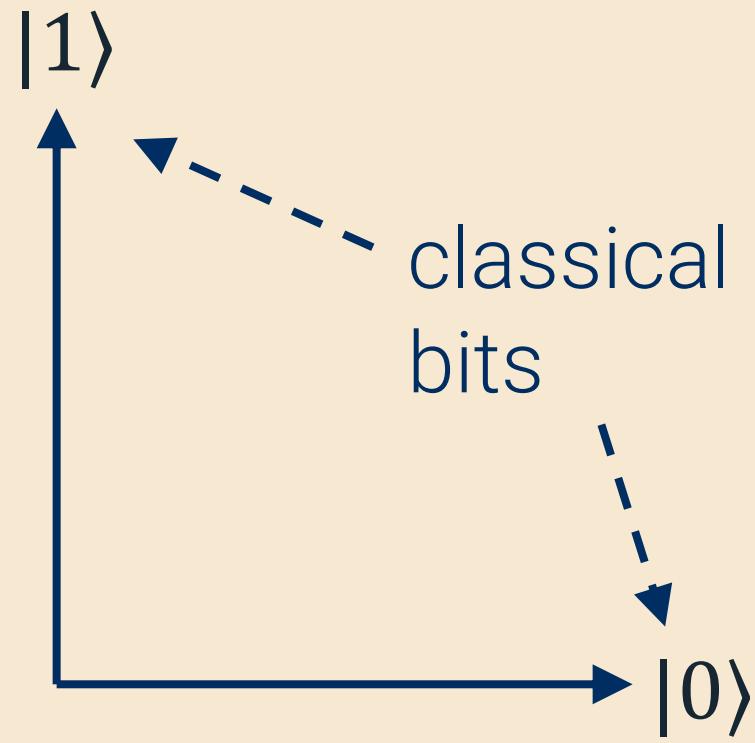
qubits: generalization of classical bits that allows **superposition**



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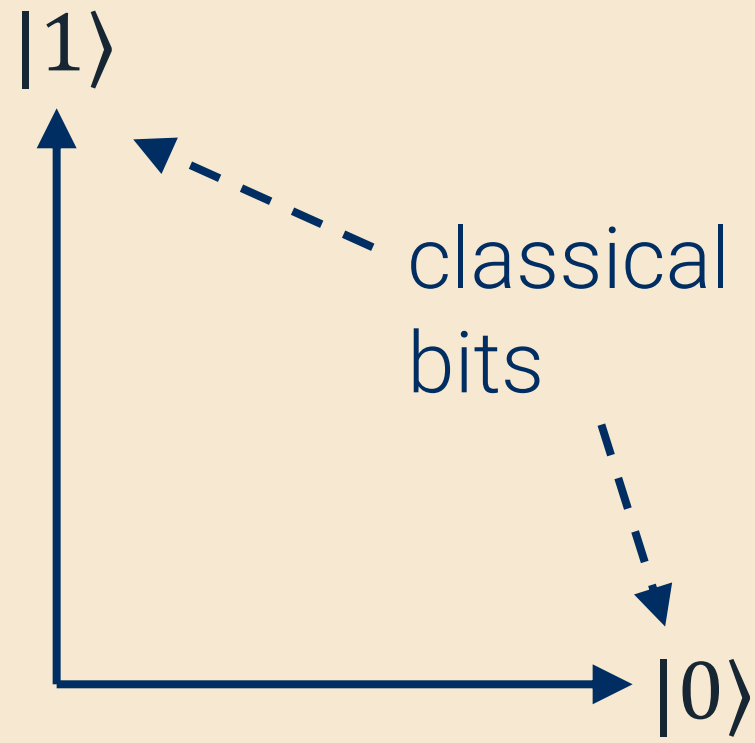
$$\alpha|0\rangle + \beta|1\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$$



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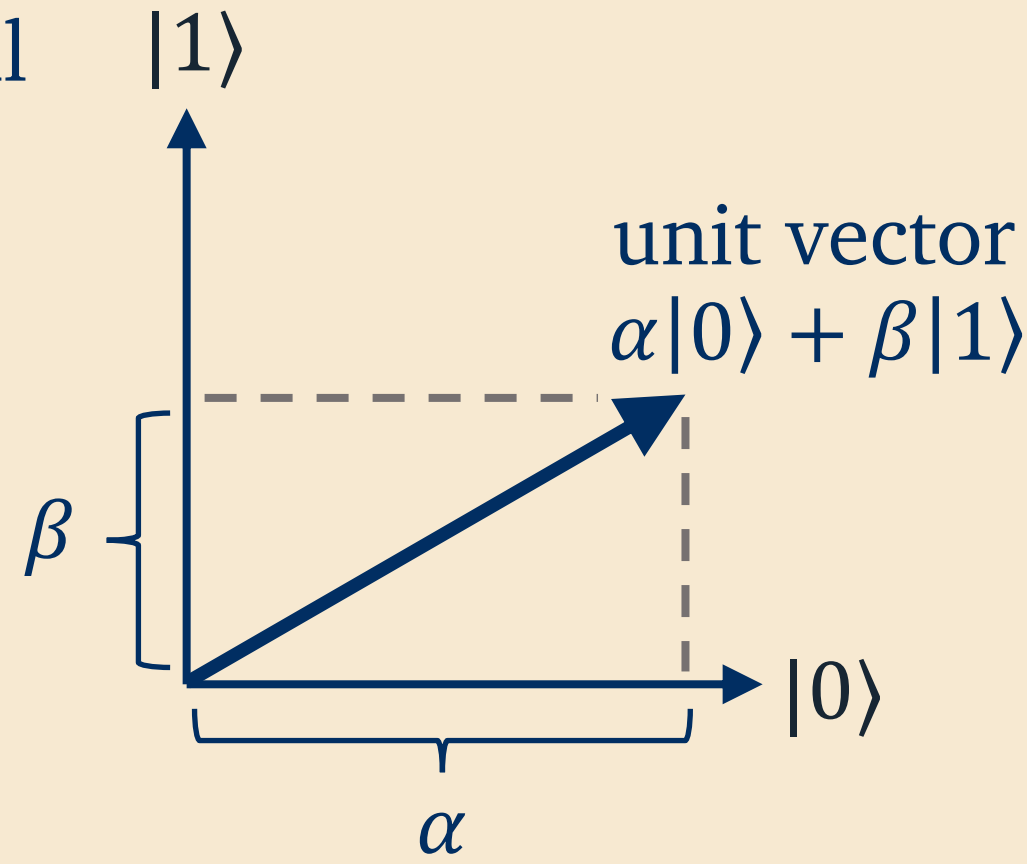

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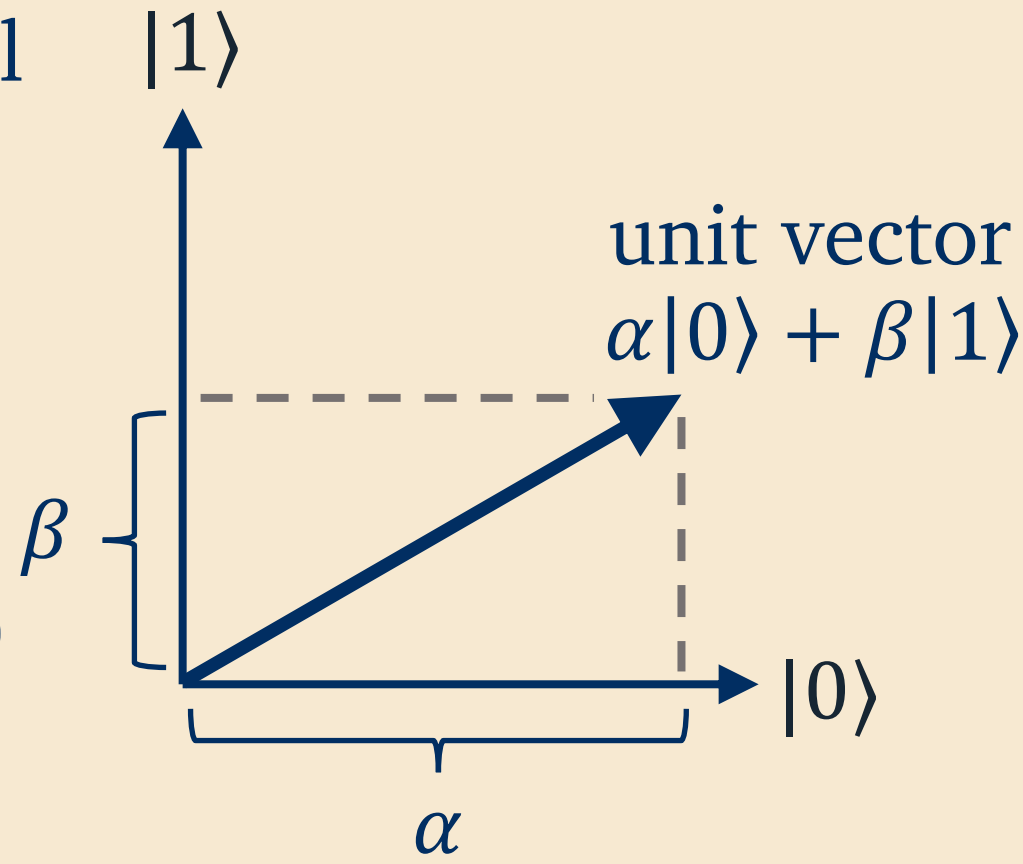

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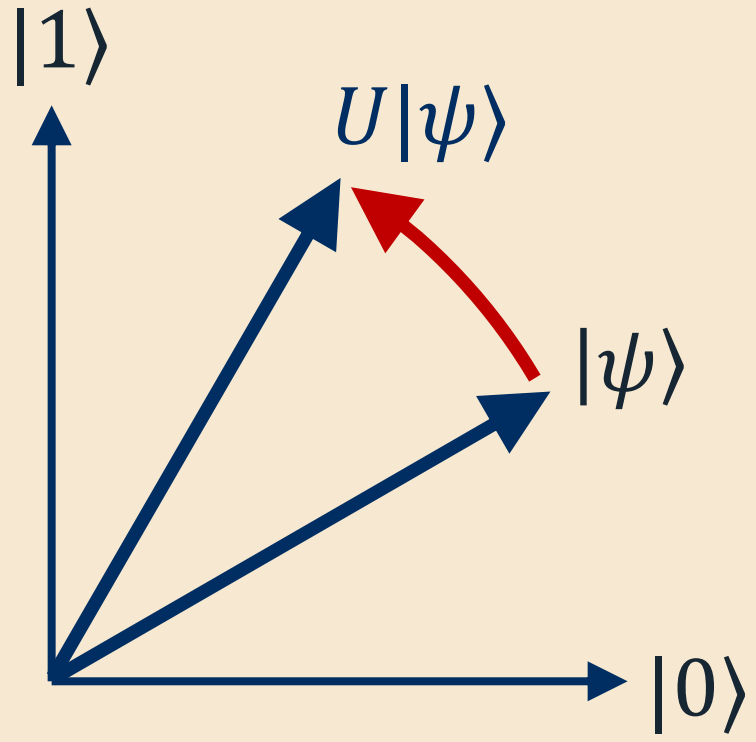

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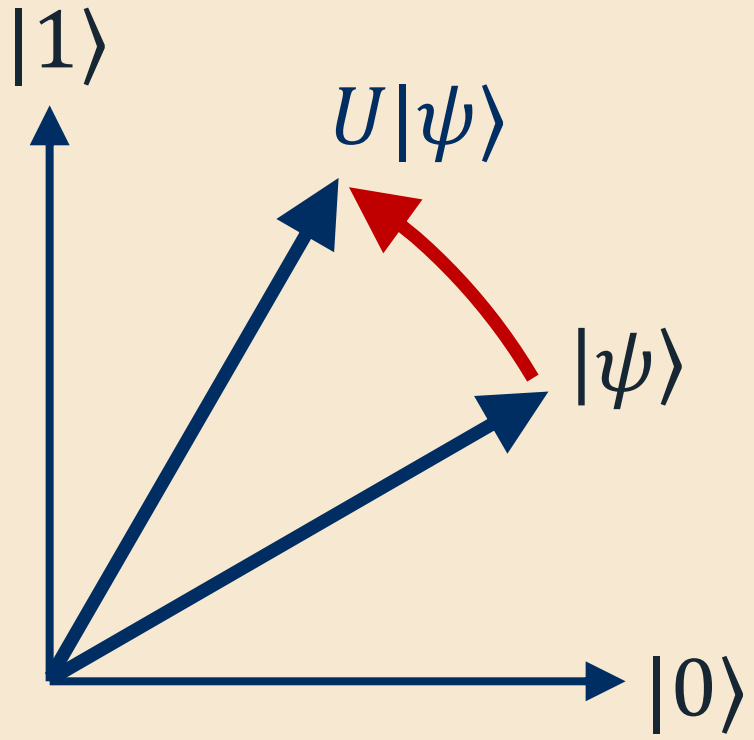

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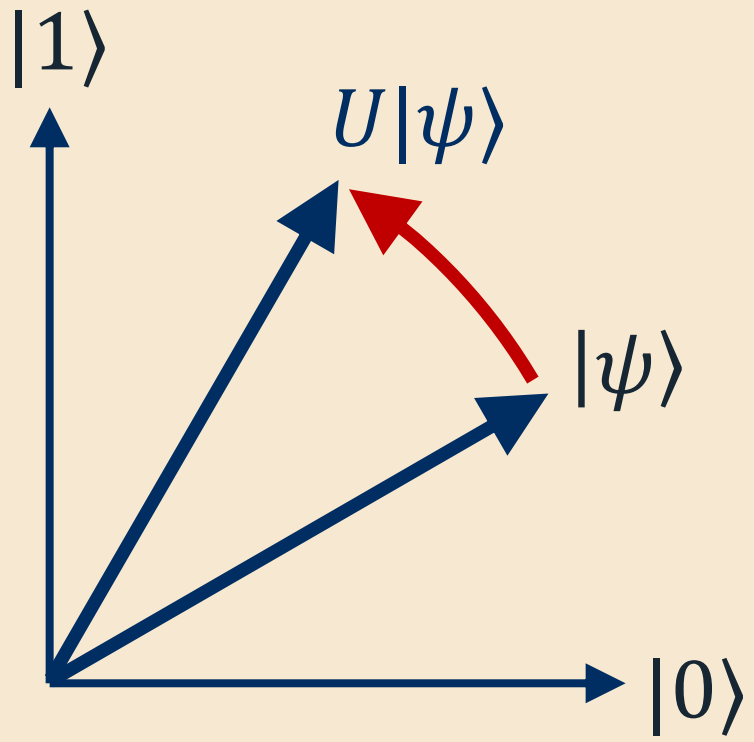

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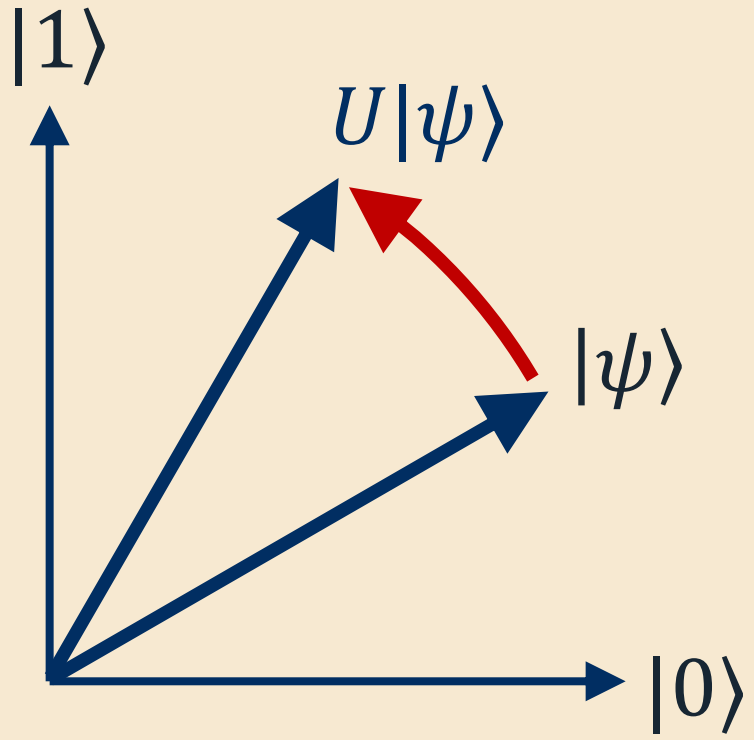

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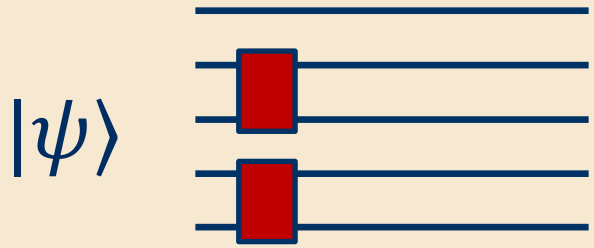

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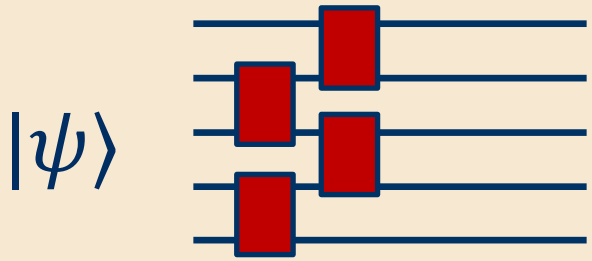

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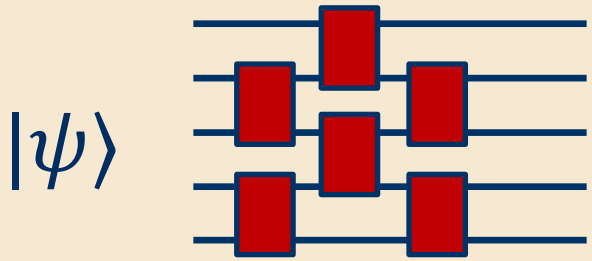

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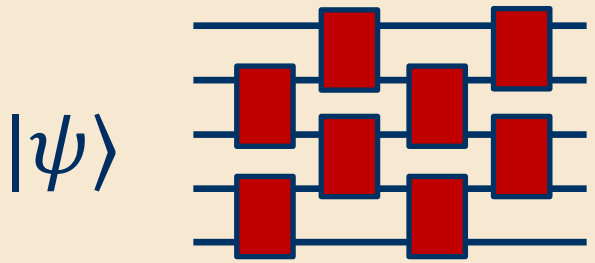

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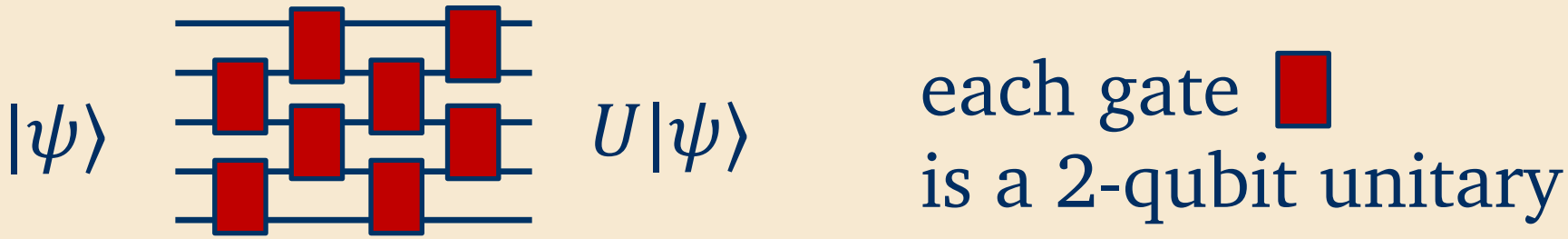

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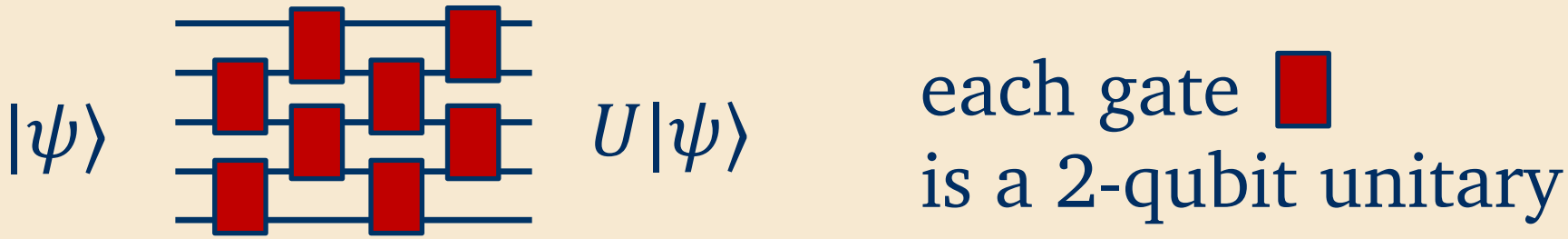

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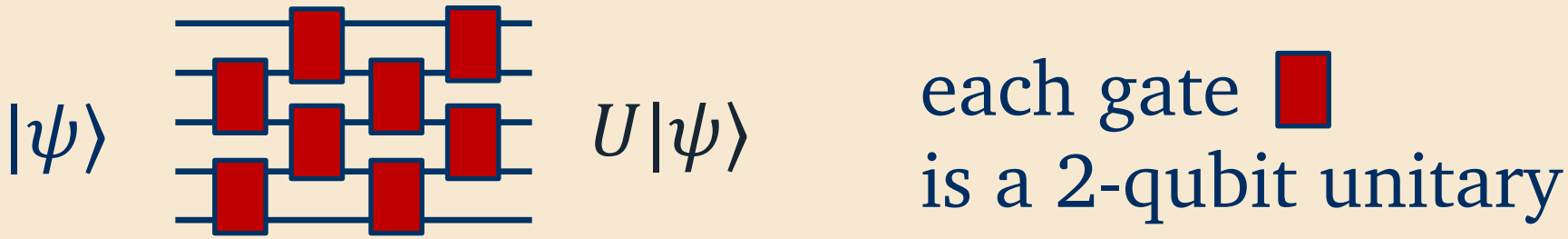

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A random unitary is **unlikely** to be efficiently computable.

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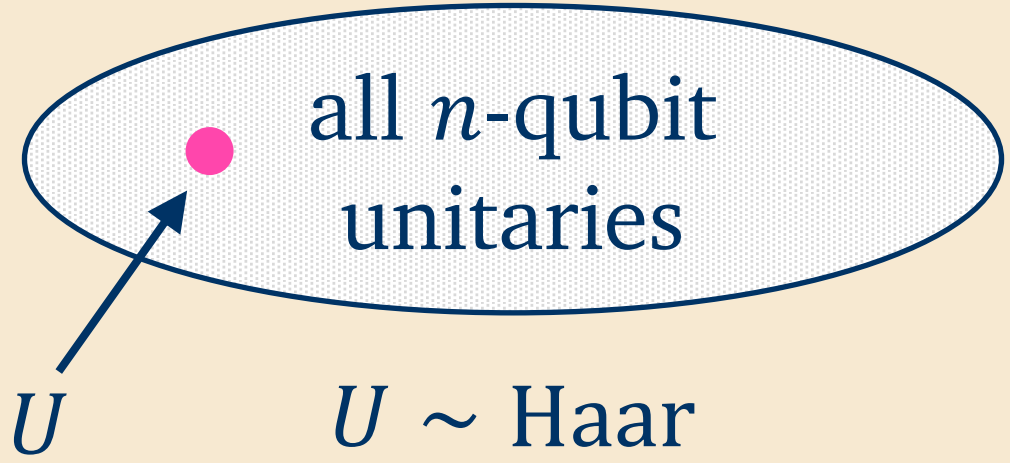

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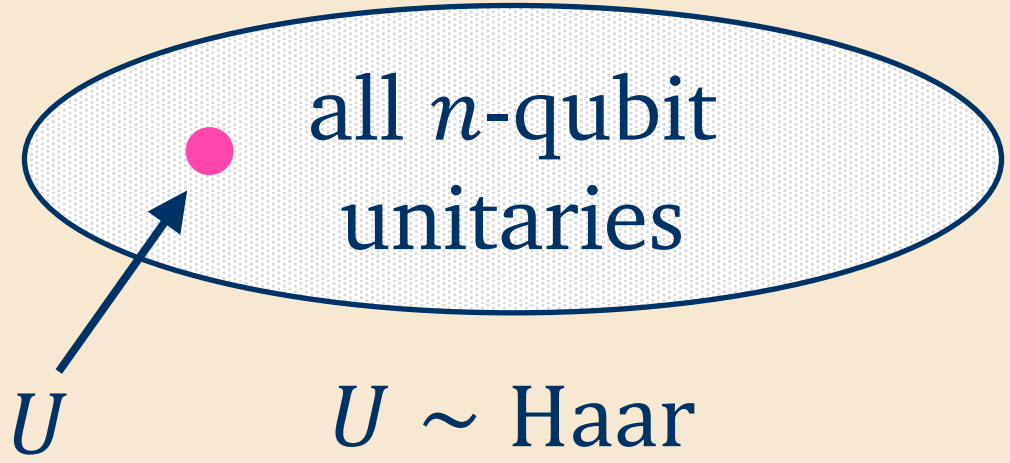

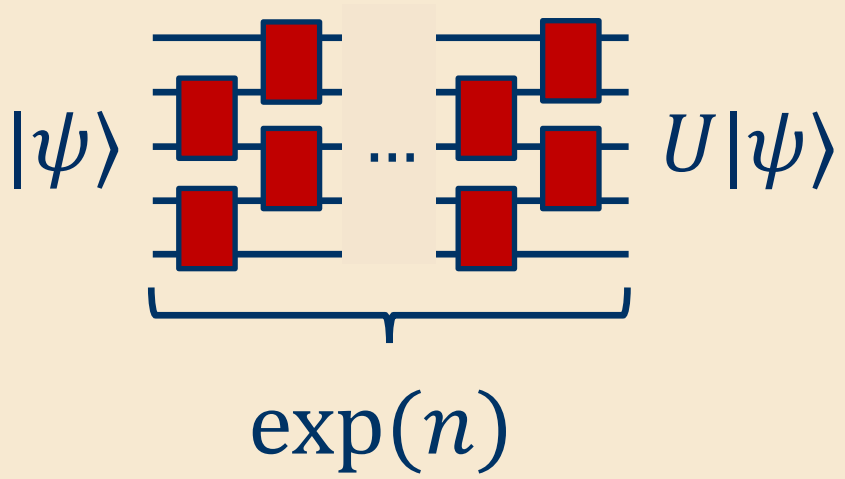

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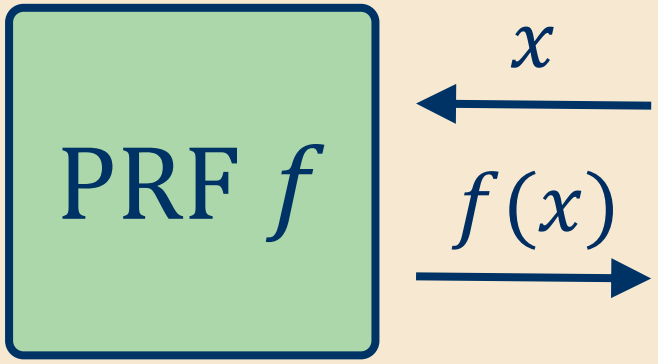

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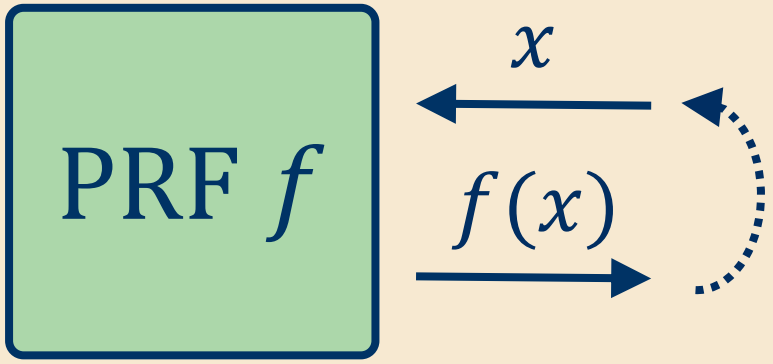

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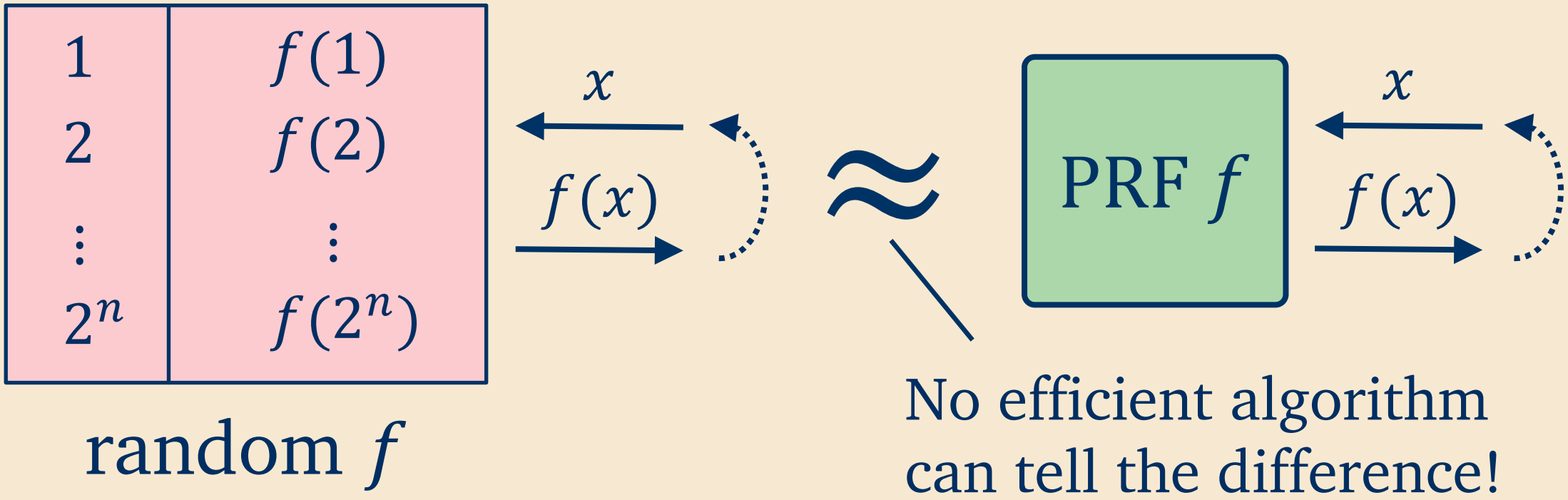

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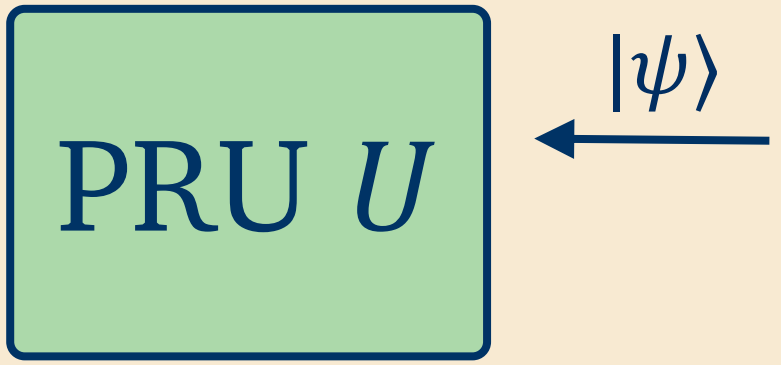

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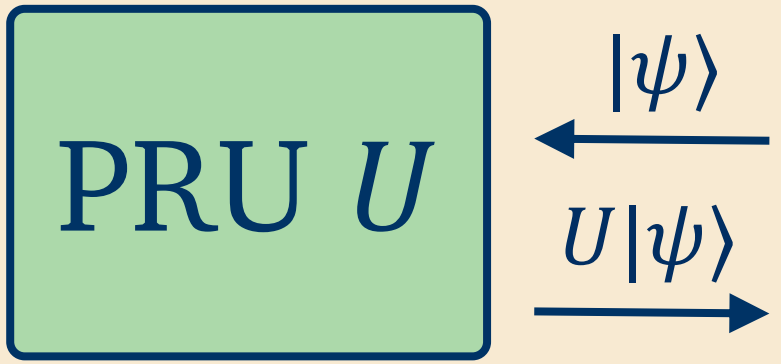

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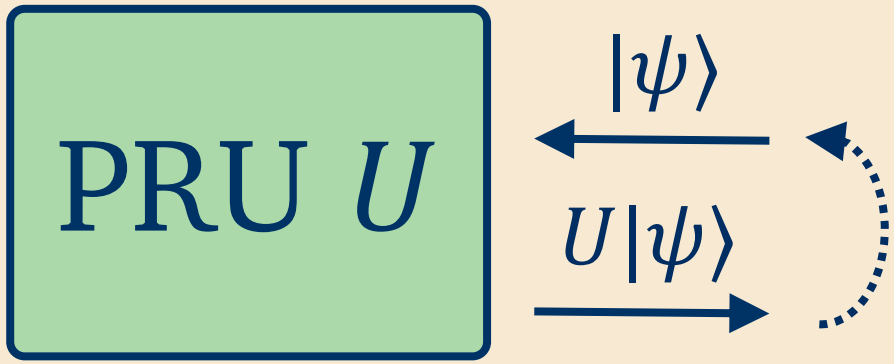

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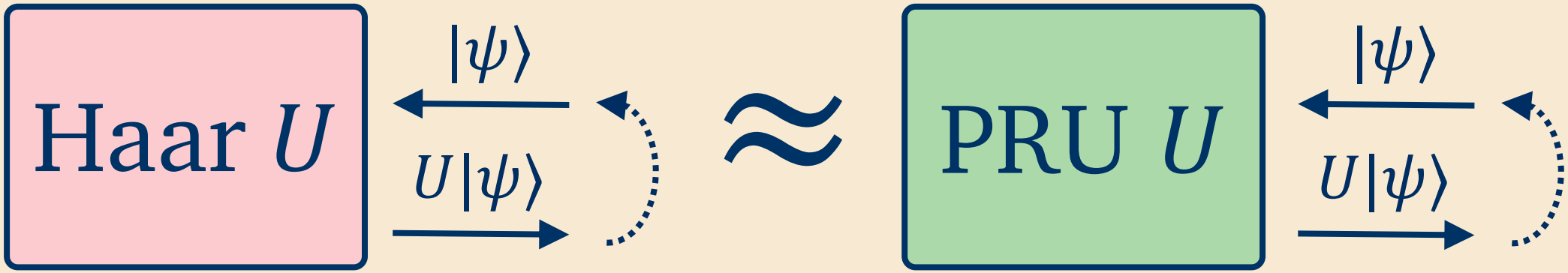

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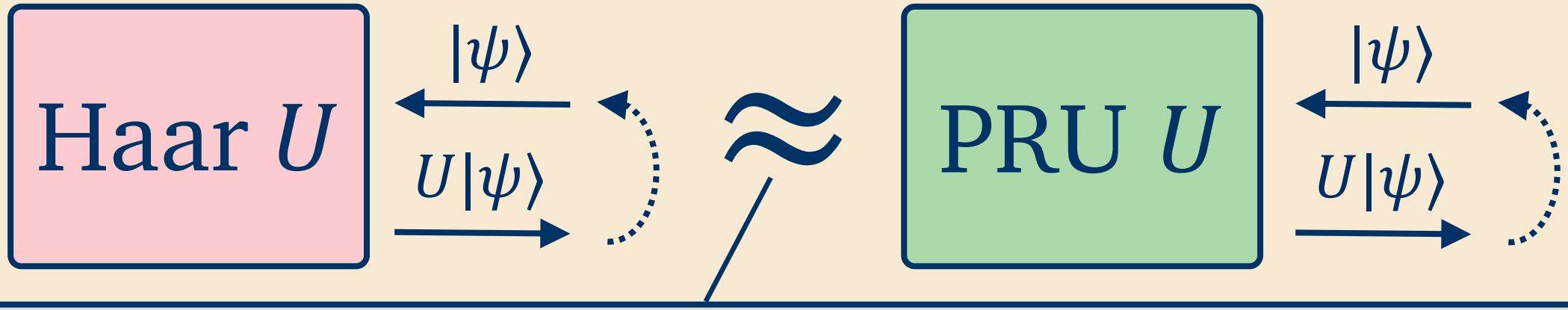

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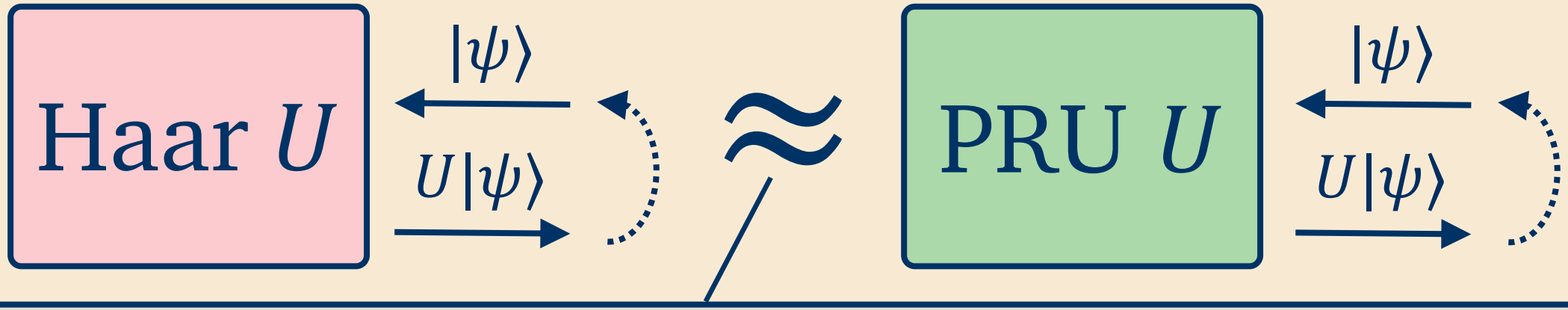

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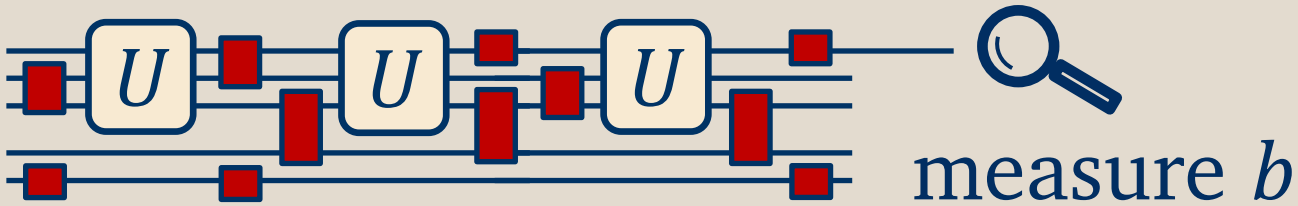

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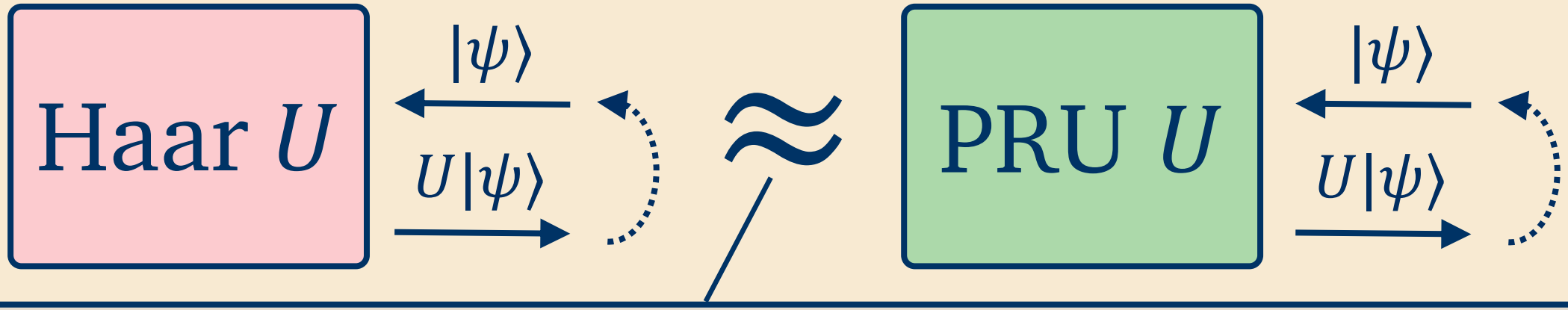

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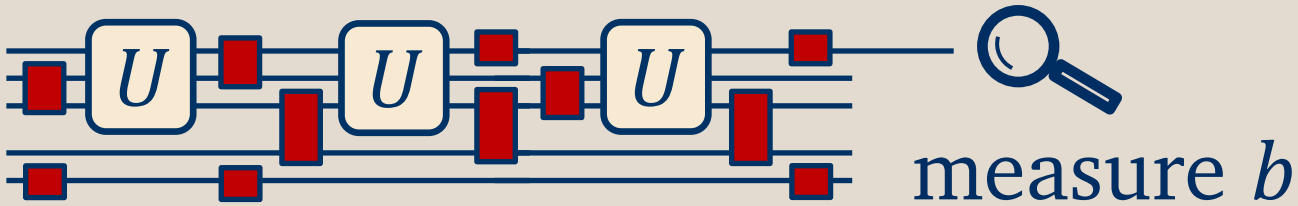

$$\Pr[b = 1 \mid U \leftarrow \text{Haar}] \approx \Pr[b = 1 \mid U \leftarrow \text{PRU}]$$

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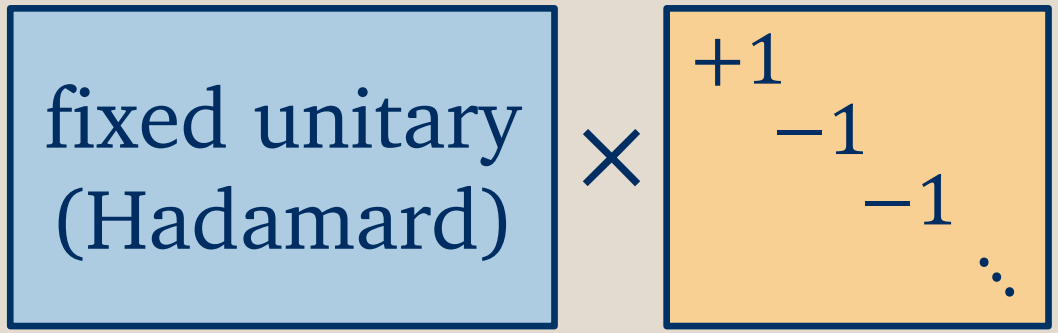

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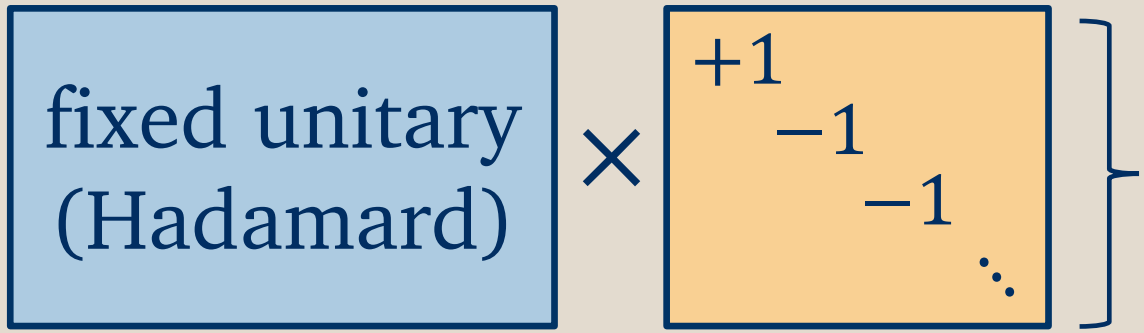

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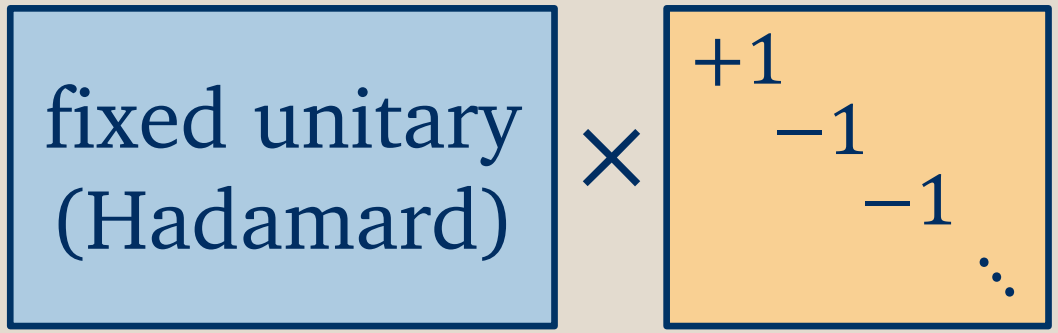

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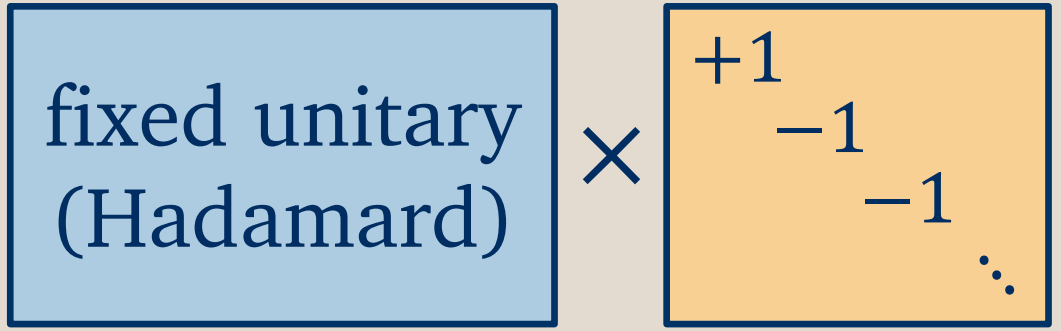

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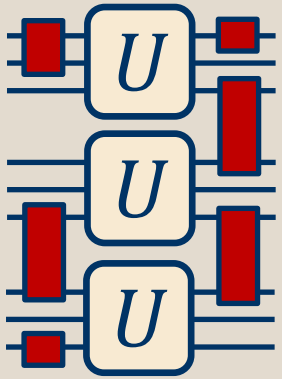

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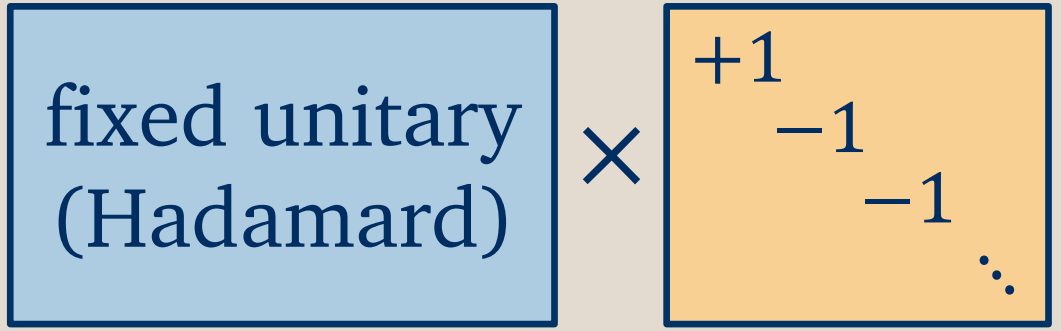

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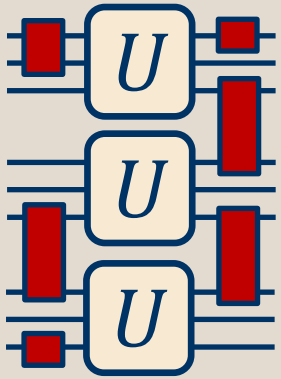

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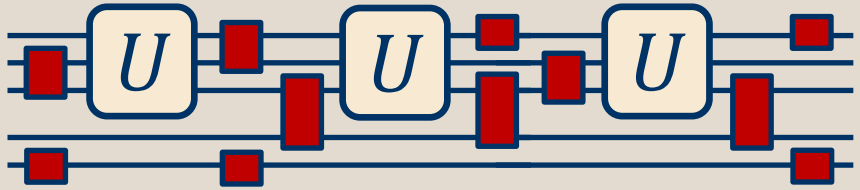

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(the construction we analyze is by [MPSY24])

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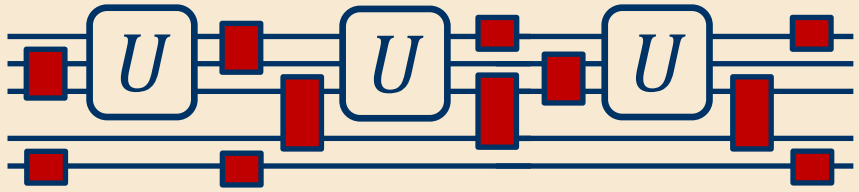

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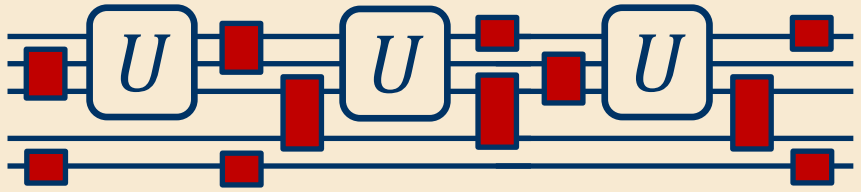

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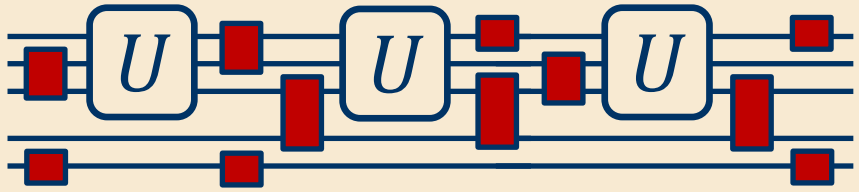

2) Mathematics of random unitaries is complicated.

- Weingarten calculus
- representation theory
- free probability

Theorem 3.1. *Let* k *be a positive integer. For any permutation* $\sigma \in \mathcal{S}_k$ *and nonnegative integer* g*, we have*

$$(k-1)^g \#P(\sigma,|\sigma|) \leq \#P(\sigma,|\sigma|+2g) \leq (6k^{7/2})^g \#P(\sigma,|\sigma|).$$

Theorem 3.2. *For any* $\sigma \in \mathcal{S}_k$ *and* $d > \sqrt{6}k^{7/4}$*,*

$$\frac{1}{1-\frac{k-1}{d^2}} \leq \frac{(-1)^{|\sigma|} d^{k+|\sigma|}\, \mathrm{Wg}^U(\sigma,d)}{\#P(\sigma,|\sigma|)} \leq \frac{1}{1-\frac{6k^{7/2}}{d^2}}.$$

In addition, the l.h.s inequality is valid for any $d \geq k$.

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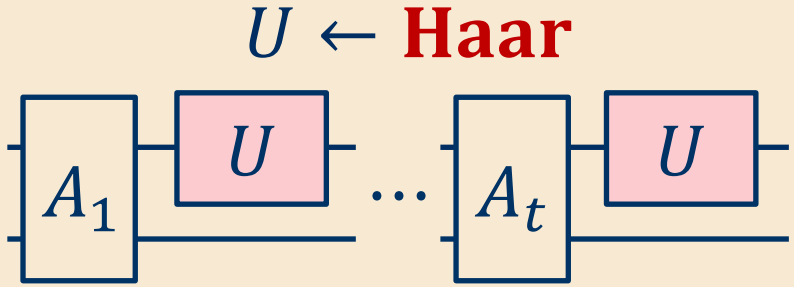

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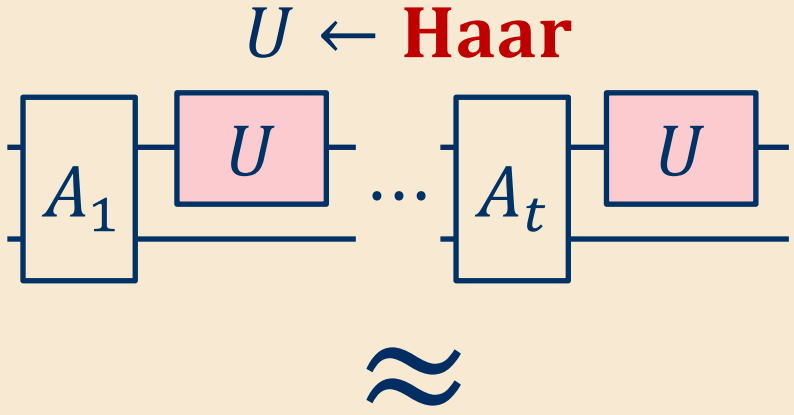

Cartoon overview of our proof

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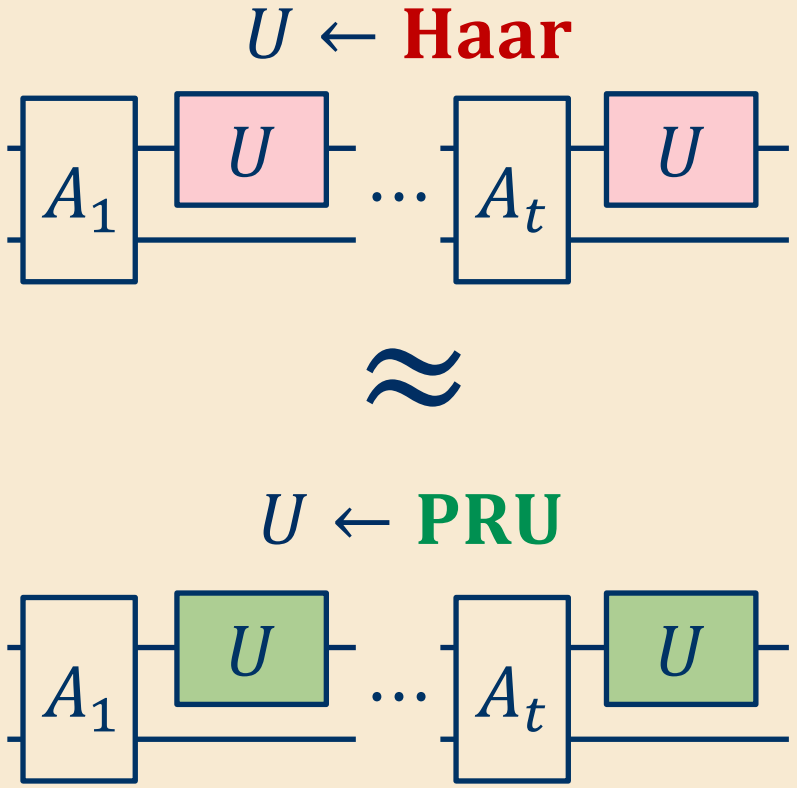

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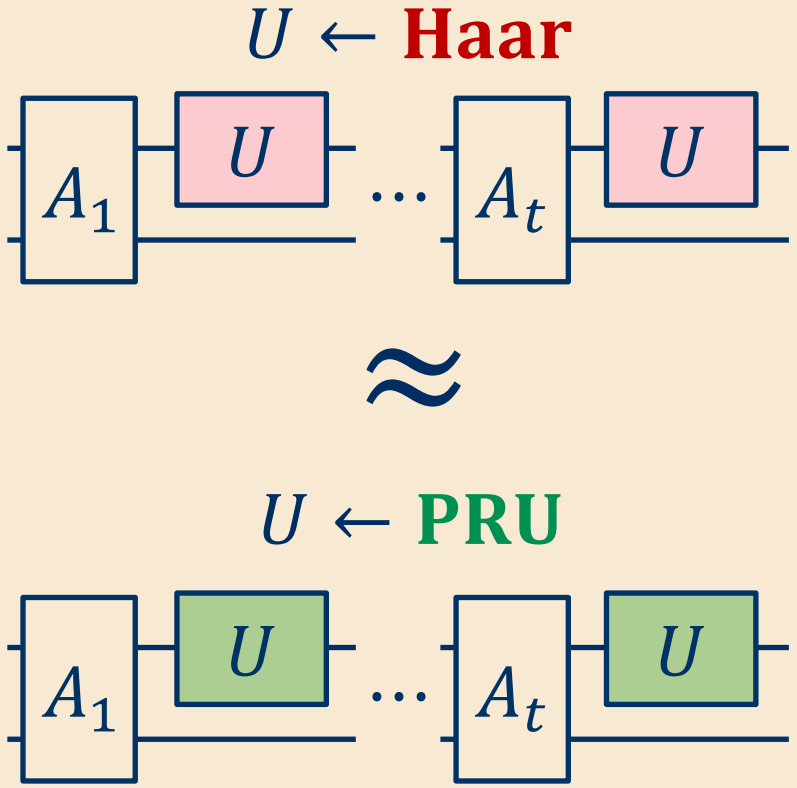

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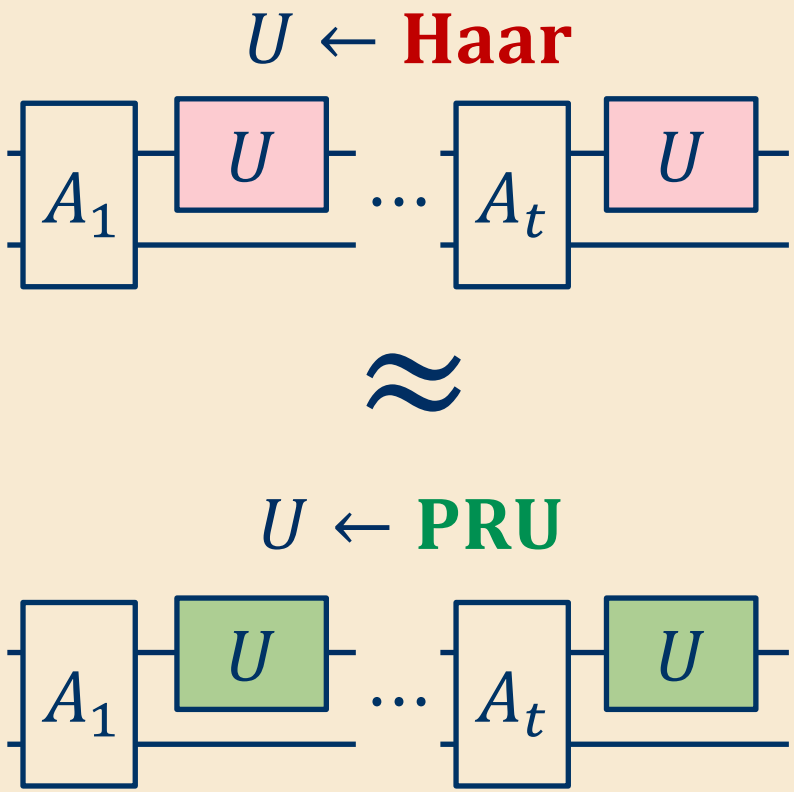

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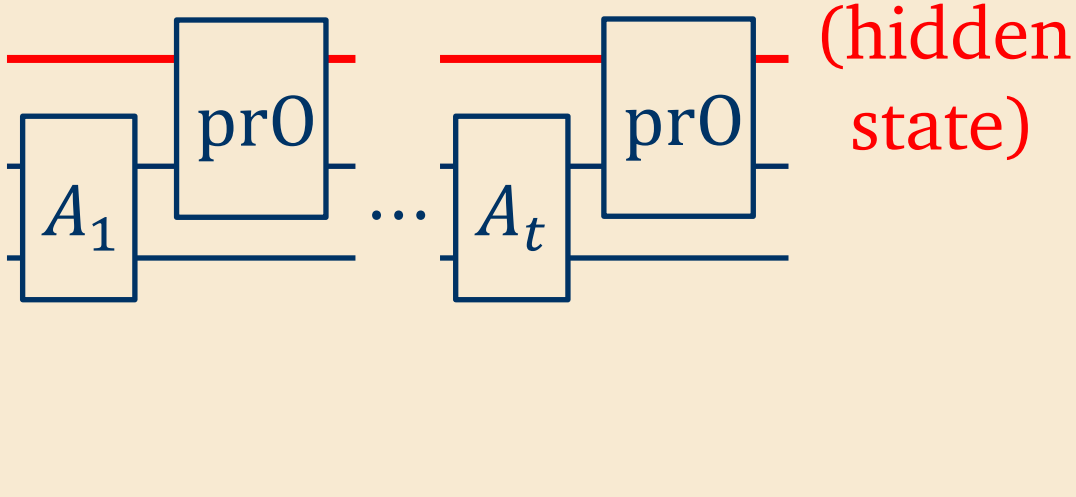

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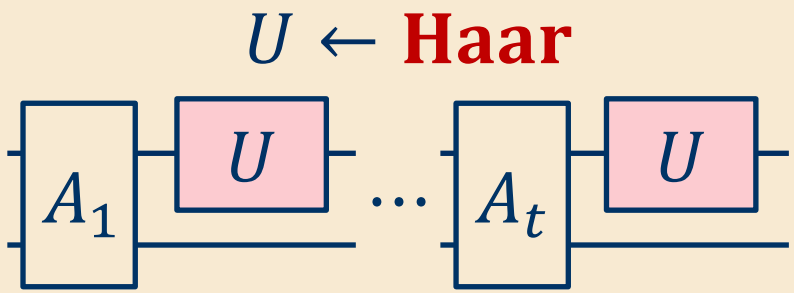

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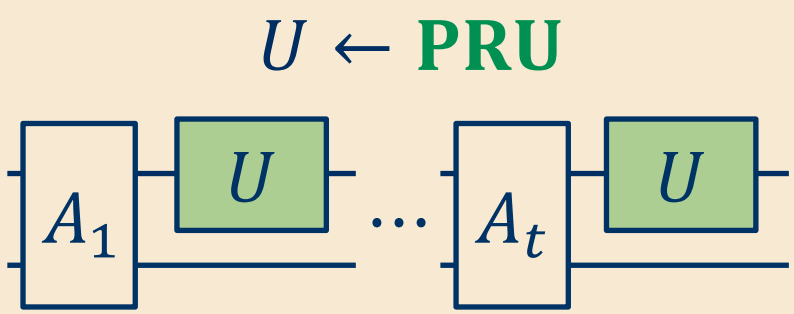

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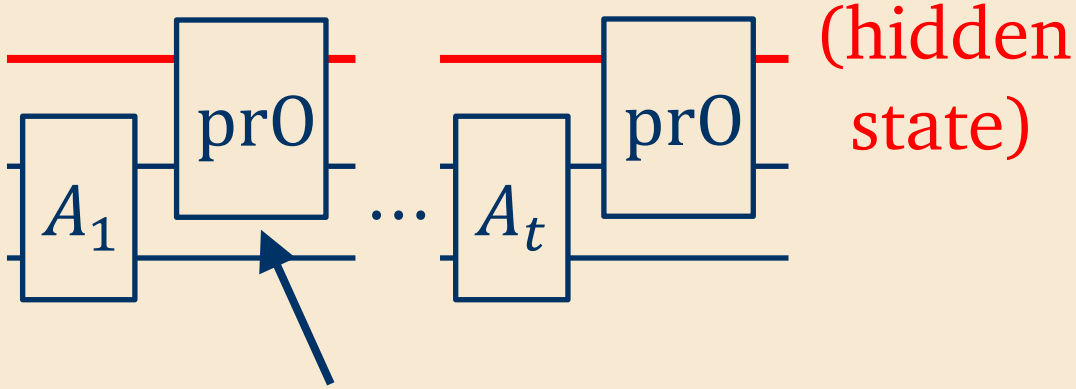

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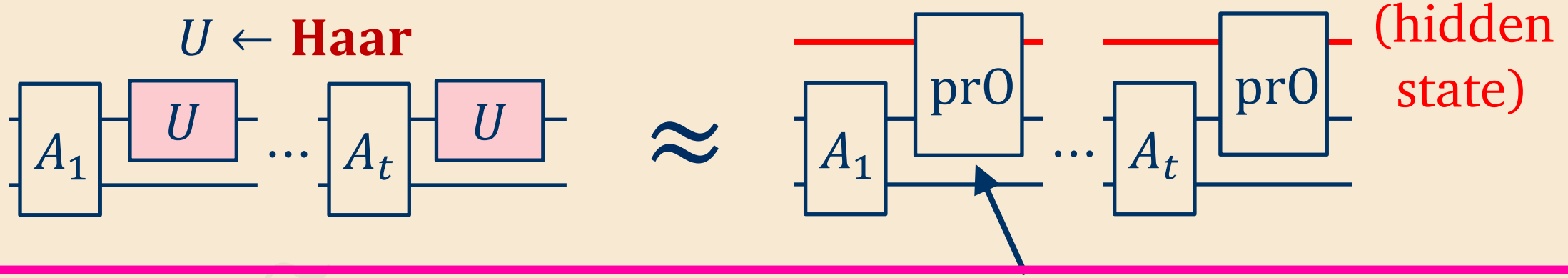
(path-recording oracle)
a data structure that performs "lazy sampling" of a Haar-random unitary

most of the proof



(path-recording oracle)
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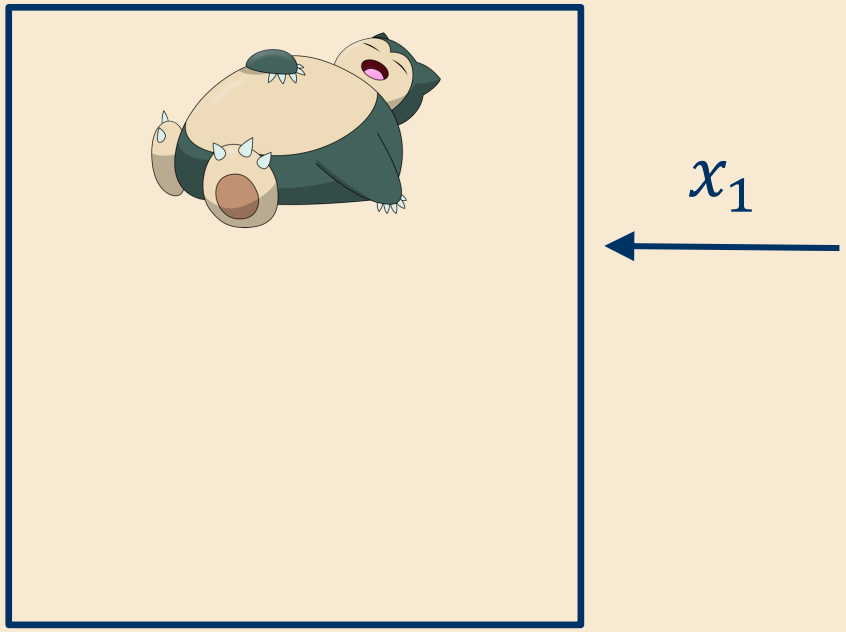

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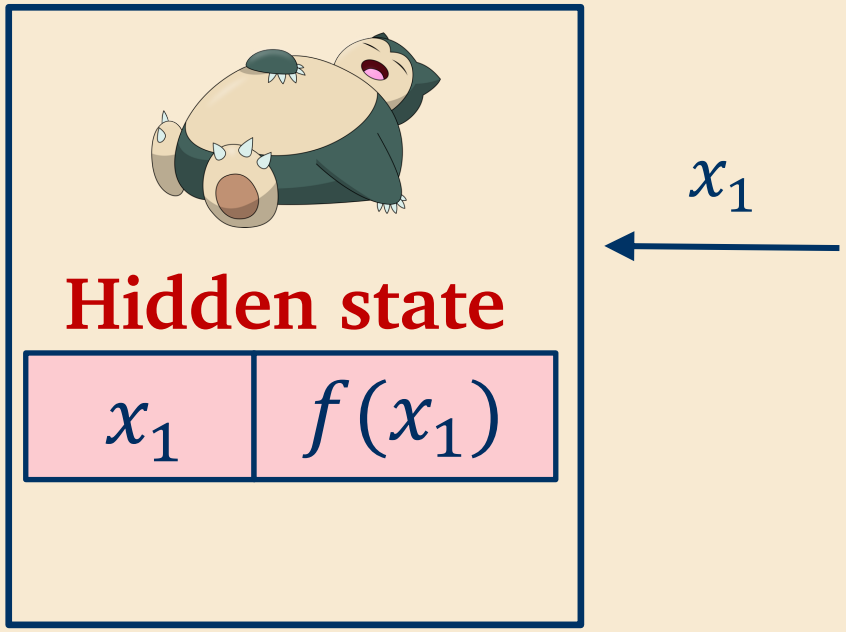

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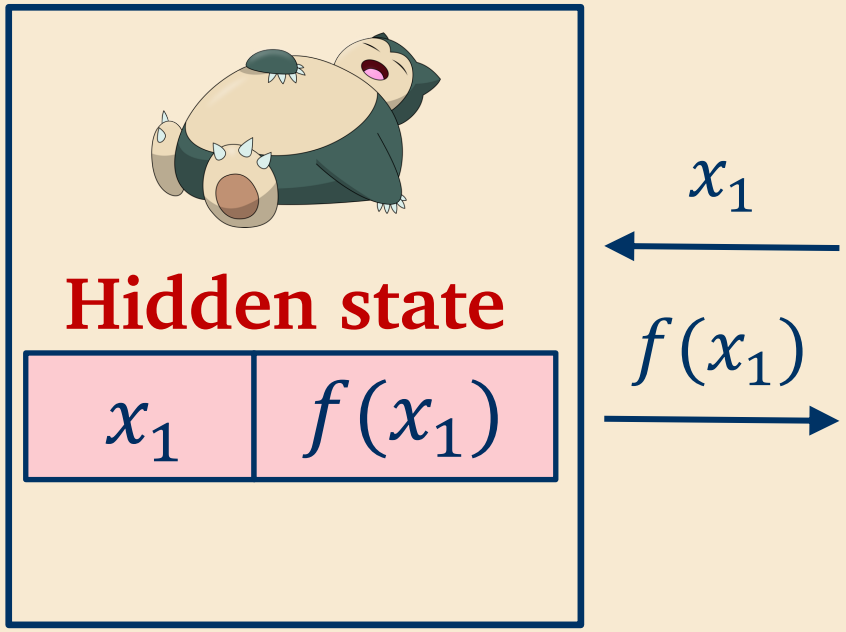

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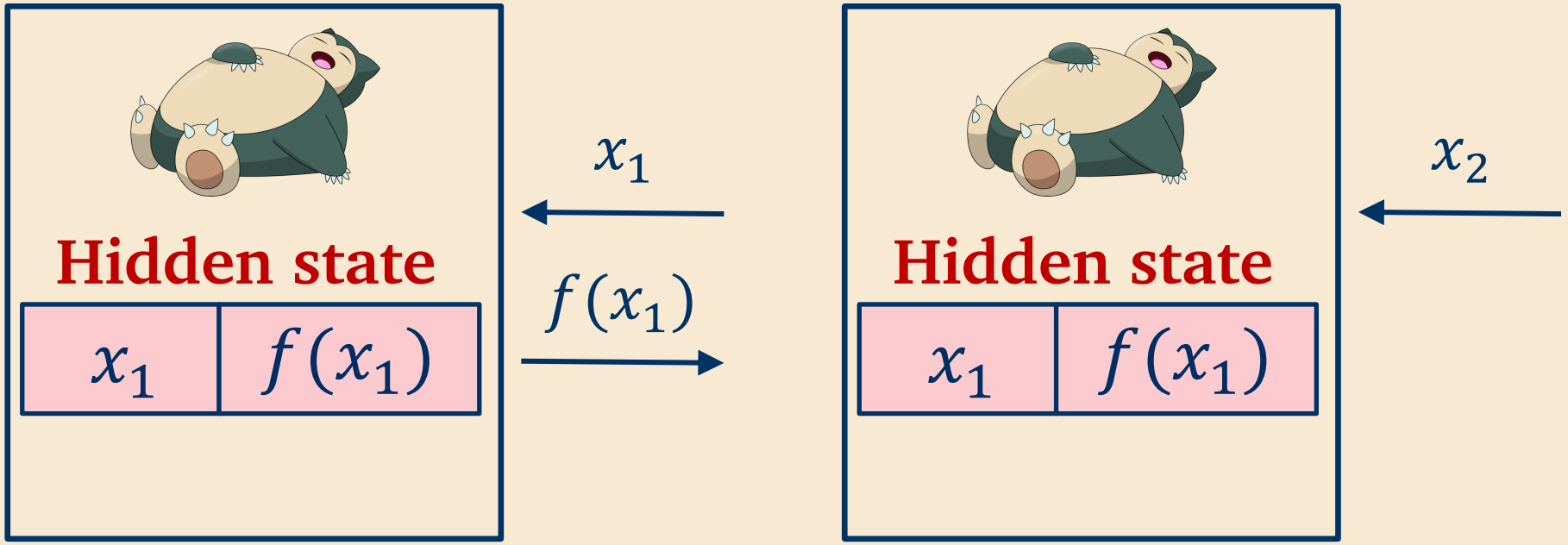

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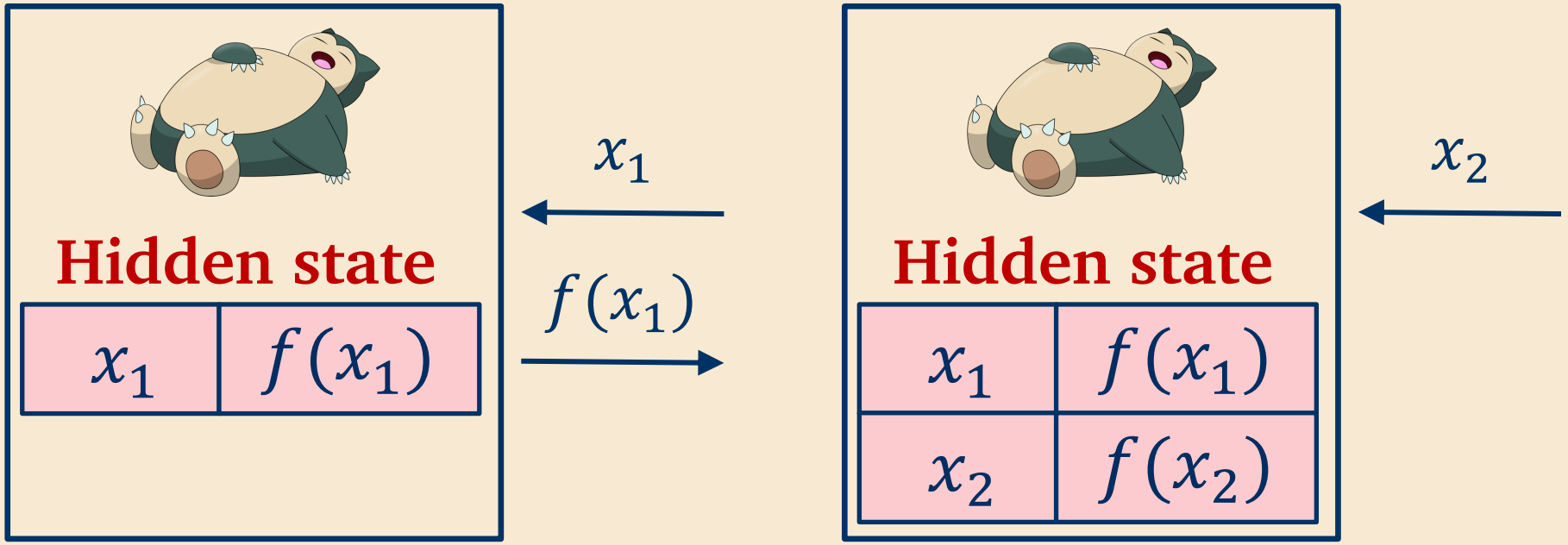

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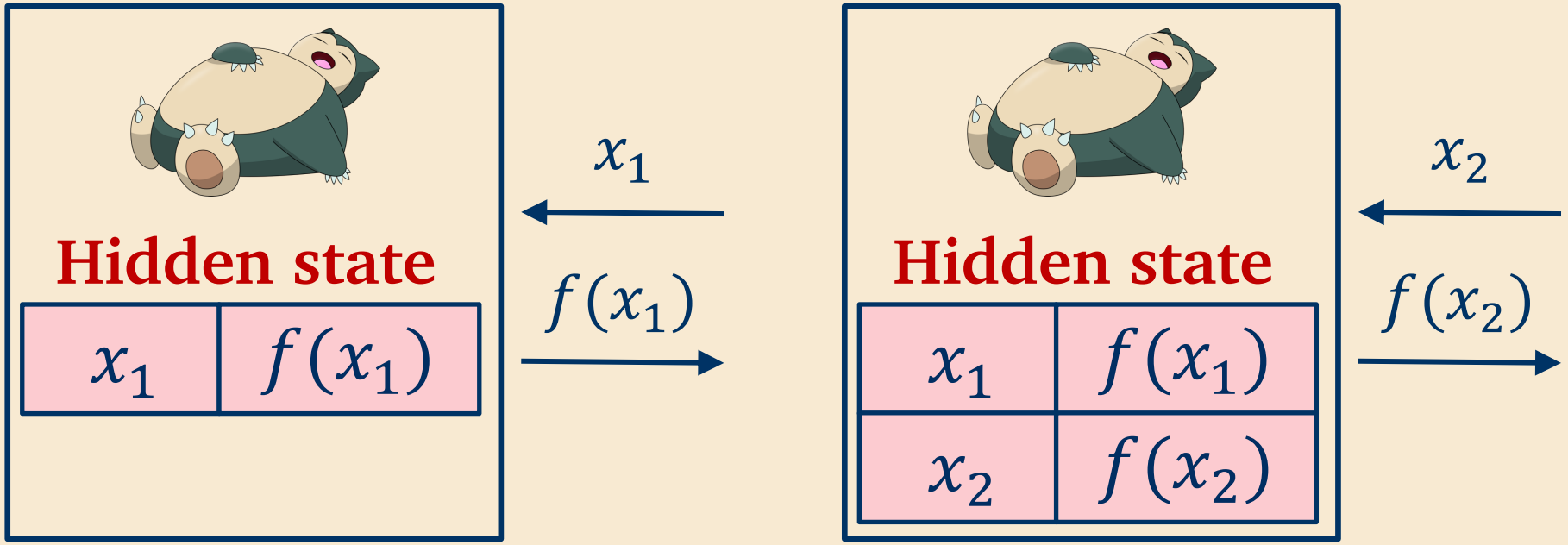

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Classical case: the data structure is the list of $(x, f(x))$ pairs.

Up next:

"Derive" the path-recording oracle through simple examples

Example 1: one query on $|0\rangle$

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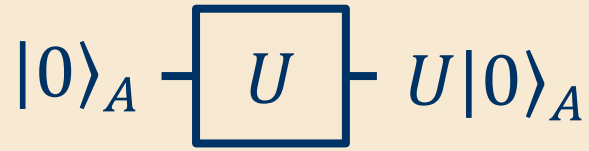

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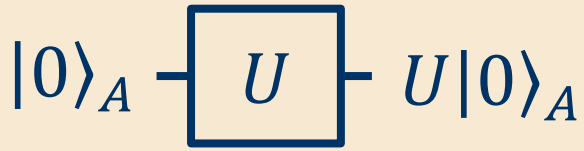

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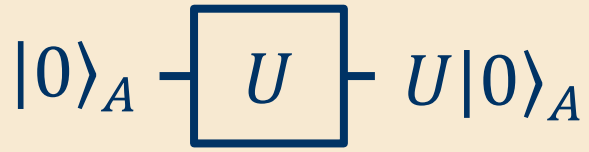

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$|0\rangle_A$ $\qquad$ $\sum_y |y\rangle_A |y\rangle_S$ $\qquad$ **(S register is hidden)**

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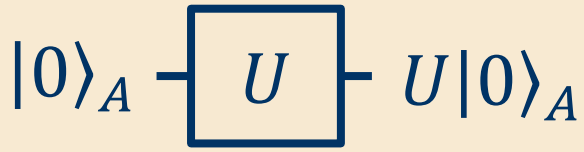

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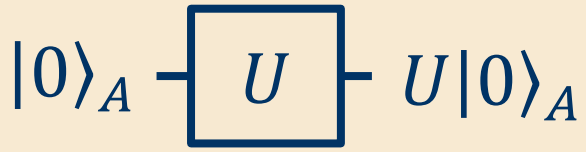

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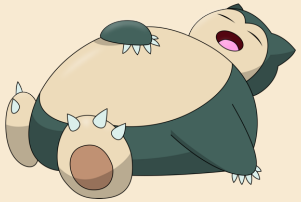

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Idea 1: entanglement with a hidden register S can simulate one query to U.

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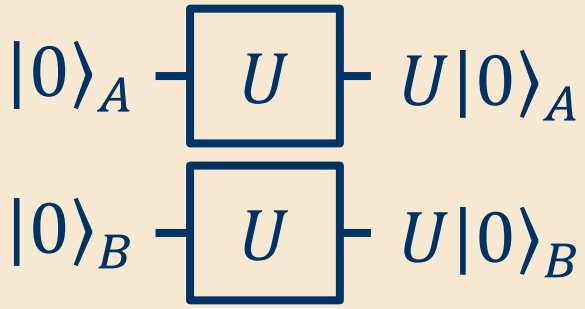

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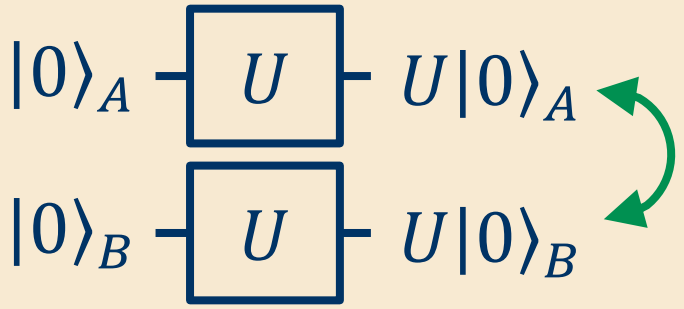

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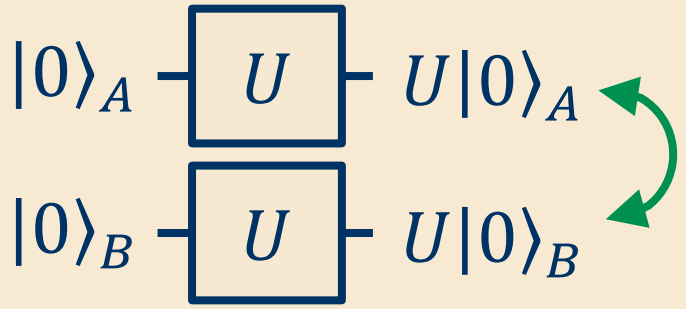

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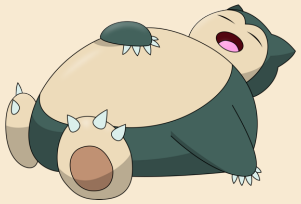

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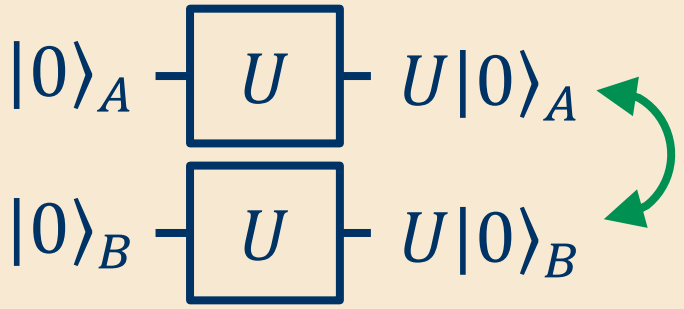

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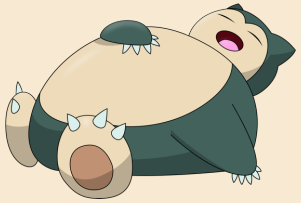

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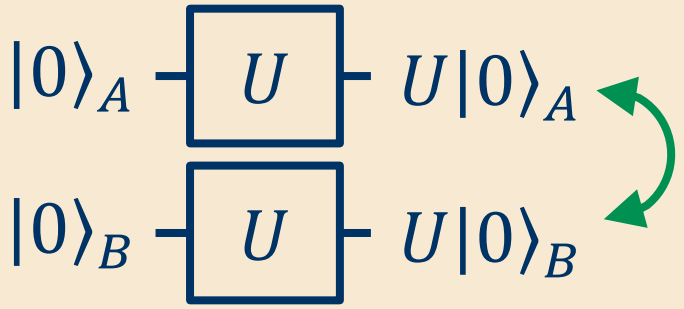

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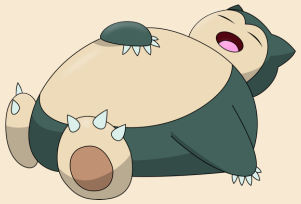

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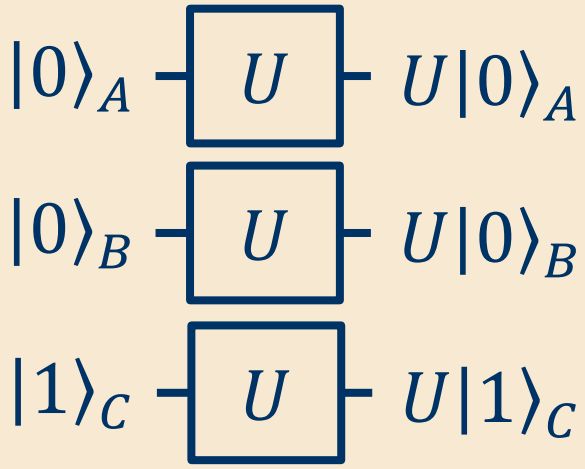

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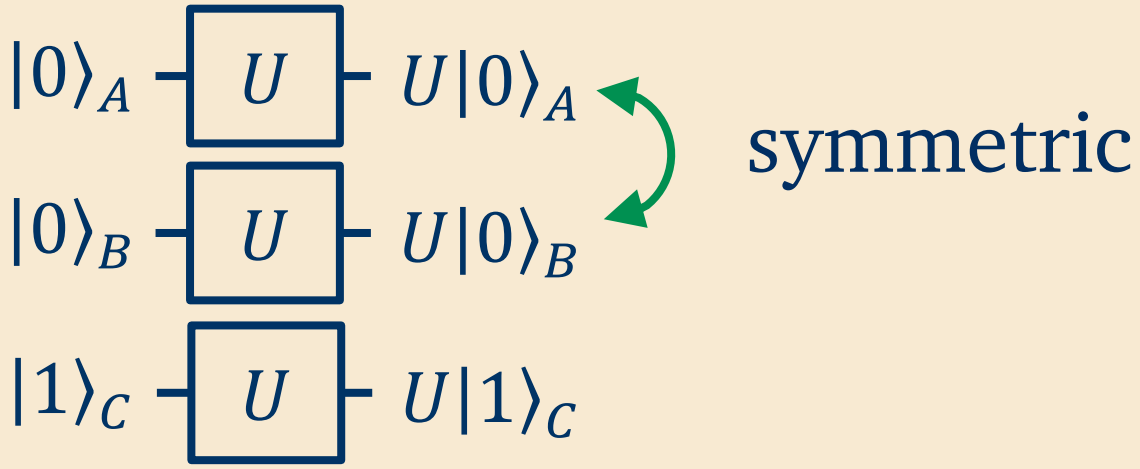

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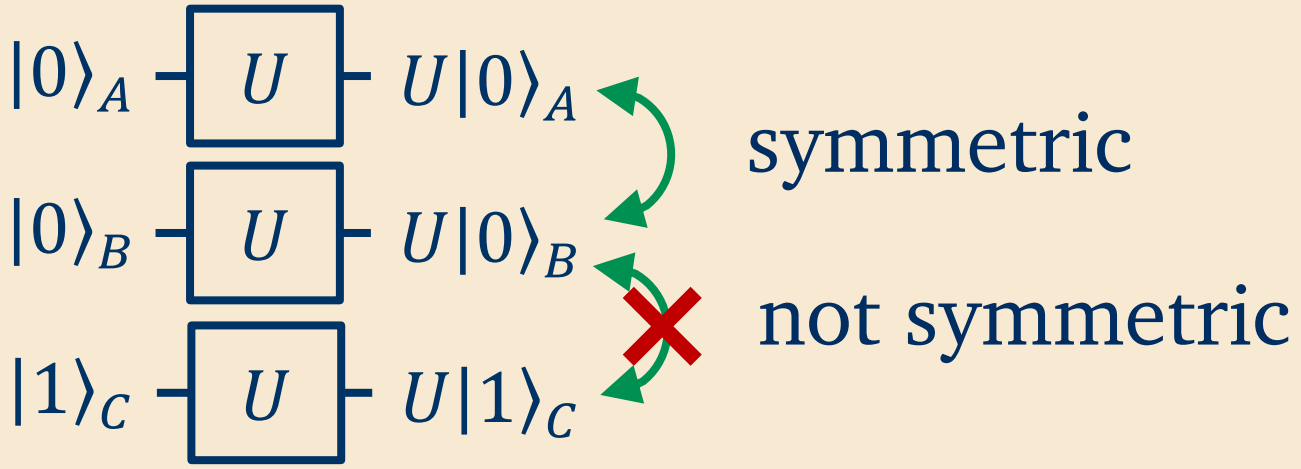

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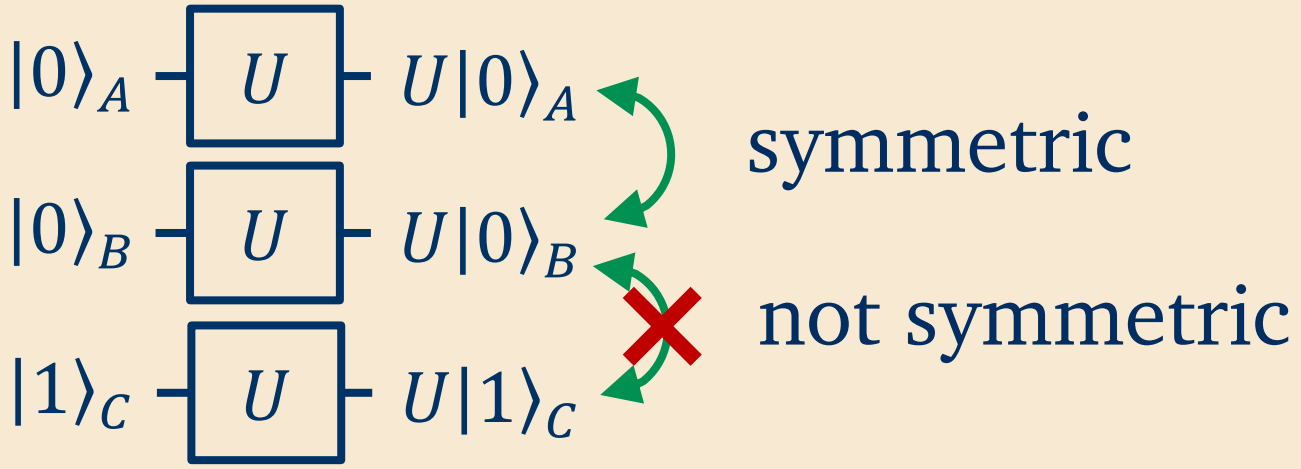

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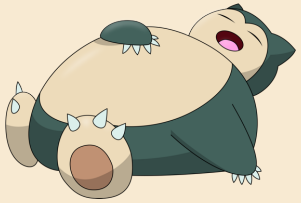

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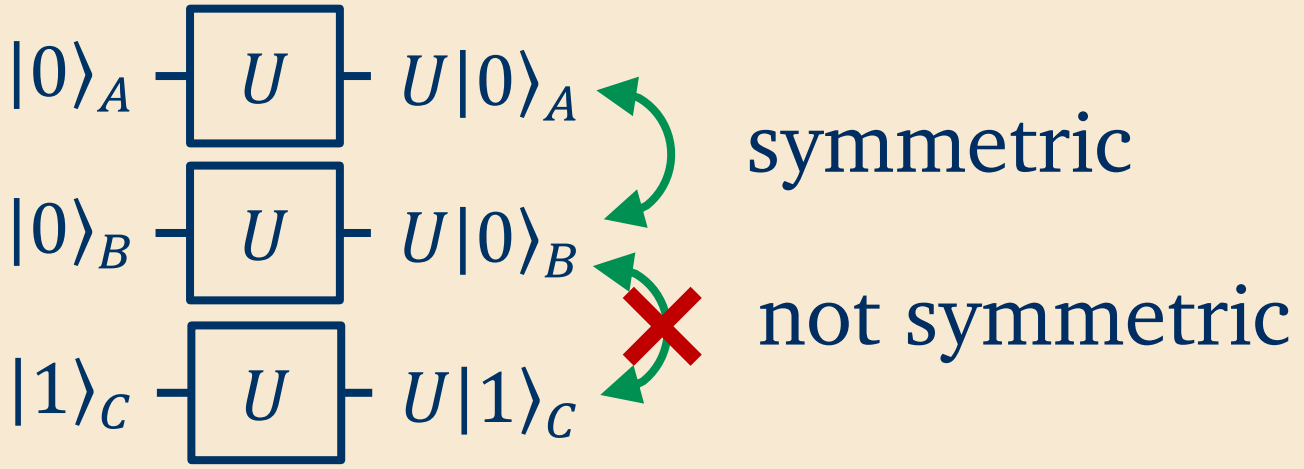

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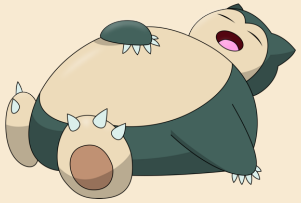

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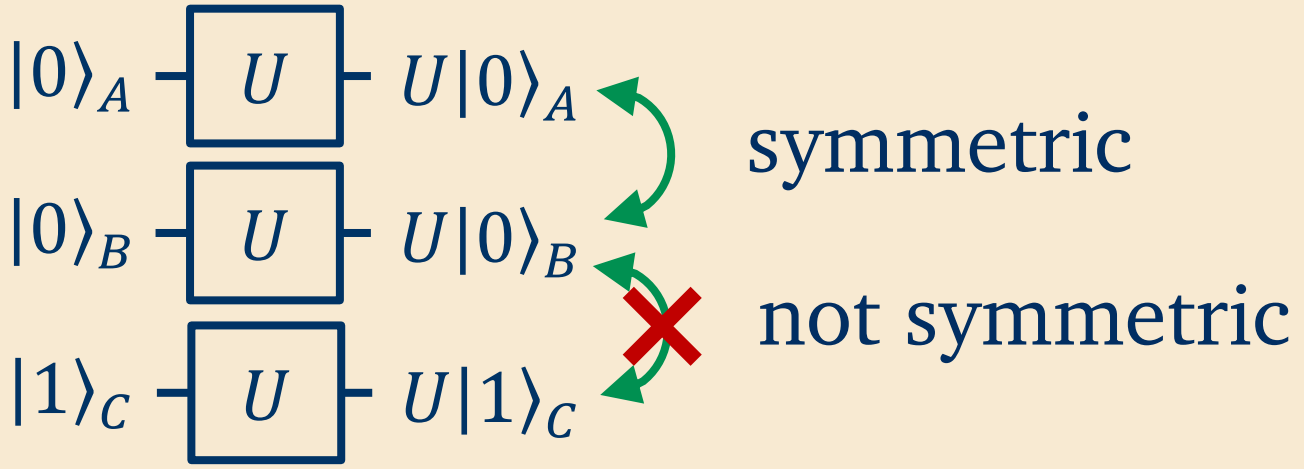

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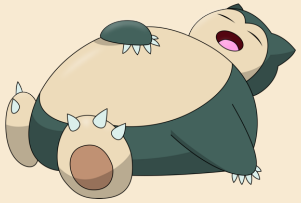

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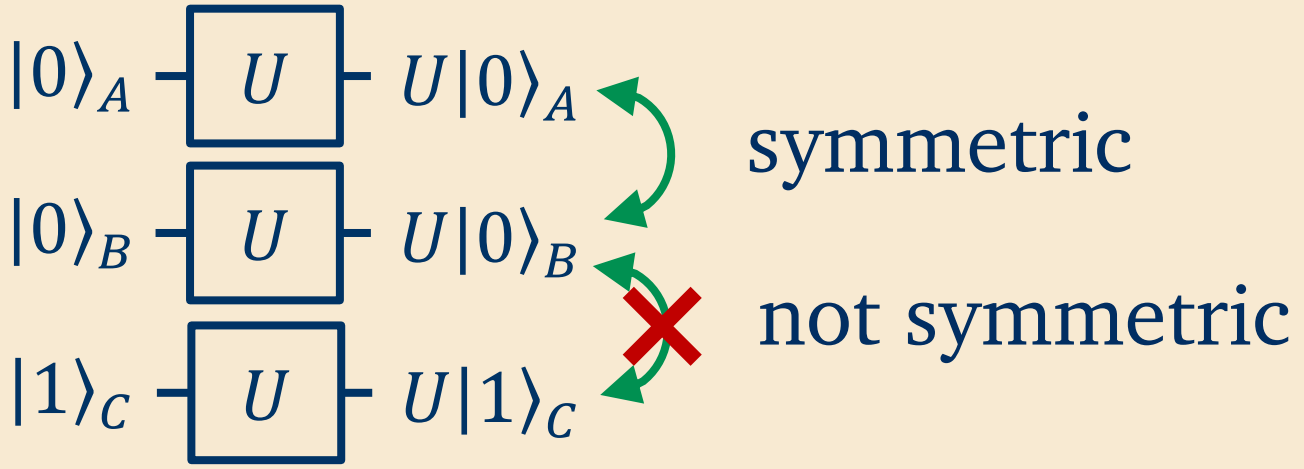

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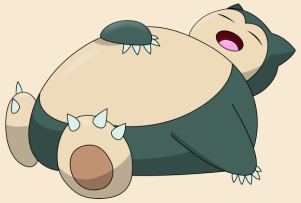

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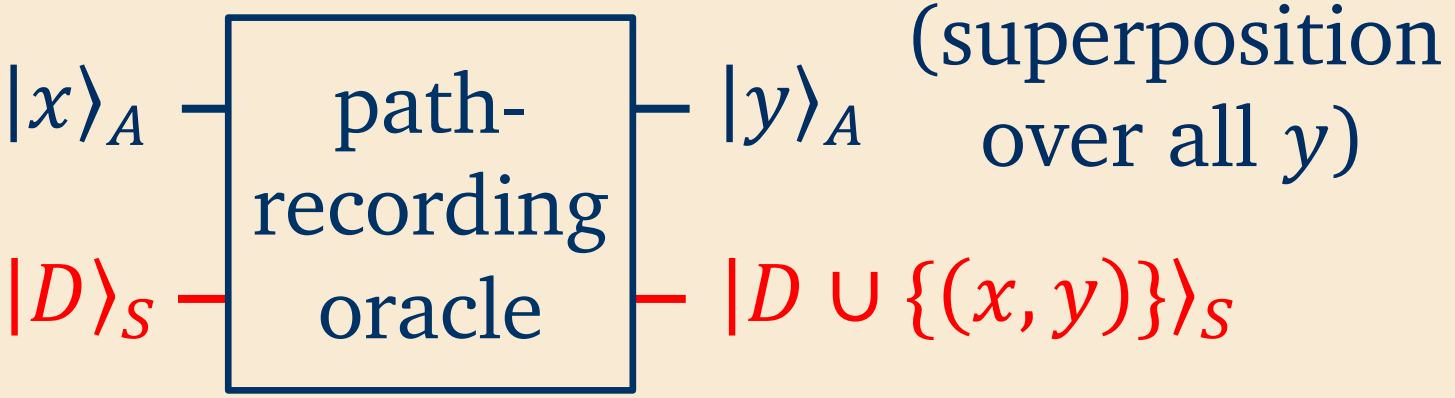

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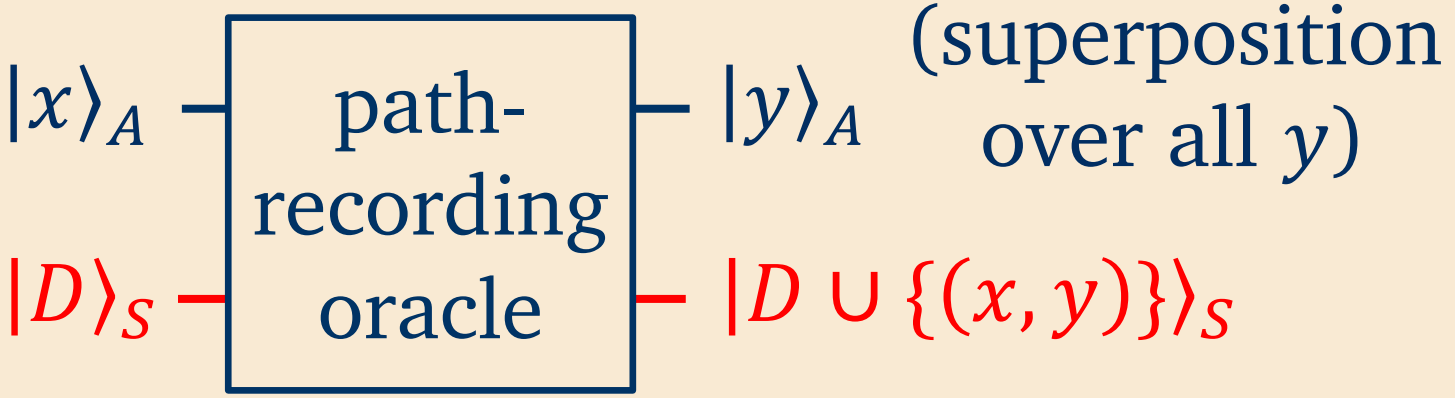

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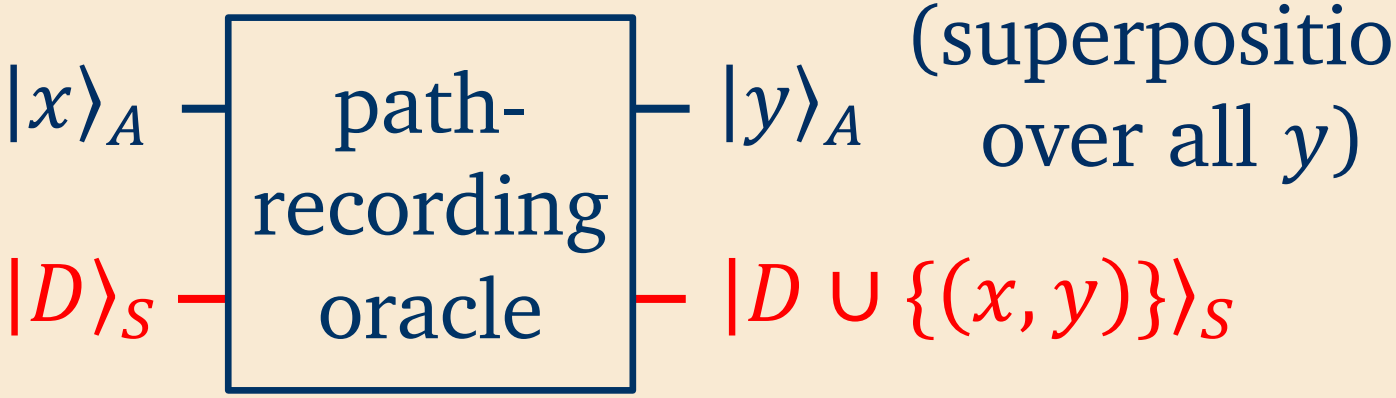
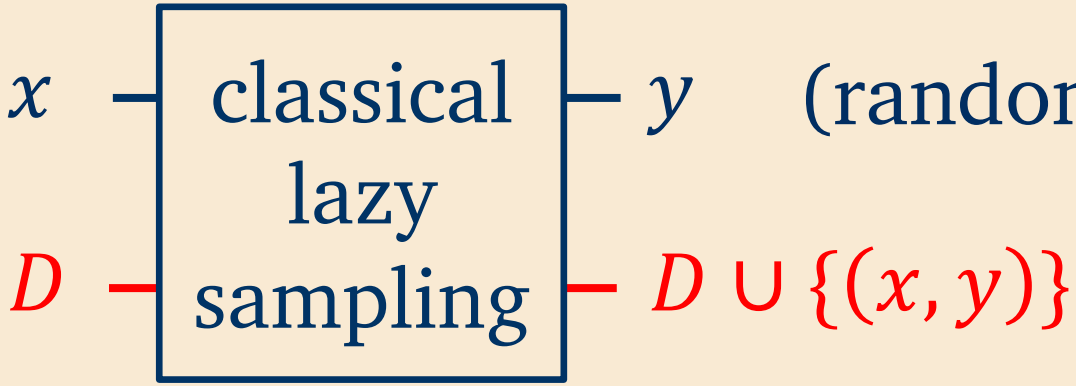


(superposition over all y)

Note the similarity to classical lazy sampling:



(random y)

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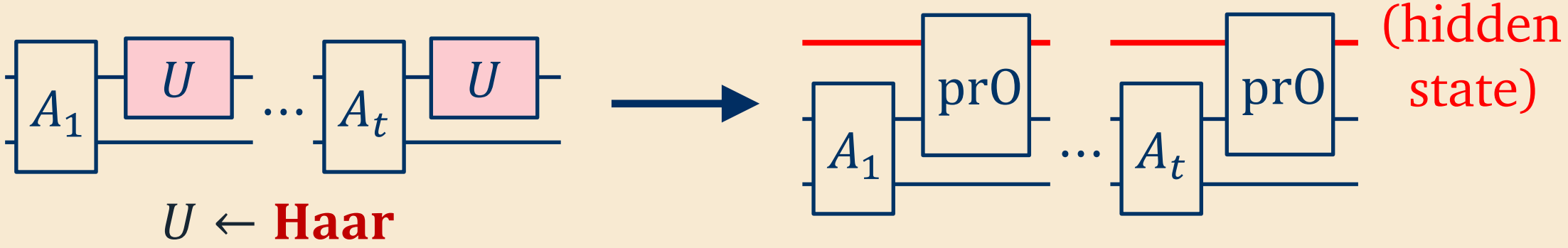

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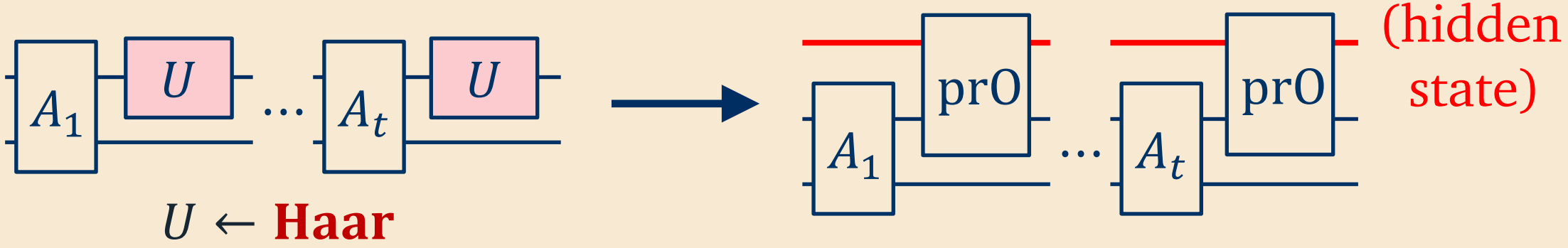

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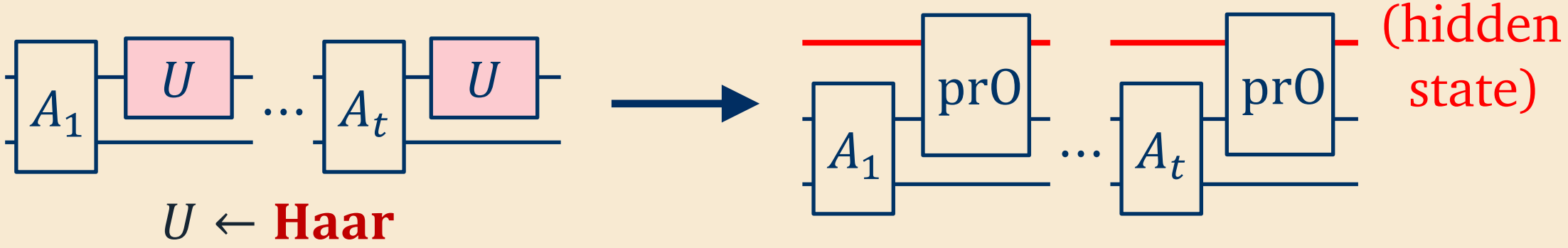

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- [**M**H24]: elementary proof of [SHH24] gluing lemma
- [ABGL24]: compress PRU key length + other results

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Pseudorandom functions [GGM84] ⟷ Pseudorandom unitaries [MH24]

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Pseudorandom functions [GGM84]	⟷	Pseudorandom unitaries [**M**H24]
[GGM84] proof: Lazy sampling of a random function	⟷	[**M**H24] proof: Lazy sampling of a random unitary

Future directions

Question 1a: Can we **unconditionally prove** that no efficient algorithm can break **quantum** cryptography?

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[LMW24]: in the quantum setting, there might be no barriers from traditional complexity theory!

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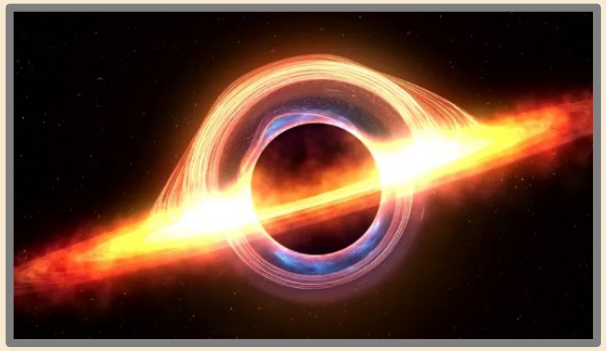
Question 1b: can PRUs "explain" why we can't unconditionally prove that quantum cryptography exists?

Question 2: Physicists model chaotic systems as pseudorandom unitaries. Is this actually justifiable?



$\overset{?}{\approx}$

PRU

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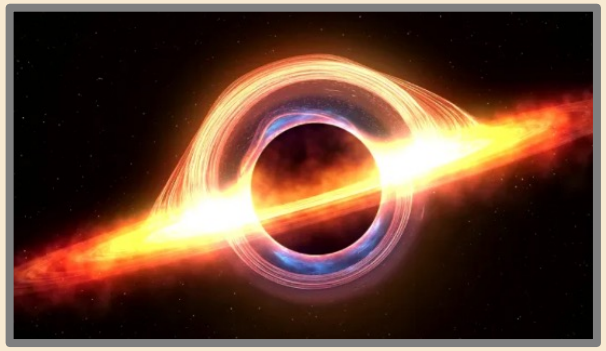

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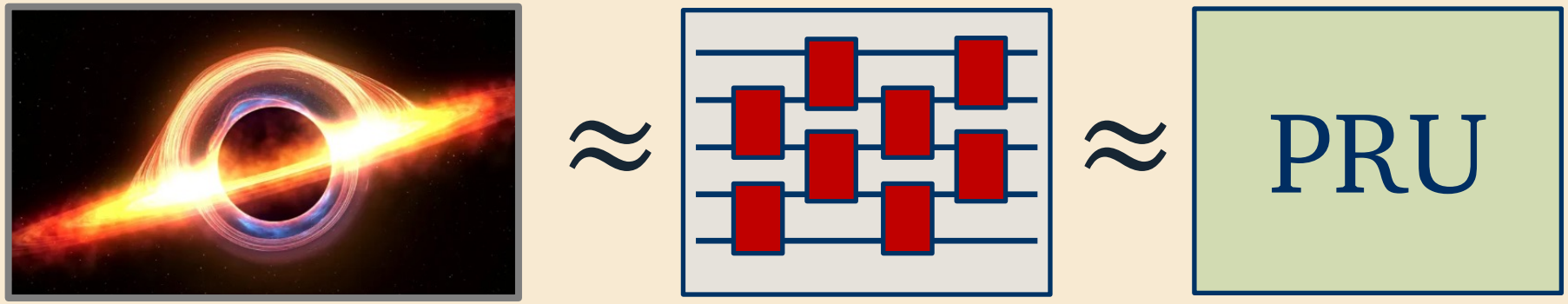

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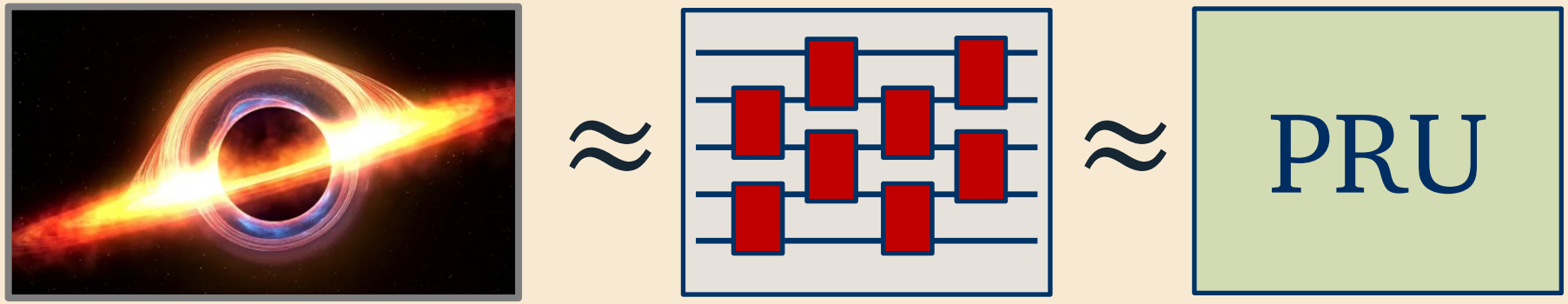

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Question 2': Is a random quantum circuit a PRU?

Upcoming work with Alex Lombardi:

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We prove that random quantum circuits are PRUs under a “structure-hiding” assumption.

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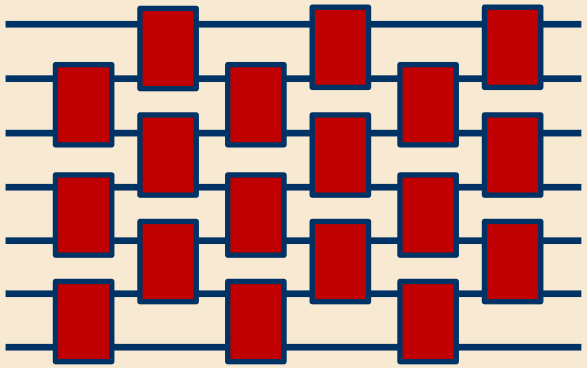
Ex: brickwork architecture



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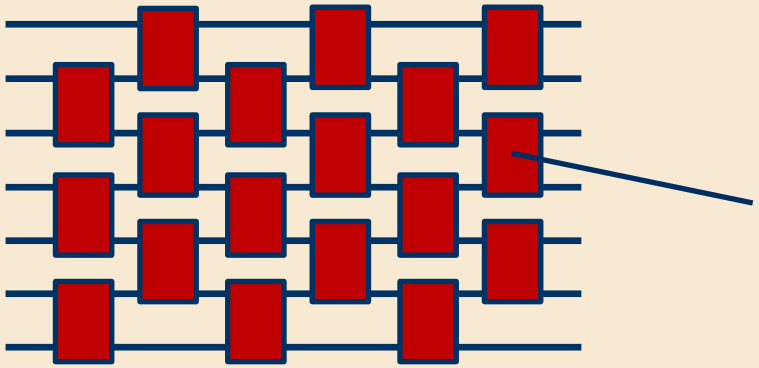

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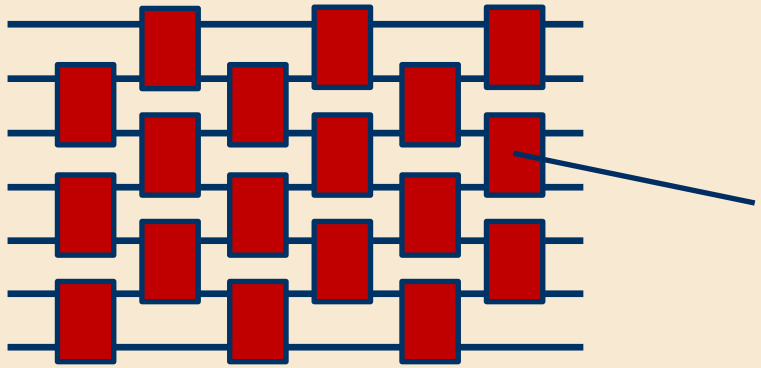

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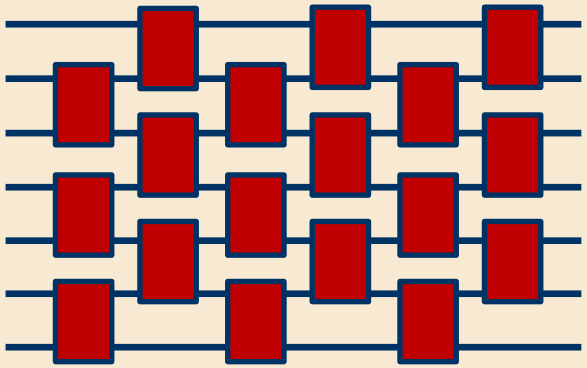

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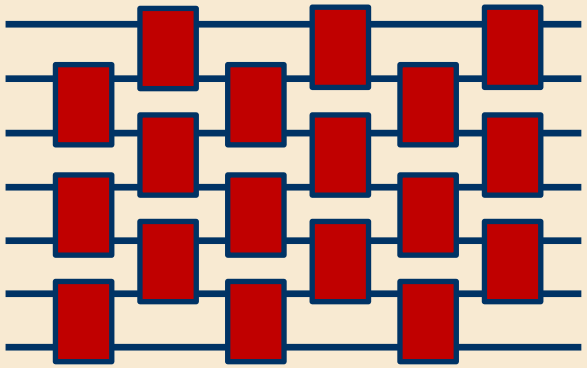

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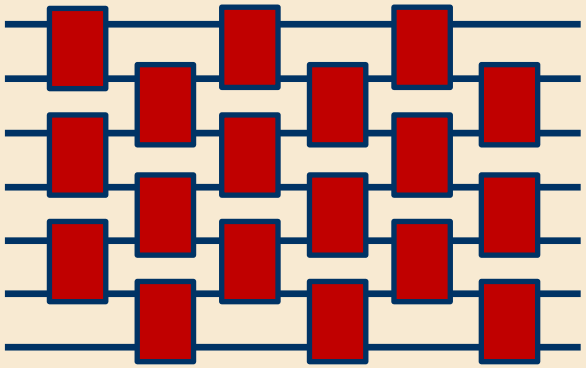
“Odd”



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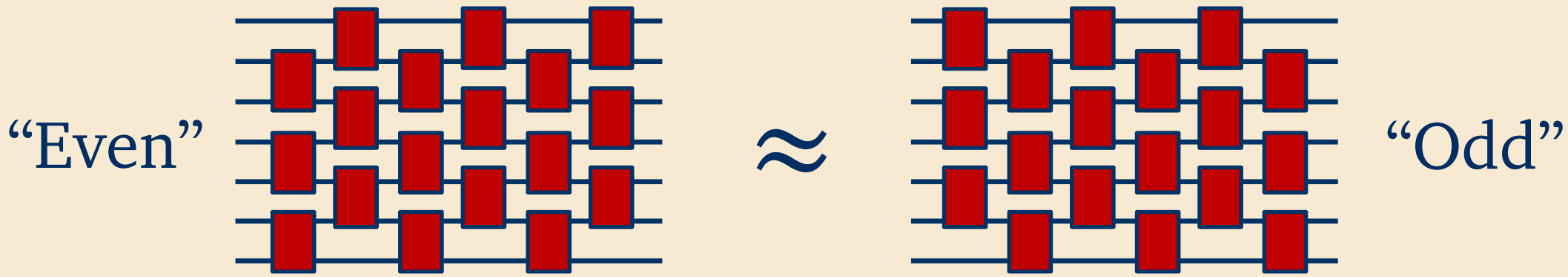

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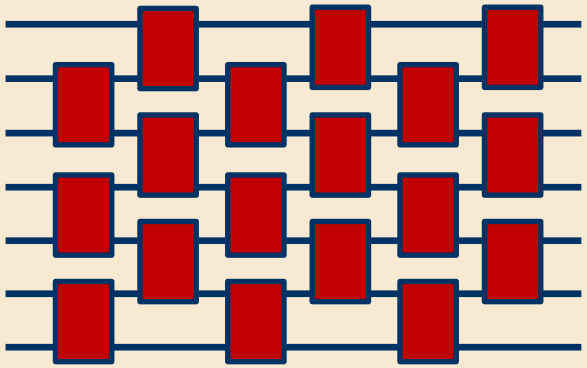
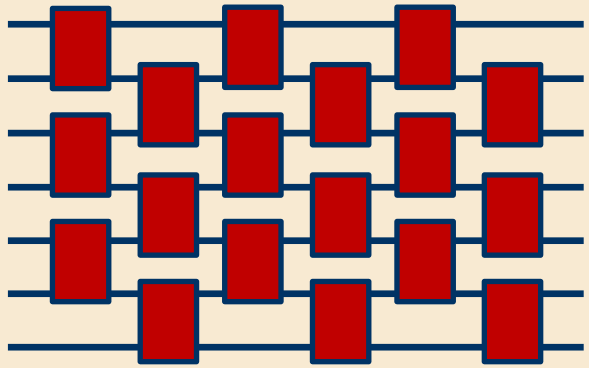
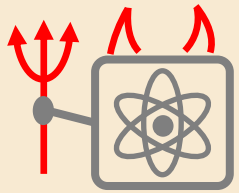
“Even”  ≈  “Odd”

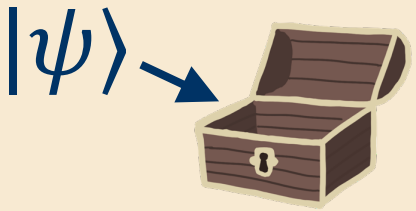
depth d depth d

…this implies a random quantum circuit is a PRU!

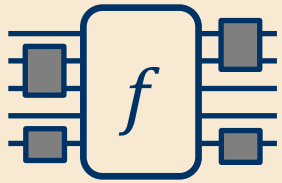
Other future directions



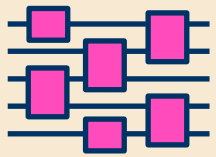
quantum security



quantum primitives



quantum hardness



pseudo-randomness

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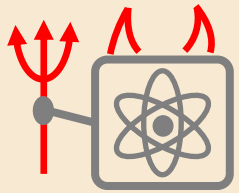
1) general framework for proving quantum security



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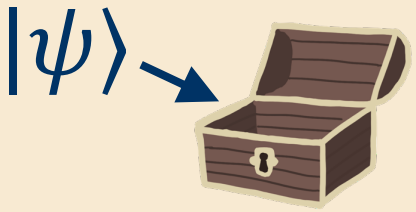

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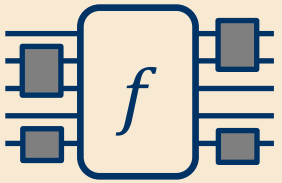

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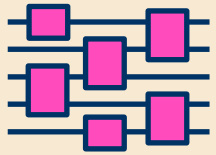

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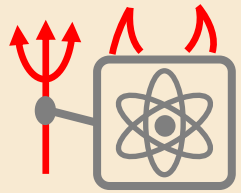

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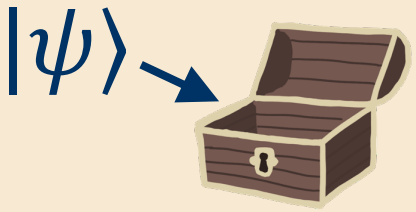

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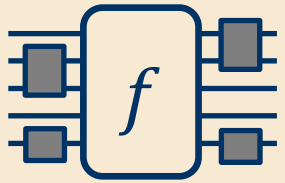

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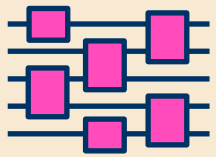

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2) “minimal” assumption for quantum crypto?

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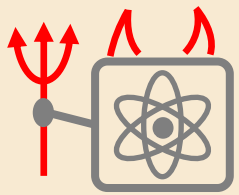

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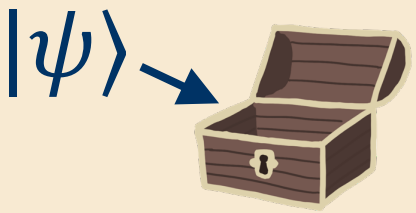

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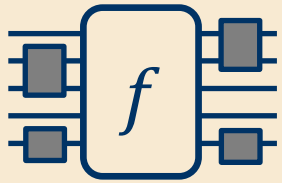

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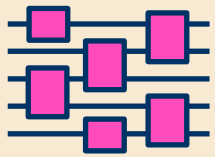

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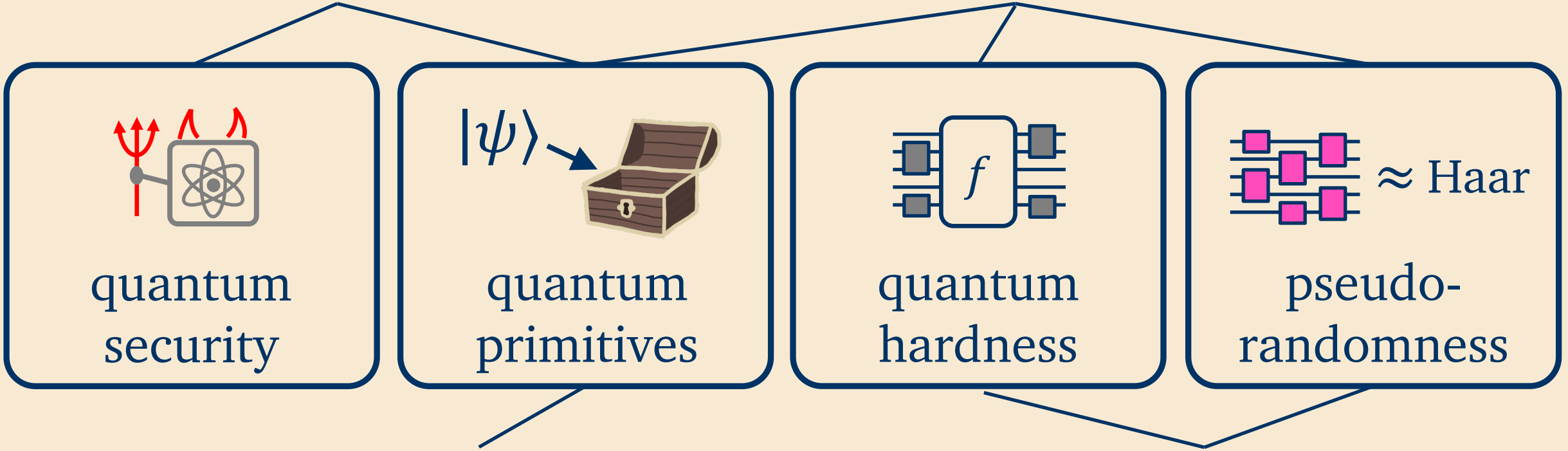

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Thanks!